The Constancy of the Speed of Light

Also, recall the *Michelson-Morley experiment*:

Result: There is no ether! \( c \) is the *same* for all inertial observers in relative motion and the speed of light \( c \) does not follow the *Galilean Velocity Transformation*. 
The Constancy of the Speed of Light

Additionally, from Maxwell’s Equations in electromagnetic theory: EM waves can be shown to travel according to the plane wave equation,

\[ \frac{\partial^2 E, B}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E, B}{\partial t^2} \]  

(Recall Sec. 32.2)

at the same speed \( c \) irrespective of the observer’s inertial ref. frame.

If we believe Maxwell’s Equations to be correct in all inertial reference frames, then we must accept that EM waves travel at the same speed \( c \) in all inertial reference frames.
Einstein’s Postulates for Special Relativity

1. All laws of physics must be the same in all inertial reference frames.
   - Specific observations might be different but the same phenomena must be described by the same physical law.
   - Not just the laws of mechanics (as in the Galilean viewpoint). *All* laws of physics include mechanics, EM, thermodynamics, QM, etc.

(a) magnet moves down

(b) coil moves up

SAME emf is induced in the coil!
Einstein’s Postulates

2. The speed of light $c$ in vacuum is the same in all inertial reference frames and is independent of the observer or the source.

This is a revolutionary statement!

One of the immediate non-intuitive consequence $\Rightarrow$ $c = c' + u$

Together with #1, SR requires us to rethink how time and space are measured!
Stating the Results First

**Time Dilation:** (moving clock runs slow)

\[ \Delta t = \gamma \Delta t_0, \quad \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \]

Measured by \( S \) \quad \text{Measured by} \ S'

\[ L = L_0 / \gamma \]

**Length Contraction:** (moving ruler get shorter)

**Simultaneity:**
Two flashes simultaneous in \( S' \) but not in \( S \).
Notes on Relative Motion

- Both observers in $S$ and $S'$ have their own measurement devices and they can also measure his/her partners devices and compare with his/her own.

- Both $S$ and $S'$ will respectively measure time dilations and length contractions from the moving clock and ruler from his/her partner.
Notes on Relative Motion

- Both observers in S and S’ have their own measurement devices and they can also measure his/her partners devices and compare with his/her own.

  - They will see different time intervals and length when they compare measurements from their own devices wrt their partners

  - But they will not disagree with each other’s observations!
- Although time duration and length contraction might depend on the observer’s inertial frame, they will agree on the following three items:
  - $c$ is the same in all frames
  - their relative speed $u$ is the same
  - all physical laws apply equally
Relativity of Time Intervals

Measuring Time Intervals with a light “clock”:

One time unit is measured by the duration of two events:
- laser light leaving (tic)
- laser light return (toc)

Consider a boxcar moving with respect to the ground and we are interested in the measurement of an interval of time by both $S$ and $S'$ from a clock placed in the boxcar.
Relativity of Time Intervals

In the $S'$ – frame:
- Mavis $O'$ is moving with the boxcar
- the clock is stationary with Mavis $O'$

distance travelled by light = $2d$
speed of light = $c$

According to Mavis $O'$,

$$\Delta t_0 = \frac{2d}{c} \quad (\text{measured in } S')$$
Relativity of Time Intervals

Now, consider the observation from Stanley’s $S$ – frame (stationary frame),

Note: speed of light is still $c$ in this frame but it now travels on a longer path!

Stanley observes the same light pulse following a diagonal path.
Relativity of Time Intervals

Thus, if $\Delta t$ is the time between the bounces of the laser light in $S$ – frame

$$\Delta t = \frac{2l}{c}$$

(light travels at the same speed $c$ in $S$ !)

then it must be longer than $\Delta t_0$

From the given geometry, we can explicitly calculate $\Delta t$:

$$d^2 + \left(\frac{u\Delta t}{2}\right)^2 = l^2 = \left(\frac{c\Delta t}{2}\right)^2$$

$$d^2 = \left(\frac{c\Delta t}{2}\right)^2 - \left(\frac{u\Delta t}{2}\right)^2 = (c^2 - u^2)\frac{\Delta t^2}{4}$$

$$\Delta t^2 = \frac{4d^2}{(c^2 - u^2)}$$

$$\Delta t = \gamma \Delta t_0$$

where $\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} > 1$
Time Dilation

This is the **time dilation** formula in SR.

