6. given: $T_1 = 291\,^\circ K$ and $T_2 = 305\,^\circ K$

$V_1 = 21 \times 10^3\ cm^3$ $V_2 = 12.7 \times 10^3\ cm^3$.

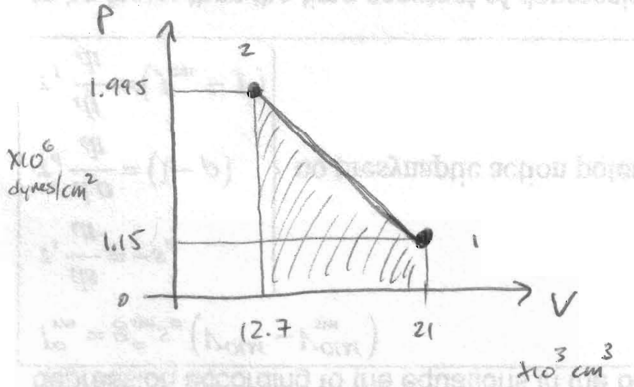
Since the gas is ideal, we can use $PV = NkT$ to get the pressures.

$$P = Nk\left(\frac{T}{V}\right) \quad P_1 = (6.02 \times 10^{23})(1.38 \times 10^{-16}\ \frac{erg}{°K})\left(\frac{291\,°K}{21 \times 10^3\ cm^3}\right)$$

$$P_1 = 1.15 \times 10^6\ dynes/cm$$

$$P_2 = N_A k\left(\frac{305\,°K}{12.7 \times 10^3\ cm^3}\right) = 1.995 \times 10^6\ dynes/cm.$$

So the PV diagram looks like this; we're told that the process is a straight line.



The work performed is $W = -\int_1^2 P dV$

$W = \int_2^1 P dV$ = area in the figure

So, W = rectangle + triangle

$$W = (21-12.7)(10^3\ cm^3)(1.15 \times 10^6\ dynes/cm^2) + \frac{1}{2}(21-12.7)(10^3\ cm^3)(1.995-1.15)(10^6\ dynes/cm^2)$$

$$W = 9.545 \times 10^9\ ergs + 3.507 \times 10^9\ ergs$$

$$W = 1.305 \times 10^{10}\ ergs = 1305\ Joules$$

This is positive, so this is work done on the gas.

to find the heat, use the 1st law: $du = \delta w + \delta Q$

so $\Delta Q = \Delta u - \Delta W$ where ΔW is what we got above.

Δu is obtained from $C_V = \frac{du}{dT}$, which we know is independent of volume.

so

$$\Delta u = \int du = \int c_V \, dT = C_V \int dT = \frac{5}{2} Nk \int_{T_1}^{T_2} dT = \frac{5}{2} Nk \, (305 - 291)°K$$

$$\Delta u = 2.91 \times 10^{9} \text{ ergs} = 291 \text{ Joules.}$$

so

$$\Delta Q = 291 \text{ Joules} - 1305 \text{ Joules} = -1014 \text{ Joules.}$$

Since this is negative, this heat
is delivered to the surroundings.

2. ~~3~~a) an adiabatic process has $\delta Q = 0$.

So, the 1st law reads (for $N = \text{const}$)

$$du = \delta Q + \delta W = \delta W$$

$$du = -p\,dV.$$

- for an ideal gas, $u = \frac{3}{2} NkT$ and $C_V = \frac{\partial u}{\partial T} = \frac{3}{2} Nk$

so

$$du = C_V\,dT = -p\,dV$$

$$\frac{3}{2} Nk\,dT = -p\,dV$$

- From the ideal gas law, $PV = NkT$, so $p = \frac{NkT}{V}$

and

$$\frac{3}{2} Nk\,dT = -NkT\frac{dV}{V}$$

$$\frac{3}{2}\frac{dT}{T} = -\frac{dV}{V}$$

$$\frac{3}{2}\ln\left(\frac{T}{T_0}\right) = -\ln\left(\frac{V}{V_0}\right)$$

$$\left(\frac{T}{T_0}\right)^{3/2} = \frac{V_0}{V}$$

$$V T^{3/2} = \text{const}$$

- from ideal gas law, $V = \frac{NkT}{p}$, so

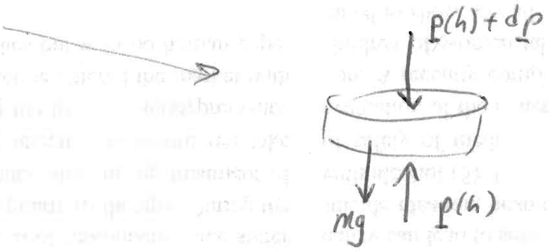
$$\frac{NKT}{P} T^{3/2} = \text{const}$$

$$\frac{T^{5/2}}{P} = \text{const}$$

or $$\boxed{\frac{T}{P^{2/5}} = \text{const}}$$

2. 3 b) consider a column of air of cross-section = 1 unit area.

more specifically, consider a slab of air as shown, located at height h and of thickness dh. Forces are as shown



The force on the lower face is $1 \times p(h)$.

The force on the upper face is $p(h)+dp$. mg is the force of gravity.

