(a) Determine Lagrange's equations of motion.
(b) What are the generalized momenta associated with any cyclic coordinates?
(c) Determine Hamilton's equations of motion.

7-28. A particle of mass m is attracted to a force center with the force of magnitude k/r^2. Use plane polar coordinates and find Hamilton's equations of motion.

7-29. Consider the pendulum described in Problem 7-15. The pendulum's point of support rises vertically with constant acceleration a.
(a) Use the Lagrangian method to find the equations of motion.
(b) Determine the Hamiltonian and Hamilton's equations of motion.
(c) What is the period of small oscillations?

7-30. Consider any two continuous functions of the generalized coordinates and momenta $g(q_k, p_k)$ and $h(q_k, p_k)$. The **Poisson brackets** are defined by

$$[g, h] \equiv \sum_k \left(\frac{\partial g}{\partial q_k}\frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k}\frac{\partial h}{\partial q_k} \right)$$

Verify the following properties of the Poisson brackets:

(a) $\dfrac{dg}{dt} = [g, H] + \dfrac{\partial g}{dt}$ **(b)** $\dot{q}_j = [q_i, H],\ \dot{p}_j = [p_j, H]$

(c) $[p_l, p_j] = 0,\ [q_l, q_j] = 0$ **(d)** $[q_l, p_j] = \delta_{lj}$

where H is the Hamiltonian. If the Poisson bracket of two quantities vanishes, the quantities are said to *commute*. If the Poisson bracket of two quantities equals unity, the quantities are said to be *canonically conjugate*. **(e)** Show that any quantity that does not depend explicitly on the time and that commutes with the Hamiltonian is a constant of the motion of the system. Poisson-bracket formalism is of considerable importance in quantum mechanics.

7-31. A spherical pendulum consists of a bob of mass m attached to a weightless, extensionless rod of length l. The end of the rod opposite the bob pivots freely (in all directions) about some fixed point. Set up the Hamiltonian function in spherical coordinates. (If $p_\phi = 0$, the result is the same as that for the plane pendulum.) Combine the term that depends on p_ϕ with the ordinary potential energy term to define as *effective* potential $V(\theta, p_\phi)$. Sketch V as a function of θ for several values of p_ϕ, including $p_\phi = 0$. Discuss the features of the motion, pointing out the differences between $p_\phi = 0$ and $p_\phi \neq 0$. Discuss the limiting case of the conical pendulum (θ = constant) with reference to the V-θ diagram.

7-32. A particle moves in a spherically symmetric force field with potential energy given by $U(r) = -k/r$. Calculate the Hamiltonian function in spherical coordinates, and obtain the canonical equations of motion. Sketch the path that a representative point for the system would follow on a surface ... by showing that th