Faraday Rotation Experiment

1 Introduction

Faraday rotation is the rotation of the direction of polarization of linearly polarized light as it traverses a transparent medium in the presence of a longitudinal magnetic field.

2 Experimental Arrangement

![Diagram of the apparatus](image_url)

Figure 1: Block diagram of the apparatus.

A light beam is polarized with a polaroid filter, and passed through the transparent sample (in our case a block of glass). A current-carrying coil creates a longitudinal magnetic field. The light out of the sample traverses another polaroid and is directed to a photodiode detector. The light beam should pass through the glass block without reflection from the sides. Use collimators, if necessary, to achieve this.

3 Principle

The polarizing filters are initially set to some angle $\theta_0$, with magnetic field $B = 0$. With an applied magnetic field, the electric field vector is rotated by an angle $\delta$. Now

$$\delta = B L V$$

(1)

where $L$ is the length of the block, and $V$ is the Verdet constant, a characteristic of the sample medium. The goal of the experiment is to observe the Faraday rotation and measure the Verdet constant. A white light source and several interference filters may be used to measure $V$ for different wavelengths.
4 Measurement of $\delta$

4.1 Law of Malus

If $\theta$ is the angle between the $E$-vector of linearly polarized light and the “pass axis” of a polarizing filter, then the transmitted intensity is related to the incident intensity via

$$I_{\text{trans}} = I_{\text{inc}} \cos^2 \theta = I_{\text{inc}} \cos^2(\theta_0 + \delta)$$  \hspace{1cm} (2)

For small $\delta$, the right-hand side of equation 2 can be expanded about $\theta_0$. This gives

$$I_{\text{trans}} \approx I_{\text{inc}}(\cos^2(\theta_0) - \delta \sin(2\theta_0) - \delta^2 \cos(2\theta_0))$$  \hspace{1cm} (3)

So when the polaroids are initially set to extinction, $\theta_0 = \pi/2$, so equation 3 gives

$$I_{\text{trans}} \approx I_{\text{inc}}\delta^2$$

and the transmitted intensity varies quadratically with $\delta$. If $\theta_0 = \pi/4$, then

$$I_{\text{trans}} \approx I_{\text{inc}}\left(\frac{1}{2} - \delta\right)$$  \hspace{1cm} (4)

The two cases are shown in the figure below. Note that for the same (small) $\delta$, the 45° arrangement gives a much larger effect.

![Figure 2: $I(\theta_0 + \delta) - I(\theta_0)$ vs. $\delta$.](image-url)
4.2 Photodiode response

In the linear range of the photodiode response, the photodiode output voltage $V_d$, is proportional to the incident light intensity. So equation 4 can be written

$$V_d = V_{d0} + a\delta$$

(5)

where $a$ is a constant which depends on the source intensity, and the sample transmission. If, with $B = 0$, we rotate the polaroids away from 45° by known angles $\phi$ and measure then change in $V_d$ for each $\phi$, then a plot of $V_d$ vs. $\phi$ would look like:

![Graph showing the relationship between photodiode voltage and delta, radians.]

The constant $a$ can be determined from the graph. Then, with the polaroids back at 45°, we turn on $B$ and measure $V_d$. Then the Faraday rotation angle is

$$\delta = \frac{V_d - V_0}{a}$$

(6)

5 Complication: AC current

In this version of the experiment, we use a 60Hz AC voltage source, (a “Variac” variable transformer), to produce current in the coil. So in all of the above, $B$ should be replaced by $B_{max}$, the amplitude of the magnetic field oscillation, $\delta$ by $\delta_{max}$, and $V_d$ by $V_{d,max}$.

6 Measurement of $B$

There are several possible methods for measurement of $B_{max}$:
1. Use a Hall probe.

2. Measure the rms current in the coil with an ammeter, convert to \( i_{\text{max}} \), and use the (approximate) expression

\[
B = \mu_0 i n_t
\]

where, in MKS units, \( \mu_0 = 4\pi \times 10^{-7} \), \( i \) is the current in amperes, and \( n_t \) is the number of turns per meter.

3. Use a test coils of known area \( A \) and number of turns \( N \). Measure the induced voltage, \( V_i \), across the test coil. Then Faraday’s Law gives

\[
V_i = -N \frac{d\Phi_B}{dt} = -NA\omega B_{\text{max}} \cos(\omega t)
\]

Since \( N, A, \) and \( \omega = 2\pi f \) are known, then \( V_{i,\text{max}} \) determines \( B_{\text{max}} \).

Note that we cannot simply measure \( B \) with the rotating coil Gaussmeter, since \( B \) is not constant in time. Remember that the MKS unit of \( B \) is the Tesla. \((1 \text{ Tesla} = 10^4 \text{ Gauss})\).

In the literature on Faraday rotation, the magnetic field strength, \( H \), is sometimes given in Oersteds \( (\text{Oe}) \). In a vacuum, if \( H = 1 \text{ Oe} \), then \( B = 1 \text{ Gauss} \).

7 What Results are Expected?

It can be shown that the Verdet constant is expected to be\(^1\)

\[
V = \frac{e}{2mc} \lambda \frac{dn}{d\lambda}
\]

The last term on the right side is the dispersion of the medium. For normal dispersion, we have \(^2\)

\[
\frac{dn}{d\lambda} \propto \frac{1}{\lambda^3}
\]

which means that we expect

\[
V \propto \frac{1}{\lambda^2}
\]

So there should be a large difference between the \( V' \)'s for red and blue light.

Some numerical values of \( V \) are tabulated below \(^3\)

<table>
<thead>
<tr>
<th>Material</th>
<th>( V(\text{min/Gauss} - \text{cm}) ), 589nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.0131</td>
</tr>
<tr>
<td>Crown Glass</td>
<td>0.0161</td>
</tr>
<tr>
<td>Flint Glass</td>
<td>0.0317</td>
</tr>
</tbody>
</table>

\(^1\)See, for example, Preston et al., The Art of Experimental Physics


8 Be sure to include in your report:

1. A graph of diode voltage vs. polarizer angle with $B = 0$.

2. An explanation of how you obtained $B$ for a given Variac setting.


4. The obtained Verdet constant(s), with uncertainties, compared with other measurements.

5. A graph of Verdet Constant vs. Wavelength, together with theoretical predictions.