Heisenberg’s Uncertainty Principle

Photons going through the slit as particles

THEN the “spreading-out” of the photons implies that the photons reaching the screen must have a small uncertainty $\Delta p_y$ as large as $\Delta y$ in the vertical ($\hat{y}$) direction of its momentum as it exits the slit.
Heisenberg Uncertainty Principle again

The Heisenberg Uncertainty Principle is not a statement on the accuracy of the experimental instruments. It is a statement on the fundamental limitations in making measurements!

To understand this, there are two important factors:

1. The wave-particle duality
2. The unavoidable interactions between the observer and the object being observed
Uncertainty Principle: Conceptual Example

Suppose we want to measure the position of a particle (originally at rest) by a laser light.

The measurement is accomplished by the scattering of photons off this particle (shining a light):
Uncertainty Principle: Conceptual Example

But, photons carry momentum and the particle after scattering will recoil!

While the scattered photon gives us a precise position of the particle, the photon will also inevitably impart momentum onto the particle.
Uncertainty Principle: Conceptual Example

If the measurement is made by a photon with wavelength $\lambda$, then at best, we can expect our position measurement to be accurate up to an uncertainty,

$$\Delta y \sim \lambda$$

But, the photon will transfer a part of its momentum to the particle along the measurement direction $y$. The amount transfer is then given by,

$$\Delta p_y \sim \frac{h}{\lambda}$$

Combining these two expressions gives: $\Delta y \Delta p_y \sim h$

NOTE: By improving $\Delta y$ by using a smaller $\lambda$ will inadvertently result in a larger uncertainty in $\Delta p_y \sim h/\lambda$ since $\Delta p_y$ is inverse proportional to $\lambda$ !.
Heisenberg’s Uncertainty Principle

We have use heuristic argument in getting to previous inequality. A more precise inequality can be derivate from full QM calculations, and it should read

\[ \Delta y \Delta p_y \geq \frac{\hbar}{2} \quad \text{where } \hbar = \frac{h}{2\pi} \]

This is called the **Heisenberg’s Uncertainty Principle**.
Uncertainty Relation for $\Delta E$ & $\Delta t$

Our discussion is not specific to the $\hat{y}$ direction. In general, we will have similar uncertainty relations for all three spatial directions, i.e.,

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}, \quad \Delta y \Delta p_y \geq \frac{\hbar}{2}, \quad \Delta z \Delta p_z \geq \frac{\hbar}{2}$$

And, additionally, there is an uncertainty relation for $\Delta E$ and $\Delta t$:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \quad \text{similar to} \quad \Delta p_x \Delta x \geq \frac{\hbar}{2}$$
Uncertainty Relation for $\Delta E \& \Delta t$

In nature, space-time coordinates are linked to its dynamical counterparts as **conjugate** variable pairs in physics.

$$(p_x, x) \quad (p_y, y) \quad (p_z, z) \quad \text{and} \quad (E, t)$$

And, most importantly, the Heisenberg’s Uncertainty Principle enforces an *inverse* proportional relation on the two *conjugate* pairs of dynamics variables:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}, \quad \Delta y \Delta p_y \geq \frac{\hbar}{2}, \quad \Delta z \Delta p_z \geq \frac{\hbar}{2}, \quad \Delta E \Delta t \geq \frac{\hbar}{2}$$

By decreasing the uncertainty in one of the variables ($x$ or $t$), its corresponding **conjugate** variable ($p_x$ or $E$) must increase accordingly!

But, there are **no** restrictions for *unconjugated* variables: $\Delta x \Delta p_y$ or $\Delta x \Delta y$, etc.
Chapter 39: Particles Behaving as Waves

- Matter Waves
- Atomic Line-Spectra and Energy Levels
- Bohr’s Model of H-atom
- The Laser
- Continuous Spectra & Blackbody Radiation
Matter Waves

As we have seen, light has a *duality* of being a wave and a particle.

By a *symmetry* argument, de Broglie in 1924 proposed that *all form of matter* should also possess this duality.

Recall for photons, we have: \( \lambda = \frac{h}{p} \)

For a massive particle with momentum \( p = mv \) (or \( \gamma mv \)) and total energy \( E \), de Broglie proposed:

\[
\lambda = \frac{h}{p} = \frac{h}{mv} \left( = \frac{h}{\gamma mv} \right) \quad f = \frac{E}{h}
\]

(de Broglie wavelength)
Electron Diffraction

Clinton Davisson and Lester Germer (1927)

1. A heated filament emits electrons.
2. The electrons are accelerated by electrodes and directed at a crystal.
3. Electrons strike a nickel crystal.
4. The detector can be moved to detect scattered electrons at any angle \( \theta \).

Nickel atoms in crystal acts as a diffraction grading,

Constructive interference is expected at:

\[ d \sin \theta = m\lambda, \quad m = 1, 2, \ldots \]
In 1928, G.P. Thomson (son of J.J. Thomson) performed another demonstrative experiment in showing electrons can act as a wave and diffract from a polycrystalline aluminum foil.

Debye and Sherrer: X-Ray diffraction experiment done a few years earlier using a similar set up.

