

## Homework #1

1. In class, we worked out the distribution function that describes the probability of finding a particle whose velocity vector  $\vec{V} = V_x \hat{x} + V_y \hat{y} + V_z \hat{z}$  has components within the intervals  $[V_x, V_x + dv_x]$ ,  $[V_y, V_y + dv_y]$ ,  $[V_z, V_z + dv_z]$  in “velocity space”. This is:

$$f(\vec{v}) = \left( \frac{m}{2\pi kT} \right)^{3/2} \exp\left( \frac{-mV^2}{2kT} \right)$$

- What is the distribution function that describes the probability of finding a particle with **speed**  $V$  within the interval  $[V, V + dV]$ ?
- Use the result of (a) to calculate the most probable speed  $V_p$ , the average speed  $\bar{V}$ , and  $V_{rms}$ . Express these in terms of  $\sqrt{kT/m}$ .
- Suppose a sample of helium gas at  $T = 300K$  contains  $10^6$  atoms, each of mass  $m = 6.65 \times 10^{-27} \text{ kg}$ . What is  $V_p$ ? Estimate how many atoms in the sample have speeds in the interval  $[V_p, V_p + 40 \text{ m/s}]$  and in the interval  $[10V_p, 10V_p + 40 \text{ m/s}]$ .

2. Show that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ .

3. Let

$$I(n) = \int_0^{\infty} e^{-ax^2} x^n dx.$$

- Evaluate  $I(0)$  and  $I(1)$ .
- Calculate  $\frac{\partial I}{\partial a}$  and obtain a recurrence relation.