Homework #6 Phys 711 Fall 2008

Hand in

1. (Approach this problem as if you didn't know about the canonical ensemble.) A substance contains a very large number N of particles, each of which can be in one of three states of energy $\varepsilon_i = (i-1)\varepsilon, \quad i = 1, 2, 3$ where ε is constant. Consider the equilibrium state of the substance at a fixed energy $U = Nu\varepsilon$ where u is the average energy per particle in units of ε. The volume of the substance is constant.

a. What are the maximum and minimum values of u?

b. Let n_i be the number of particles in the i^{th} state, and let n_i^* denote their equilibrium values. By following the argument in Greiner (pages 147-148), show that at equilibrium the n_i^* are in geometric progression:

$$\frac{n_3^*}{n_2^*} = \frac{n_2^*}{n_1^*} = x.$$

c. Obtain an expression for x in terms of u.

d. Obtain an expression for s, the entropy per particle. A compact expression is obtained if s is left as a function of x and u, without eliminating x in favor of a function of u. Show on a plot the qualitative behavior of s vs. u.

e. Obtain an expression for the temperature T in terms of x.

f. What is the range of T? Show on a plot the qualitative behavior of T vs. u. How would you interpret T<0?

g. Show that

$$\frac{n_i^*(T)}{N} = \frac{e^{-\varepsilon_i/KT}}{\sum_{j=1}^{3} e^{-\varepsilon_i/KT}}.$$

2. Establish that the probability density of finding a system in the canonical ensemble with energy E,

$$p(E) = \frac{1}{Z} g(E) e^{-\beta E}$$

has a Gaussian shape in the vicinity of its most likely value E^*, i.e., that

$$p(E) = p(E^*) \exp\left\{ -\frac{(E-E^*)^2}{2kT^2 C_V} \right\}, \text{ near } E^*.$$