G.P. Thomson’s electron diffraction experiment produced a qualitatively similar result.
Electron Two-Slit Interference

Similar to the two-slit experiment with photons, electrons demonstrate *both* particle (detection of single e’s on film) and wave (interference pattern) characteristics.

The probability of arrival vs. $y$ can only be explained by two *interfering matter waves*!

de Broglie wavelength: an electron vs. a baseball

\[ \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^7 \text{ m} / \text{s})} \]

\[ = 7.28 \times 10^{-11} \text{ m} \]

(experimentally assessable)

\[ \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(145 \times 10^{-3} \text{ kg})(40.0 \text{ m} / \text{s})} \]

\[ = 1.14 \times 10^{-34} \text{ m} \]

(immeasurably small !)

Since \( h \) is so small, typical daily object’s wave characteristic is insignificant!
The Electron Microscope

- Similar to a light beam, a beam of electrons can be bent by reflection and refraction using electric and/or magnetic fields.

- Using electric/magnetic fields as “lens”, an electron beam can be used as a microscope to form magnified images of an object.

- Resolution of a microscope is limited by diffraction effects!

→ Electron microscope is better than regular microscope since $\lambda_{\text{electron}} \ll \lambda_{\text{visible light}}$
The Electron Microscope (Example 39.3)

What accelerating voltage is needed to produce $\lambda = 0.010\text{nm}$?

From the electron diffraction experiment, we know how to relate the wavelength for a fast moving electron to the accelerating voltage $V_a$.

$$eV_a = \frac{p^2}{2m}$$
$$p = \frac{h}{\lambda}$$

$$V_a = \frac{h^2}{2me\lambda^2}$$

$$V_a = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2 \left( 9.109 \times 10^{-31} \text{ kg} \right) \left( 1.602 \times 10^{-19} \text{ C} \right) \left( 10 \times 10^{-12} \text{ m} \right)^2}$$

$$= 1.5 \times 10^4 V = 15,000V$$
Electric Discharge Tube with Diluted Gas

Diluted gas containing trace elements such as H, He, Na, Hg, …

Observation: Energetic electrons from cathode excite gaseous atoms in the tube, light can be emitted.

Gas is diluted so that the emission process is by individual atoms. The spectrum of light emitted are sets of unique lines characteristic of the specific type of atoms in the gas.
Emission Line Spectra

https://youtu.be/N_mwHxEugVE
Emission Line Spectra

- Nitrogen
- Xenon
- Neon
- Krypton
- Argon
- Mercury
- Helium
- Hydrogen
Emission Line Spectra

- Hydrogen
- Helium
- Neon
- Sodium
- Mercury
Continuous vs. Line Spectra

When light emitted by a hot solid (e.g. filament in a light bulb) or liquid, the spectrum (from a diffraction grating or prism) is continuous.

But light from the spectrum of a gas discharge tube is composed of sharp lines. Atoms/molecules in their gaseous state will give a set of unique wavelengths.

Understanding the reason why they are different…

Will require our understanding of:

- light behaving like particles
- electron behaving like waves
The Nuclear Atom (a bit of history)

What were known by 1910:

- 1897: J.J. Thomson discovered $e$ and measured $e/m$ ratio.
- 1909: Millikan completed the measurement of the electron charge $-e$.
- Size of atom is on the order of $10^{-10}$ m.
- Almost all mass of an atom is associated with the $+$ charge.

But, the mass and charge distributions inside the atom were not known. The leading assumption was by J.J. Thomson and an atom was modeled as a sphere with positive charge and the electrons were thought to be embedded within it like raisins in a muffin.

Ernest Rutherford designed the first experiment to probe the interior structure of an atom in 1910-1911. (together with Hans Geiger and Ernest Marsden)
Rutherford Scattering Experiments

*Alpha particles*: high energy charged helium nuclei. It can travel several centimeter through air and ~0.1mm through solid matter.

*Target*: Thin gold, silver, or copper foils.

The alpha particle is about 7300 times heavier than an electron. So, by momentum considerations alone, it will only minimally interact with the much *lighter* electron. Only the *positive* charge within the atom associated with the majority of its mass can produce significant deflections in scattering the alpha particles.
Rutherford Scattering Experiments

Thomson’s model of the atom: An alpha particle is scattered through only a small angle.

Positive charge inside atom is distributed uniformly throughout the volume so that electric field inside the atom is expected to be diffused (small) and the force that it exerts on the $\alpha$ will also be small $\rightarrow$ deflection angles are expected to be small.

BUT, there were large scattering angles ($\sim 180^\circ$, almost backward) observed!

Indicating that Thomson’s model was not correct.

Rutherford’s model of the atom: An alpha particle can be scattered through a large angle by the compact positively charged nucleus (not drawn to scale).

Nucleus
Rutherford Scattering Experiments

These large angle deflections can only be possible if the positive charge within an atom is concentrated in a small and dense space called the nucleus.

The following is a computer simulation of the deflection angles with two different nucleus size (done much later). The experimental results agreed with simulation data using small (~10^{-15} m) nucleus sizes.

