(1) (20 points)
Consider a system whose Hamiltonian $H$ and an operator $C$ are given by the following matrices:

$$
H = \epsilon \begin{pmatrix}
1 & -1 & 0 \\
-1 & 1 & 0 \\
0 & 0 & -1
\end{pmatrix}
$$

$$
C = c \begin{pmatrix}
0 & 4 & 0 \\
4 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}
$$

where $\epsilon$ has the dimensions of energy.

(a) If we measure the energy, what values will we obtain?

(b) Suppose that when we measure energy, we obtain a value of $-\epsilon$. Immediately afterwords, we measure $C$. What values will we obtain for $C$ and what are the probabilities corresponding to each value?

(2) (20 points)
Consider a system of three non-interacting particles (each of mass $m$) confined in a two-dimensional potential $V(x, y) = (x^2 + y^2)$. Calculate the total ground state energy and the
first excited state energy of the system if, (a) Particles are non-interacting neutrons. (b) Particles are non-interacting He atoms. (c) Two particles are electrons and the other is a positron.

(3) **(10 points)**
Wave function of a particle moving in three dimension is \( \psi(r, \theta, \phi) = Ae^{-r} \). Calculate the value of \( r \) where the particle is most likely to be found.

(4) **(20 points)**
At \( t = 0 \), a particle of mass \( m \) is equally likely to be in the ground and the first excited state of the system described by \( V(x) = 2x^2 \) for \( x > 0 \), \( V(x) = \infty \) at \( x = 0 \). What is the wave function of the system? at \( t = 0 \). What is the wave function of the particle at \( t = 1 \) sec.

(5) **(15 points)**
An electron moving in a harmonic potential \( V(x) = 2x^2 \) is subjected to a constant electric field \( E \). What is the ground state energy and the ground state wave function of the system.

(6) **(15 points)**
A particle in a spherically symmetric potential is described by a wave function, \( \psi(x, y, z) = A(x + z) \) where \( A \) is a normalization constant. Calculate the possible angular momentum quantum numbers of the system and the probability of being found in those states.