Homework #4 Phys 711 Fall 2008

Hand in

1. Consider a rubber band from a thermodynamic point of view. A rubber band is essentially a bundle of long-chain polymer molecules. Quantities of macroscopic interest are the length L (analogous to volume), the tension τ (analogous to negative pressure), and the energy. It is experimentally observed that at constant length, the tension surprisingly increases with temperature. It is also observed that the energy is essentially independent of the length L, as long as $L<L_1$, the elastic limit length. Thus,

$$U = cL_0T$$
$$\tau = bT^n\left(\frac{L-L_0}{L_1-L_0}\right), \quad L_0 < L < L_1$$

   where L_o is the unstretched length.

   (a) Study the equation of dS in terms of dU and dL, and show that this constrains the exponent n.

   (b) If the rubber band is stretched by the amount dL at constant T, find the heat transfer dQ to the rubber band, the work dW done, and the relation between them.

   (c) Obtain an expression for S, and note the dependence of S on $(L-L_0)$.

2. If the specific heat C_v of a substance over a certain temperature range varies as (1/T),

   (a) Find the dependence of U on T at fixed V
   (b) Find the equilibrium temperature T_f if equal masses at temperature T_1 and T_2 are brought into thermal contact.

3. Start with $S = S(U,V)$ for fixed N.

(a) Show that $\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_V$ and $\frac{P}{T} = \left.\frac{\partial S}{\partial V}\right|_U$.

(b) Define $m \equiv \frac{1}{T}$ and $q \equiv \frac{P}{T}$. Do a Legendre transformation from S to $B = B(m,q)$ and thus obtain a formula for B in terms of S, m, and q. By writing out dB, show that B really is only a function of m and q.

(c) The formula for the entropy of an ideal gas is given by

$$S = S(U,V) = Nk\left[s_0 + \ln\left\{ \left(\frac{U}{U_0}\right)^{3/2} \left(\frac{V}{V_0}\right) \right\} \right].$$

Obtain the analogous formula for B in terms of (only!) m, q, and constants.

(d) By taking appropriate derivatives of B, obtain the two ideal gas equations of state.