Lecture 8
Introduction to Dynamical Systems

Systems
- Newton's Equations
- Schroedinger's Equations

Types of Motions
- Regular motions
- Irregular motions

Solutions to equations
- Broad band spectrum
- Chaotic systems

System with many
- Fluids
- Solids
- Networks
- Neurons

Degrees of freedom
- Traditional view point
- Irregular motion

3D plane
- Clock like motion
- Circular pendulum
- Linear pendulum

\( r(t) \)

\( \phi(t) \)
~ 1975: Chaotic Evolution

There exists a third kind of motion!

○ "chaos"
  - erratic, aperiodic
  - broad frequency spectrum

☆ BUT, the system can be extremely simple with only a few degrees of freedom!

Before 1975: Poincaré 1890 – three body systems

\[\begin{align*}
\text{Birkhoff} & \quad 1920 - 1960 \\
\text{Cartwright} & \\
\text{Littlewood} & \\
\text{Levinson} & \\
\text{Kolmogorov} & \\
\text{Smale} &
\end{align*}\]

A mathematician groundworks in understanding this type of motion

After 1975: Advent of personal computers & computer graphics
Scientists realized that this type of motion is prevalent in all physical, biological, chemical processes.

"Erratic and complicated board band behaviors can be possible for system with a few degrees of freedom."

Irregular motions from experiments previously attributed to noise & error may now be explained & analyzed in terms of this new type of motion.

Many system with many interacting particles might not be untrackable.

- Many degrees of freedom might coerce into type C with a trackable number of degree of freedom.

E.g. Fluid - Rayleigh-Benard flow
**Type c motion provides a link in understanding the transition from type A to C!**

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**Dynamical Systems**

**Def:**
- Set of equations giving the time evolution of the state of a system.

- **Continuous time:** \( \dot{x}(t) = F(x) \) (Flow)
- **Discrete time:** \( x_{n+1} = G(x_n) \) (Map)

- **State space or phase space**
- \( \mathbf{x} = (x^1, x^2, ..., x^k) \)
- \( \{x(t)\} \) orbit or trajectory
Flow to Map : Poincare surface of Section

$A_i$ - upward/downward crossing of flow thru $S$

$A_i = \frac{y_i}{z_i} = (y_i^1, y_i^2)$

$\text{Flow} |_S \rightarrow \begin{align} y_i+1 &= G(y_i) \end{align}$

* Dynamics is "preserved" in $S$
  - chaotic in full $\rightarrow$ chaotic in $S$
  - periodic in full $\rightarrow$ periodic in $S$

- actually $\dot{x} = F(x)$ uniquely determines

$y_i+1 = G(y_i)$

- because we can integrate from $A_i$ to $A_{i+1}$ using $F$!