Chapter 39 - Particles Behaving as Waves

1. [39.61] The star Betelgeuse has a surface temperature of 3000 K and is 600 times the diameter of our sun. (a) If Betelgeuse were to radiate all of its energy at the peak-intensity wavelength, how many photons per second would it radiate? (b) Find the ratio of the power radiated by Betelgeuse to the power radiated by our sun (at 5800 K).

2. [39.65] When a photon is emitted by an atom, the atom must recoil to conserve momentum. This means that the photon and the recoiling atom share the transition energy. (a) For an atom with mass m, calculate the correction $\Delta\lambda$ to the photon's wavelength due to recoil. Assume that without a recoil the photon's wavelength would be λ. (b) Evaluate the correction for a hydrogen atom in which an electron in the n^{th} energy level returns to the ground state.

   - Let the energy released in the transition be ΔE. Without recoil, all this energy goes to the photon:

     $$\Delta E = \frac{hc}{\lambda}$$

     With recoil, the energy is shared by the atom and the photon:

     $$\Delta E = \frac{hc}{\lambda'} + \frac{p^2}{2m}$$

     where p is the atom's acquired momentum. We are interested in $\Delta\lambda = \lambda' - \lambda$. Assuming that the atom was initially at rest, the total momentum must be zero after the transition:

     $$\frac{h}{\lambda'} = p$$

     Therefore:

     $$\Delta E = \frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{p^2}{2m} = \frac{hc}{\lambda'} + \frac{h^2}{2m\lambda'^2} = \frac{hc}{\lambda'}\left(1 + \frac{h}{2mc\lambda'}\right)$$

     $$\lambda' = \lambda\left(1 + \frac{h}{2mc\lambda'}\right) = \lambda\left(1 + \frac{h}{2mc\lambda\left(1 + \frac{h}{2mc\lambda'}\right)}\right) = \cdots \approx \lambda\left(1 + \frac{h}{2mc\lambda}\right)$$

     $$\Delta\lambda = \lambda' - \lambda \approx \frac{h}{2mc}$$

3. [39.81] The radii of atomic nuclei are of the order of 5.0×10^{-15} m. (a) Estimate the minimum uncertainty in the momentum of an electron if it is confined within a nucleus. (b) Take this momentum uncertainty to be an estimate of the typical magnitude of a confined electron's momentum. Estimate the kinetic energy of an electron confined inside a nucleus. (c) Compare the estimated kinetic energy to the magnitude of the Coulomb potential energy of a proton and an electron separated by 5.0×10^{-15} m. Could there be electrons within the nucleus?

   (a) $\Delta p \Delta x \sim h \quad \Rightarrow \quad \Delta p \sim h/\Delta x \sim 1.3 \times 10^{-19}$ kg m/s ... You may want to use $\hbar/2$ instead of h here - it's more accurate, but Heisenberg uncertainty is anyway useful only for making order-of-magnitude estimates.

   (b) $T = \sqrt{(\Delta p\, c)^2 + (m_e c^2)^2} - m_e c^2 = 3.9 \times 10^{-11}$ J

   (c) $U \sim -(e^2/4\pi\epsilon_0)/\Delta x = 4.6 \times 10^{-14}$ J ... Kinetic energy is orders of magnitude larger than potential energy, so the electron is free.

4. [39.85] If your wavelength were 1.0 m, you would undergo considerable diffraction in moving through a doorway. (a) What must your speed be for you to have this wavelength? Assume that your mass is 60.0 kg. (b) At the speed calculated in part (a), how many years would it take you to move 0.8 m (one step)? Will you notice diffraction effects as you walk through doorways?

   (a) Note how this is related to Compton effect (Compton wavelength is $\lambda_c = h/mc$):

$$p = \frac{h}{\lambda} = mv \quad \Rightarrow \quad v = \frac{h}{m\lambda} = 1.1 \times 10^{-35} \text{ m/s}$$

   (b) $t = s/v = 7.2 \times 10^{34}$ s $= 2.3 \times 10^{27}$ years. Even if you could maintain coherence to enable difraction, you could not observe anything at such low speeds and long time scales.