Balancing forces,

$$p(h) - [p(h)+dp] - mg = 0$$

$$dp = -mg$$

in terms of the density of the air, and the volume of the slab, This is

$$dp = -\rho(1)\,dh\,g.$$

The density can be expressed via the ideal gas law, $PV = NkT$, as

$$\rho = \frac{MN}{V} = \frac{M N P}{NkT} = \frac{MP}{kT}$$

where M is the average mass of a particle of air.

$$\text{so} \quad dP = -\frac{Mg}{kT} P\,dh$$

$$\text{or} \quad \underline{\frac{dP}{P} = -\frac{Mg}{kT}dh} = d(\ln P).$$

Recall from part (a) that $\frac{T}{P^{2/5}} = \text{const}$, so $T = C\,P^{2/5}$

That's $\ln T = \ln C + \frac{2}{5} \ln P$

and $\frac{dT}{T} = \frac{2}{5}\frac{dP}{P} \Rightarrow \frac{dP}{P} = \frac{5}{2}\frac{dT}{T}$

plug this in above

$$\frac{dP}{P} = \frac{5}{2}\frac{dT}{T} = -\frac{M g}{kT}\,dh$$

$$\boxed{\frac{dT}{dh} = -\frac{2}{5}\frac{M g}{k}}$$

given 1 mole of particles of air $= N_a = 28.88\,g$

$\Rightarrow M = \frac{28.88}{N_a}$ grams

So

$$-\frac{2}{5}\left(\frac{28.88(\text{grams})}{N_a}\right)\left(\frac{9.8\,\cancel{m}}{s^2}\right)\left(\frac{{}^{o}K}{1.38\times10^{-16}\,\cancel{\text{ergs}}}\right)\left(\frac{\cancel{\text{erg}}}{\cancel{\text{dyne}}\,\text{cm}}\right)\left(\frac{100\,\text{cm}}{1\,\cancel{m}}\right)$$

$$\cdot\left(\frac{\cancel{\text{dyne}}\,s^2}{(\text{gram})\,\text{cm}}\right)\left(\frac{100\,\text{cm}}{1m}\right)$$

$$= -1.36\times10^{-2}\,\frac{{}^{o}K}{m}\cdot\left(\frac{1000\,m}{1\,km}\right) = \boxed{-13.6\,\frac{{}^{o}K}{km}}$$

note: The assumption that air is monoatomic is not good since it's mostly Nitrogen gas (N_2) and oxygen (O_2).

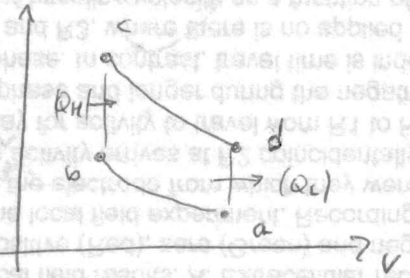
Because a diatomic gas has extra degrees of freedom,

The result quoted in part (a) should be replaced with

$T = C p^{2/7}$ This results in an answer to part (b)

of ≈ -9.8 °K/km.

3. 1. in the OTTO cycle, heat is exchanged only during the constant-volume processes bc and da; the other paths are adiabatic.



- let $|Q_H|$ be the amount of heat added in bc
- let $|Q_L|$ be the amount of heat ejected in da.

- Since this occurs at const. volume,

$$|Q_H| = C_V (T_c - T_b)$$

$$|Q_L| = C_V (T_d - T_a)$$

(where the subscripts on the T_s identify the position on the PV diagram)

- So the efficiency is

$$\eta = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \left(\frac{T_d - T_a}{T_c - T_b}\right).$$

- for an adiabatic process with an ideal gas, $PV^{5/3} = \text{const}$

or, since $P = \frac{NkT}{V}$, $\frac{NkT}{V} V^{5/3} = NkT V^{2/3} = \text{const}$

i.e. $\boxed{TV^{2/3} = \text{const}}$

- So, since cd and ab are adiabatic processes,

$$T_c V_c^{2/3} = T_d V_d^{2/3} \quad \text{and} \quad T_a V_a^{2/3} = T_b V_b^{2/3}$$

- write

$$T_d = T_c \left(\frac{V_c}{V_d}\right)^{2/3} \quad \text{and} \quad T_a = T_b \left(\frac{V_b}{V_a}\right)^{2/3}$$

- so,

$$\eta = 1 - \left(\frac{T_d - T_a}{T_c - T_b}\right) = 1 - \left(\frac{T_c \left(\frac{V_c}{V_d}\right)^{2/3} - T_b \left(\frac{V_b}{V_a}\right)^{2/3}}{T_c - T_b}\right)$$

- note that $V_c = V_b$ and $V_d = V_a$, so

$$\eta = 1 - \left(\frac{T_c \left(\frac{V_b}{V_a}\right)^{2/3} - T_b \left(\frac{V_b}{V_a}\right)^{2/3}}{T_c - T_b}\right)$$

$$\eta = 1 - \left(\frac{V_b}{V_a}\right)^{2/3}$$

i.e. $$\boxed{\eta = 1 - \left(\frac{V_a}{V_b}\right)^{-2/3}}$$