Faraday Rotation Experiment

1 Introduction

Faraday rotation is the rotation of the direction of polarization of linearly polarized light as it traverses a transparent medium in the presence of a longitudinal magnetic field.

2 Experimental Arrangement

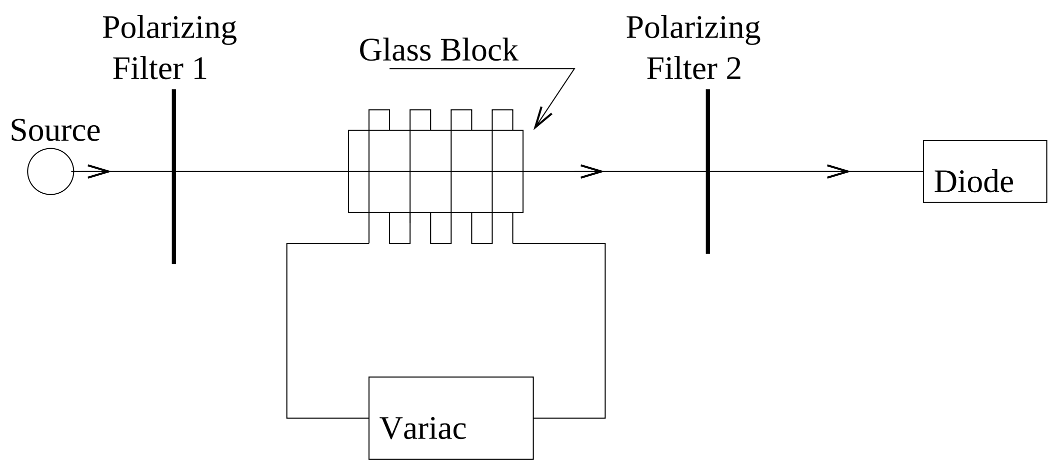


Figure 1: Block diagram of the apparatus.

A light beam is polarized with a polaroid filter, and passed through the transparent sample (in our case a block of glass). A current- carrying coil creates a longitudinal magnetic field. The light out of the sample traverses another polaroid and is directed to a photodiode detector. The light beam should pass through the glass block without reflection from the sides. Use collimators, if necessary, to achieve this.

3 Principle

The polarizing filters are initially set to some angle θ_0 , with magnetic field $B = 0$. With an applied magnetic field, the electric field vector is rotated by an angle δ. Now

$$\delta = BLV \tag{1}$$

where L is the length of the block, and V is the Verdet constant, a characteristic of the sample medium. **The goal of the experiment** is to observe the Faraday rotation and measure the Verdet constant. A white light source and several interference filters may be used to measure V for different wavelengths.

4 Measurement of δ

4.1 Law of Malus

If θ is the angle between the E-vector of linearly polarized light and the "pass axis" of a polarizing filter, then the transmitted intensity is related to the incident intensity via

$$I_{trans} = I_{inc} \cos^2 \theta = I_{inc} \cos^2(\theta_0 + \delta) \tag{2}$$

For small δ, the right-hand side of equation 2 can be expanded about θ_0. This gives

$$I_{trans} \approx I_{inc}(\cos^2(\theta_0) - \delta \sin(2\theta_0) - \delta^2 \cos(2\theta_0)) \tag{3}$$

So when the polaroids are initially set to extinction, $\theta_0 = \pi/2$, so equation 3 gives

$$I_{trans} \approx I_{inc}\delta^2$$

and the transmitted intensity varies quadrically with δ. If $\theta_0 = \pi/4$, then

$$I_{trans} \approx I_{inc}(\frac{1}{2} - \delta) \tag{4}$$

The two cases are shown in the figure below. Note that for the same (small) δ, the 45° arrangement gives a much larger effect.