(Note: both observers $S$ and $S'$ will agree on this relationship between time intervals as long as they are both looking at the same clock in $S'$.)

$$\Delta t = \gamma \Delta t_0 = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$$

Since $u$ is strictly less than $c$,

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}} > 1$$

and $\Delta t > \Delta t_0$ always!
Proper Time

\[ \Delta t = \gamma \Delta t_0 \]

\( \Delta t_0 \) is called the proper time and it is a “special” (or “proper”) time interval since it is the time interval of the clock measured by an observer stationary with respect to that clock, i.e., the two events (tic & toc) occur at the same location.

\( \Delta t \) is the measurement of this same pair of tic-toc events by another observer in relative motion with respect to the clock.

All observers have his/her own proper time and all other observers measuring other observer’s clocks will not necessary be proper.

The proper time will always be the shortest time interval among all observers.
Unreality Check

If speed of light changes according to Galilean Velocity Transformation, then

$$\sqrt{u^2 + c^2}$$

Then, $\Delta t = 2l/\sqrt{u^2 + c^2}$

Following the same calculation as previously, we have

$$d^2 + \left(\frac{u\Delta t}{2}\right)^2 = l^2 = \left(\sqrt{u^2 + c^2} \frac{\Delta t}{2}\right)^2$$

$$d^2 = \left(\sqrt{u^2 + c^2}\right)^2 - u^2 \left(\frac{\Delta t}{2}\right)^2 = \frac{c^2 \Delta t^2}{4}$$

$\Delta t^2 = \frac{4d^2}{c^2} = \Delta t_0^2$

no time dilation: all inertial observers measure the same time interval
Relativity of Length

→ Distance between two points on a rigid body $P$ & $Q$ can be measured by a light signal’s round-trip time.

$l$ can be measured by the time interval: $t_2 - t_1$, $2l = c(t_2 - t_1) \implies l = \frac{c}{2}(t_2 - t_1)$

→ As we have seen, $\Delta t$ will be different for different inertial observers, $l$ will also!
Proper Length

Similar to the concept of proper time $\Delta t_0$ which is the measured time interval of a clock which is at rest with the observer, proper length $l_0$ is the measured length of an object at rest with the observer.

Observer $O'$ in $S'$- frame will measure proper time and proper length for the clock and ruler shown.
Length Contraction (parallel to $\mathbf{u}$)

Let consider a ruler *at rest* in the moving frame ($S''$) and it lays *parallel* to the direction of the relative motion between $S$ and $S''$ (as shown previously).

Within $S''$- frame, the ruler is at rest with Mavis.

The length of the ruler is measured using a light-clock by measuring the time interval between *two events* (light leaving the laser and light arrives back to the source). In this measurement process, light pulse travels a distance of a total of $2l_0$ within a time interval of $\Delta t_0$. 
Length Contraction (parallel to \( \mathbf{u} \))

Since both measurement events are at rest (same location) within \( S' \)-frame, \( \Delta t_0 \) is the proper time measurement and we have,

\[
\Delta t_0 = \frac{2l_0}{c}
\]

and \( l_0 \) is the proper length of the ruler.

Now, let consider the description according to Stanley in \( S \)-frame.

The ruler according to Stanley will have a length of \( l \) and let the time of travel for the light pulse from the source to the mirror be \( \Delta t_1 \).

(note: Since Stanley is measuring these from afar and he is in relative motion with respect to Mavis, his measurement of \( \Delta t_1 \) will NOT be proper.)
Length Contraction (parallel to $u$)

Within the time interval $\Delta t_1$, the mirror will have moved a distance of $u \Delta t_1$ so that the actual distance that the laser light has to travel is,

$$d = l + u\Delta t_1$$

Since the speed of light is also $c$ in Stanley frame, we can also write,

$$d = c\Delta t_1$$