(a) A gold nucleus with radius 7.0 \times 10^{-15} m gives large-angle scattering.
(b) A nucleus with 10 times the radius of the nucleus in (a) shows no large-scale scattering.

The nucleus occupies only about 10^{-12} of the total volume but it contains 99.95% of the total mass.
Rutherford’s Atomic Model

Following his discovery of the atomic nucleus, Rutherford also suggested that the negatively charged electron might revolve around the positively charged nucleus similar to a planet orbiting the sun in classical physics.
Failure of Classical Physics

Following his discovery of the atomic **nucleus**, Rutherford also suggested that the negatively charged electron might revolve around the positively charged nucleus similar to a planet orbiting the sun in classical physics.

However, there are conceptual problems with the classical circular orbits idea:

- According to classical EM, an *accelerating* electron in circular orbit around the nucleus will *radiate* EM radiations.
- Thus, its energy will *decrease* continuously and spiral rapidly toward the nucleus.
- The radiated frequency will also depend on the frequency of revolution and since its orbit is expected to decay continuously, the radiated spectrum will also be *continuous* with a mixture of frequencies.
Photon Emission by Atoms

In order to explain the observed \textit{discrete} spectra lines from atomic emissions, Niels Bohr in 1913 combined the following two central ideas in his model:

- \textit{electrons as waves}
- \textit{light as packets of energy}

\textit{(energy levels in atoms)}

\textit{(photons)}

\textbf{Energy Levels} in a typical atom

Each atom has a \textit{specific set} of possible internal energy states. An atom can possess any one of these levels but cannot take on any \textit{intermediate} values.

\[ hf = \frac{hc}{\lambda} = E_i - E_f \]

An atom drops from an initial level \(i\) to a lower-energy final level \(f\) by emitting a photon with energy equal to \(E_i - E_f\).
The Hydrogen Spectrum

Hydrogen is the simplest atom and it also has the easiest spectrum to analyze. In 1885, Johann Balmer first analyzed its spectrum and derived an empirical relation to accurately describe the wavelengths in the spectrum.

\[
\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad n = 3, 4, 5, \ldots
\]

\[ R = 1.097 \times 10^7 \, m^{-1} \] is called the Rydberg constant which was experimentally found to match with observed data.
The Hydrogen Spectrum

The formula for the Balmer series can be interpreted in terms of Bohr’s hypothesis. Multiplying Balmer’s empirical equation by $hc$, we have

$$\Delta E = hc \frac{1}{\lambda} = hcR \left( \frac{1}{2^2} - \frac{1}{n^2} \right), \quad n = 3, 4, \ldots$$

Comparing this with Bohr’s equation for the energy of an emitted photon from an excited atom, we can identify the term on the right as the difference between two energy levels $i$ and $f$,

$$hf = \frac{hc}{\lambda} = E_i - E_f$$

$$E_i = -\frac{hcR}{n^2}$$

$$E_f = -\frac{hcR}{2^2}$$
The Hydrogen Spectrum

This suggests that the hydrogen atom has a series of discrete energy levels,

\[ E_n = -\frac{hcR}{n^2} \quad n = 1, 2, 3, 4, 5, \ldots \]

Then, according to Bohr’s explanation, the various lines from the Balmer series correspond to the transition of an excited atom from \( n = 3 \) or above to the \( n = 2 \) level.

The constant \( hcR \) has the following numerical value: \( hcR = 2.19 \times 10^{-18} \text{ J} = 13.6 \text{ eV} \)

For a hydrogen atom, the lowest possible energy level is given by \( E_1 = -13.6 \text{ eV} \). \( n = 1 \) called the ground level (ground state) and all the higher levels \( (n > 1) \) are called the excited levels (excited states).
The Hydrogen Spectrum

Other special series corresponding to different transitions have also been described:

**Lyman Series** \[ \frac{1}{\lambda} = R \left( \frac{1}{1^2} - \frac{1}{n^2} \right) \quad (n = 2, 3, 4, \cdots) \]

**Balmer Series** \[ \frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad (n = 3, 4, 5, \cdots) \]

**Paschen Series** \[ \frac{1}{\lambda} = R \left( \frac{1}{3^2} - \frac{1}{n^2} \right) \quad (n = 4, 5, 6, \cdots) \]

**Brackett Series** \[ \frac{1}{\lambda} = R \left( \frac{1}{4^2} - \frac{1}{n^2} \right) \quad (n = 5, 6, 7, \cdots) \]

**Pfund Series** \[ \frac{1}{\lambda} = R \left( \frac{1}{5^2} - \frac{1}{n^2} \right) \quad (n = 6, 7, 8, \cdots) \]
The Hydrogen Spectrum

Energy-level diagram for hydrogen, showing some transitions corresponding to the various series:

- **Lyman series**
- **Paschen series**
- **Brackett series**
- **Pfund series**

- **Balmer series**
  - (visible light and ultraviolet)
- **Paschen series**
  - (infrared)
- **Brackett series**
  - (infrared)
- **Pfund series**
  - (infrared)

“Permitted” orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

- **Lyman series** (ultraviolet)
- **Balmer series**
- **Paschen series**
- **Brackett series**
- **Pfund series**