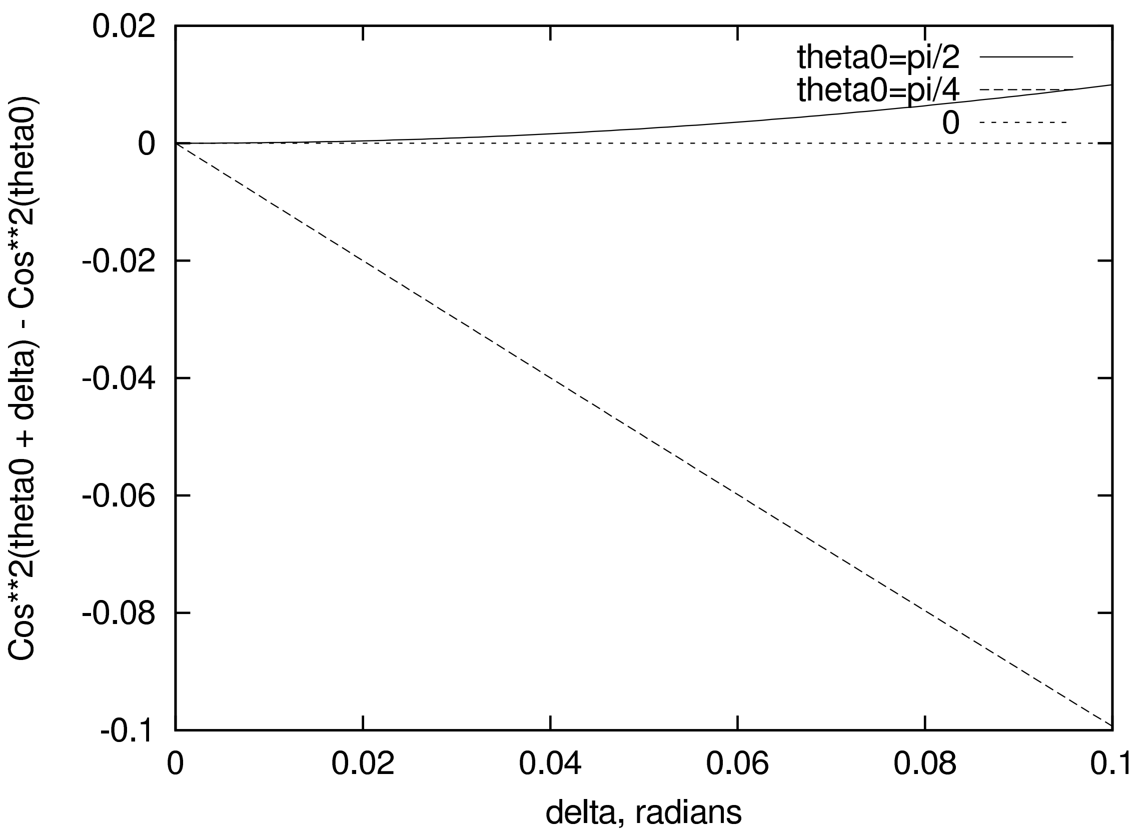


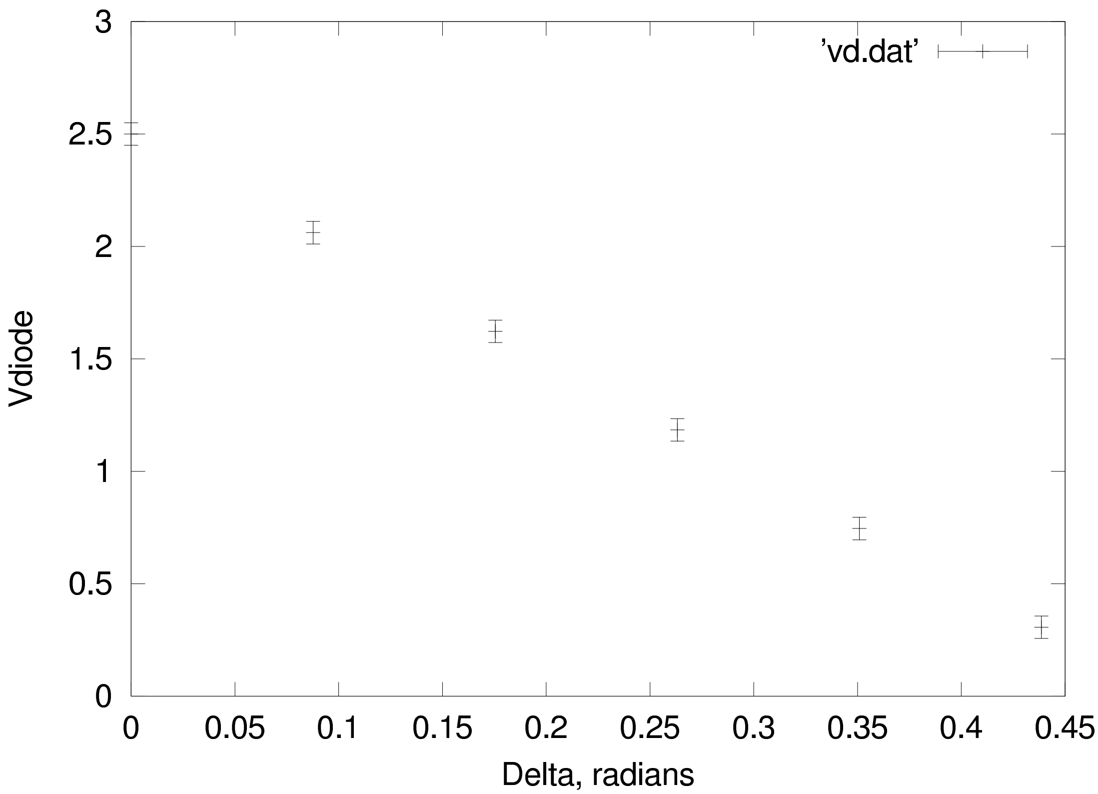
Figure 2: $I(\theta_0 + \delta) - I(\theta_0)$ vs. δ

