1. 2.

a) $dF = (3y + e^x)\,dx + (3x + \cos y)\,dy$

it's exact if $\frac{\partial}{\partial y}(3y + e^x) \overset{?}{=} \frac{\partial}{\partial x}(3x + \cos y)$

$3 = 3$ yes, it's exact.

So, there exists a function $F = f(x,y)$ s.t.

$$\frac{\partial f}{\partial x} = 3y + e^x \quad \text{and} \quad \frac{\partial f}{\partial y} = 3x + \cos y.$$

from the first of these,

$$f = 3yx + e^x + h(y) + C$$

Therefore, taking $\frac{\partial}{\partial y}$,

$$\frac{\partial f}{\partial y} = 3x + \frac{dh}{dy} = 3x + \cos y$$

So $\frac{dh}{dy} = \cos y \rightarrow h = \sin y$

therefore, $\boxed{f(x,y) = 3xy + e^x + \sin y + C}$

Check:

$\frac{\partial f}{\partial x} = 3y + e^x$ yup.

$\frac{\partial f}{\partial y} = 3x + \cos y$. yup.

1. ~~2~~ b) $dF = (3x^2y + 8xy^2)\,dx + (x^3 + 8x^2y + 12y^2)\,dy$

if its exact,

$$\frac{\partial}{\partial y}(3x^2y + 8xy^2) \overset{?}{=} \frac{\partial}{\partial x}(x^3 + 8x^2y + 12y^2)$$

$$3x^2 + 16xy \overset{?}{=} 3x^2 + 16xy + 0$$

yes, it's exact.

Thus, $\exists$ a fn $f = f(x,y)$ s.t.

$$\frac{\partial f}{\partial x} = 3x^2y + 8xy^2 \quad \text{and} \quad \frac{\partial f}{\partial y} = x^3 + 8x^2y + 12y^2$$

from the first,

$$f = x^3y + 4x^2y^2 + h(y) + C$$

from the second,

$$\frac{\partial f}{\partial y} = x^3 + 8x^2y + \frac{dh}{dy} = x^3 + 8x^2y + 12y^2$$

so $\frac{dh}{dy} = 12y^2$

so $h(y) = 4y^3$

Thus $\boxed{F = x^3y + 4x^2y^2 + 4y^3 + C}$

Check : $\frac{\partial f}{\partial x} = 3x^2y + 8xy^2$ yes

$\frac{\partial f}{\partial y} = x^3 + 8x^2y + 12y^2$ yes.

~~2~~.c) 1.

$$df = \left(4x^3 e^{x+y} + x^4 e^{x+y} + 2x\right) dx + \left(x^4 e^{x+y} + 2y\right) dy$$

if it's exact, Then

$$\frac{\partial}{\partial y}\left(4x^3 e^{x+y} + x^4 e^{x+y} + 2x\right) \stackrel{?}{=} \frac{\partial}{\partial x}\left(x^4 e^{x+y} + 2y\right)$$

$$4x^3 e^{x+y} + x^4 e^{x+y} \stackrel{?}{=} x^4 e^{x+y} + e^{x+y} 4x^3$$ yes, it's exact!

So, $\exists$ a fn $f = f(x,y)$ s.t.

$$\frac{\partial f}{\partial x} = 4x^3 e^{x+y} + x^4 e^{x+y} + 2x$$

and

$$\frac{\partial f}{\partial y} = x^4 e^{x+y} + 2y$$

from the 2^{nd}

$$f = x^4 e^{x+y} + y^2 + h(x) + C$$

so

$$\frac{\partial f}{\partial x} = x^4 e^{x+y} + 4x^3 e^{x+y} + \frac{dh}{dx} = 4x^3 e^{x+y} + x^4 e^{x+y} + 2x$$

$$\frac{dh}{dx} = 2x$$

$$h(x) = x^2$$

so $\boxed{f(x,y) = x^4 e^{x+y} + y^2 + x^2 + C}$

Check: $\frac{\partial f}{\partial x} = 4x^3 e^{x+y} + x^4 e^{x+y} + 2x$ yep.

$\frac{\partial f}{\partial y} = x^4 e^{x+y} + 2y$ yep.