5. [39.55] The negative muon has a charge equal to that of an electron but a mass that is 207 times as great. Consider a hydrogen-like atom consisting of a proton and a muon. (a) What is the reduced mass of the atom? (b) What is the ground-level energy (in electron volts)? (c) What is the wavelength of radiation emitted in the transition from the $n = 2$ level to the $n = 1$ level?

   (a) $m_p = 1.67 \times 10^{-27}$ kg, $m_e = 9.1 \times 10^{-31}$ kg, $m_m = 207 m_e$

$$\frac{1}{\mu} = \frac{1}{m_p} + \frac{1}{m_m} \quad \Rightarrow \quad \mu = \frac{m_p m_m}{m_p + m_m} = 1.69 \times 10^{-28} \text{ kg}$$

   (b) For a normal hydrogen atom, the ground-level energy (negative ionization energy) is:

$$E_0 = -13.6 \text{ eV}$$

   We need the dependence of E_0 on (reduced) mass m - and trace-out the derivation of Bohr's model to obtain it. For a circular trajectory of a particle in a potential $U(r) \propto 1/r$, it is well known that kinetic T and potential U energy are related as $U = -2T$:

$$U(r) = -\frac{A}{r} \quad \Rightarrow \quad F(r) = -\frac{dU}{dr} = -\frac{A}{r^2}$$

$$\frac{mv^2}{r} = |F(r)| = \frac{A}{r^2} \quad \Rightarrow \quad T = \frac{mv^2}{2} = \frac{1}{2}\frac{A}{r} = -\frac{1}{2}U$$

   The constant A is $Ze^2/4\pi\epsilon_0$, but we don't need to keep track of details. The total energy of a Bohr's hydrogen atom is:

$$E_0 = U + T = -T = -\frac{mv^2}{2}$$

   Electron's speed v must depend on its (reduced) mass because the angular momentum $L = mvr = n\hbar$ is quantized ($n = 1$ for the ground level, and $\hbar = h/2\pi$). Using again $U = -2T$, we find:

$$-\frac{mv^2}{2} = -\frac{A}{r} \quad \Rightarrow \quad 2A = mv^2 r = (mvr)v = (n\hbar)v \quad \Rightarrow \quad v = \frac{2A}{n\hbar}$$

   and therefore:

$$E_0 = -\frac{mv^2}{2} \propto \frac{m}{n^2}$$

   The ground state energy ($n = 1$) is simply proportional to mass. In general, this is the reduced mass: when the two-body problem is expressed in the center-of-mass reference frame, it is equivalent to the problem of a single particle with reduced mass orbiting about the fixed center (of mass). So, the energy of the muon-hydrogen atom is:

$$E_m = \frac{\mu}{m} E_0 = -2.52 \text{ keV}$$

(c) ...

$$\frac{hc}{\lambda} = |E_m| \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \quad \Rightarrow \quad \lambda = \frac{4hc}{3E_m} = 6.53 \times 10^{-10} \text{ m}$$

6. [39.59] A sample of hydrogen atoms is irradiated with light with wavelength 85.5 nm, and electrons are observed leaving the gas. (a) If each hydrogen atom was initially in its ground level, what would be the maximum kinetic energy in electron volts of these photoelectrons? (b) A few electrons are detected with energies as much as 10.2 eV greater than the maximum kinetic energy calculated in part (a). How can this be?

   (a) The minimum energy needed to ionize a hydrogen atom in its ground state is:

   $$\Delta E = (13.6 \text{ eV}) \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = 13.6 \text{ eV}$$

   A photon with $\lambda = 85.5$ nm carries energy:

   $$E_\gamma = \frac{hc}{\lambda} = 14.5 \text{ eV}$$

   Therefore, a photon has enough energy to ionize an atom and impart an extra kinetic energy on the electron:

   $$T = E_\gamma - E_0 = 0.9 \text{ eV}$$

   Some of this amound may be lost on the recoil to the atom, so this is the maximum kinetic energy that a single photon absorption can generate.

   (b) More kinetic energy needs absorption of more photons. One photon can excite an electron to a high energy level and give a part of its energy to recoil the atom, and another photon can then ionize this excited atom and push the electron out with higher speed. Such a process is not very likely, but possible.