4.2 Photodiode response

In the linear range of the photodiode response, the photodiode output voltage V_d, is proportional to the incident light intensity. So equation 4 can be written

$$V_d = V_{d0} + a\delta \tag{5}$$

where is a is a constant which depends on the source intensity, and the sample transmision. If, with $B = 0$, we rotate the polaroids away from 45° by known angles ϕ and measure then change in V_d for each ϕ, then a plot of V_d vs. ϕ would look like:



The constant a can be determined from the graph. Then, with the polaroids back at 45°, we turn on B and measure V_d. Then the Faraday rotation angle is

$$\delta = \frac{V_d - V_0}{a} \tag{6}$$

5 Complication: AC current

In this version of the experiment, we use a $60Hz$ AC voltage source, (a "Variac" variable transformer), to produce current in the coil. So in all of the above, B should be replaced by B_{max}, the amplitude of the magnetic field oscillation, δ by δ_{max}, and V_d by $V_{d,max}$.

6 Measurement of B

There are several possible methods for measurement of B_{max}:

1. Use a Hall probe.

2. Measure the rms current in the coil with an ammeter, convert to i_{max}, and use the (approximate) expression

$$B = \mu_0 i n_l$$

where, in MKS units, $\mu_0 = 4\pi \times 10^{-7}$, i is the current in amperes, and n_l is the number of turns per meter.

3. Use a test coils of known area A and number of turns N. Measure the induced voltage, V_i, across the test coil. Then Faraday's Law gives

$$V_i = -N\frac{d\Phi_B}{dt}$$
$$= -NA\omega B_{max}\cos(\omega t)$$

Since N, A, and $\omega = 2\pi 60Hz$ are known, then $V_{i,max}$ determines B_{max}.

Note that we cannot simply measure B with the rotating coil Gaussmeter, since B is not constant in time. Remember that the MKS unit of B is the Tesla. ($1Tesla = 10^4 Gauss$). In the literature on Faraday rotation, the magnetic field strength, H, is sometimes given in **Oersteds** (Oe). In a vacuum, if $H = 1Oe$, then $B = 1Gauss$.

7 What Results are Expected?

It can be shown that the Verdet constant is expected to be[1]

$$V = \frac{e}{2mc}\lambda\frac{dn}{d\lambda}$$

The last term on the right side is the dispersion of the medium. For normal dispersion, we have [2]

$$\frac{dn}{d\lambda} \propto 1/\lambda^3$$

which means that we expect

$$V \propto 1/\lambda^2$$

So there should be a large difference between the $V's$ for red and blue light.

Some numerical values of V are tabulated below [3]

Material	$V(min/Gauss - cm), 589nm$
Water	0.0131
Crown Glass	0.0161
Flint Glass	0.0317

[1] See, for example, Preston et al. , The Art of Experimental Physics
[2] See, for example, Jenkins and White, Fundamentals of Optics, 3rd Edition, McGraw-Hill, 1957.
[3] From F. Pedrotti and L. Pedrotti, Introduction to Optics, 2nd Edition, Prentice-Hall 1993, p 554.

8 Be sure to include in your report:

1. A graph of diode voltage vs. polarizer angle with $B = 0$.
2. An explanation of how you obtained B for a given Variac setting.
3. A graph of Faraday rotation angle vs. B.
4. The obtained Verdet constant(s), with uncertainties, compared with other measurments.
5. A graph of Verdet Constant vs. Wavelength, together with theoretical predictions.