~~3.~~ note that

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$$\left(\frac{y^2}{2}+2ye^x\right)dx + \left(y+e^x\right)dy$$

is not exact, since

$$\frac{\partial}{\partial y}\left(\frac{y^2}{2}+2ye^x\right) \overset{?}{=} \frac{\partial}{\partial x}\left(y+e^x\right)$$

$$y+2e^x \neq e^x.$$

So, include the integrating factor, and try to solve for it. let $g=g(x,y)$ in general.

$$g\left(\frac{y^2}{2}+2ye^x\right)dx + g\left(y+e^x\right)dy$$

is exact if

$$\frac{\partial}{\partial y}\left[g\left(\frac{y^2}{2}+2ye^x\right)\right] = \frac{\partial}{\partial x}\left[g\left(y+e^x\right)\right]$$

$$g\left(y+2e^x\right) + \frac{\partial g}{\partial y}\left(\frac{y^2}{2}+2ye^x\right) = g\left(e^x\right) + \frac{\partial g}{\partial x}\left(y+e^x\right)$$

$$g\left(y+e^x\right) + \frac{\partial g}{\partial y}\left(\frac{y^2}{2}+2ye^x\right) = \frac{\partial g}{\partial x}\left(y+e^x\right)$$

let's see if assuming that $g=g(x)$ only works. Then

$$g(x)\left(y+e^x\right) = \frac{dg}{dx}\left(y+e^x\right)$$

$$g(x) = \frac{dg}{dx} \longrightarrow x = \ln g + c$$

or $\boxed{g(x) = ce^x}$

to confirm that this works, check that with $g = ce^x$,

$$df = ce^x\left(\frac{y^2}{2} + 2ye^x\right)dx + ce^x\left(y + e^x\right)dy$$

is exact. That is,

$$\frac{\partial}{\partial y}\left[\frac{cy^2}{2}e^x + 2cye^{2x}\right] \overset{?}{=} \frac{\partial}{\partial x}\left[ce^x y + ce^{2x}\right]$$

$$cye^x + 2ce^{2x} \overset{?}{=} cye^x + 2ce^{2x}$$

Confirmed! so df is exact.

With this integrating factor, we can now get F.

Since $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$,

$$\frac{\partial f}{\partial x} = \frac{c}{2}y^2e^x + 2cye^{2x}.$$

So

$$f = \frac{c}{2}y^2e^x + cye^{2x} + h(y)$$

and so

$$\frac{\partial f}{\partial y} = cye^x + ce^{2x} + \frac{dh}{dy} = ce^x y + ce^{2x}$$

i.e. $\frac{dh}{dy} = 0$, and $h = \text{const.}$

So,

$$\boxed{F = c_1\left[\frac{y^2}{2}e^x + ye^{2x}\right] + c_2}$$

for the integrating factor $g(x) = c_1 e^x$.

1. $\delta w = -p\,dV = -p\Delta V$ in this case, since $p = \text{const}.$

3

$$\delta w = -(2.34\text{ atm})(4.01 - 3.12\text{ L}) = -2.08\text{ L atm}$$

$$\delta w = -2.08\text{ L}\left(\frac{10^3\text{ cm}^3}{1\text{ L}}\right)\text{atm}\left(\frac{1013.25\times10^3\ \frac{\text{dynes}}{\text{cm}^2}}{1\text{ atm}}\right) = -2.108\times10^9\text{ cm dynes}$$

units: $\text{cm dyne} = \text{cm}\ \frac{\text{gm}}{\text{cm}^2} = \text{ergs}$

$$1\text{ erg} = 10^{-7}\text{ Joules}$$

So $\delta w = -2.108\times10^9\text{ ergs}\cdot\frac{10^{-7}\text{ J}}{1\text{ erg}} = -2.108\times10^2\text{ Joules}$

$\simeq -211$ Joules.

The negative means that the gas performs this amount of work on the surroundings.

2. Treating the gas as ideal, $pV = NkT$.

4

30 grams of H_2 = 15 moles, so $N = 15\,N_A = 15\times6.02\times10^{23}$

$$p = \frac{NkT}{V} = \frac{(15)(6.02\times10^{23})(1.380662\times10^{-16}\text{ ergs}/{}^\circ\text{K})(18^\circ + 273.15^\circ)({}^\circ\text{K})}{1\text{ m}^3\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3}$$

$$p = 3.628\times10^5\text{ ergs/cm}^3 = 3.628\times10^5\ \frac{\text{cm dyne}}{\text{cm}^3} \Rightarrow \text{dynes/cm}^2$$

$$p = 3.628\times10^5\ \frac{\text{dynes}}{\text{cm}^2}\left(\frac{\text{atm}\ \text{cm}^2}{1013.25\times10^3\text{ dynes}}\right) = 0.36\text{ atm}$$

~~3.~~ Treat the gas as ideal. Note that N_2 has atomic mass 28.

5

Suppose there are x grams of N_2 gas.

$$x \text{ grams } N_2 \cdot \frac{1 \text{ mole } N_2}{28 \text{ gram } N_2} \cdot \frac{6.02\times10^{23} \text{ particles}}{1 \text{ mole}} = \frac{x N_a}{28} \text{ particles} = N.$$

ideal gas: $PV = NkT = \frac{x N_a}{28} kT$

density $= \frac{x}{V} = \frac{28P}{N_a kT}$ lets assume one atmosphere of pressure.

$$\text{density} = \frac{28\,(1013.25\times10^{3} \text{ dynes}/\text{cm}^2)}{(6.02\times10^{23})(1.38\times10^{-16} \text{ ergs}/{}^\circ\text{K})(273.15\,{}^\circ\text{K})}$$

$$\text{density} = 1.25\times10^{-3} \frac{\text{dynes}}{\text{ergs}\cdot\text{cm}^2} \rightarrow \frac{1}{\text{cm}^3}$$

so since x is the # of grams, density $= 1.25\times10^{-3}$ grams$/\text{cm}^3$

or $1.25\times10^{-3} \frac{\text{grams}}{\text{cm}^3} \cdot \frac{N_a}{28} = 2.69\times10^{19}$ particles$/\text{cm}^3$.

note that specific volume is volume per unit mass $= (\text{density})^{-1}$.

~~4~~. Assume the gas is ideal. $T = \text{const} = 20°C$, since it's an isothermal expansion.

6

$$W = -\int p\,dV = -\int_{V_1}^{V_2} \frac{NkT}{V}\,dV = -NkT\int_{V_1}^{V_2} \frac{dV}{V} = NkT \ln\left(\frac{V_1}{V_2}\right)$$

using $PV = NkT$. Note that since $NkT = \text{const}$ here, $\frac{V_1}{V_2} = \frac{P_2}{P_1}$.

So $W = NkT \ln\left(\frac{P_2}{P_1}\right)$.

Now, O_2 has atomic weight 32 so 10 grams has

$\left(10 \text{ grams } O_2\right) \cdot \frac{N_a}{32}$ particles.

So

$$W = \left(10\,\frac{N_a}{32}\right)\left(1.38\times10^{-16}\,\frac{\text{ergs}}{°K}\right)\left(20+273.15\right)(°K)\ln\left(\frac{0.3}{1}\right)$$

$$\underline{W = -9.16\times10^{9}\text{ ergs}} = -9.16\times10^{9}\text{ ergs}\cdot\frac{10^{-7}\,J}{1\text{ erg}} = \underline{-916\text{ Joules}}$$

The minus sign means that the gas does work on the surroundings.

~~5.~~ Assume that one calorie of heat changes the temperature of

7

1 gm of water by 1° C throughout this temp. range.

10° C → 10 calories per gram. There are 1000 grams of water,

So That's 10^4 calories. $= 10^4 \, c \cdot \left(\frac{4.184 \, J}{1 \, c}\right) = 4.184 \times 10^4$ Joules.

Set This equal to mgh, and solve for h.

$$h = \frac{4.184 \times 10^4 \text{ Joules}}{(1 \, kg)(9.8 \, m/s^2)} = 4270 \, \frac{N m}{kg \, m/s^2} \to 4270 \, \frac{\cancel{kg \, m} \; m \, \cancel{s^2}}{\cancel{s^2} \; \cancel{kg \, m}}$$

$h = 4270$ meters

$h = 4.27$ kilometers

(so, approximating the accel. due to gravity as g is fine.)