Chapter 38 - Photons: Light Waves Behaving as Particles

1. [38.41] A photon with wavelength 0.1100 nm collides with a free electron that is initially at rest. After the collision the wavelength is 0.1132 nm. (a) What is the kinetic energy of the electron after the collision? What is its speed? (b) If the electron is suddenly stopped (for example in a solid target), all of its kinetic energy is used to create a photon. What is the wavelength of this photon?

   - $\lambda = 0.1100$ nm, $\lambda' = 0.1132$ nm, $m_e c^2 = 511$ keV

   (a) ...

   $$E = \frac{hc}{\lambda} \quad , \quad E' = \frac{hc}{\lambda'} \quad \Rightarrow \quad T_e = E - E' = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = 318.3 \text{ eV}$$

   Exact electron's speed:

   $$T_e = (\gamma - 1)m_e c^2 \quad \Rightarrow \quad \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = 1 + \frac{T_e}{m_e c^2}$$

   $$v = c\sqrt{1 - \frac{1}{\gamma^2}} = c\sqrt{1 - \left(1 + \frac{T_e}{m_e c}\right)^{-2}} = 0.0353c = 1.058 \times 10^7 \text{ m/s}$$

   Approximate electron's speed: $\gamma \approx 1$

   $$T_e = \frac{m_e v^2}{2} \quad \Rightarrow \quad v = \sqrt{\frac{2T_e}{m_e}} = 1.059 \times 10^7 \text{ m/s}$$

   (b) ...

   $$T_e = \frac{hc}{\lambda''} \quad \Rightarrow \quad \lambda'' = \frac{hc}{T_e} = \left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)^{-1} = \frac{\lambda\lambda'}{\lambda' - \lambda} = 3.891 \text{ nm}$$

2. [38.37] A photon with wavelength 0.1050 nm is incident on an electron that is initially at rest. If the photon scatters at an angle 60^o from its original direction, what is the magnitude and direction of the linear momentum of the electron just after the collision with the photon?

   - $\lambda = 0.1050$ nm, $\theta = 60^o$, $m_e = 9.1 \times 10^{-31}$ kg
   - Before the collision, the momentum projections p_x and p_y on the x (along the photon's path) and y (perpendicular to the photon's path) directions respectively, and the total energy E (of the electron at rest and the photon) are:

   $$p_x = \frac{h}{\lambda} \quad , \quad p_y = 0 \quad , \quad E = \frac{hc}{\lambda} + m_e c^2$$

   - After the collision, the scattered photon has wavelength:

   $$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta) = 0.1062\,\text{nm}$$

   Energy and momentum projections are conserved:

   $$p_x = \frac{h}{\lambda'}\cos\theta + p_e\cos\theta' \quad , \quad p_y = \frac{h}{\lambda'}\sin\theta - p_e\sin\theta' \quad , \quad E = \frac{hc}{\lambda'} + \sqrt{(p_e c)^2 + (m_e c^2)^2}$$

   Electron's momentum (by magnitude) can be obtained from:

   $$E = \frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + \sqrt{(p_e c)^2 + (m_e c^2)^2}$$

$$(p_e c)^2 + (m_e c^2)^2 = \left(\frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2\right)^2$$

$$p_e^2 = \left(\frac{h}{\lambda} - \frac{h}{\lambda'} + m_e c\right)^2 - m_e^2 c^2 = \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)\left(\frac{h}{\lambda} - \frac{h}{\lambda'} + 2m_e c\right) \approx 2m_e c\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)$$

$$p_e \approx 11.68 \text{ keV} = 6.24 \times 10^{-24} \text{ kg m/s}$$

To compare with the SI units, think of $p_e c$ (an energy scale) being 11.68 keV. The direction of the electron is given by the angle θ' from the initial photon's direction. We can obtain it from $p_y = 0$:

$$p_y = \frac{h}{\lambda'} \sin\theta - p_e \sin\theta' = 0 \quad \Rightarrow \quad \theta' = \arcsin\left(\frac{h}{p_e \lambda'} \sin\theta\right) = 59.95^o$$

3. [38.15] Protons are accelerated from rest by a potential difference of 4.00 kV and strike a metal target. If a proton produces one photon on impact, what is the minimum wavelength of the resulting X rays? How dows the answer compare to the minimum wavelength if 4.00 kV electrons are used instead? Why do X-ray tubes use electrons rather than protons to produce X rays?

- $V = 4.00$ kV
- Proton's kinetic is $T_p = eV = 4.00$ keV (in the units of electronvolts, it equals the numerical value of voltage), where $e = 1.6 \times 10^{-19}$ C is proton's absolute charge. When the proton stops, at best all of its kinetic energy goes to the photon:

$$T_p = eV \geq \frac{hc}{\lambda} \quad \Rightarrow \quad \lambda \geq \frac{hc}{eV} = 3.1 \times 10^{-10} \text{ m}$$

- The above result does not depend on the proton's mass. Since an electron has the same (absolute) charge as a proton, the minimum photon's wavelength obtained by stopping electrons is the same.
- Electrons are used in X-ray tubes simply because they are easier to extract from materials (e.g. by the photoelectric effect). Protons are strongly bound by nuclear forces in nuclei, and the nuclei are heavy and deeply bound by chemical bonds to neighboring nuclei in molecules and crystals.

4. [38.25] An electron and a positron are moving toward each other and each has speed $0.5000c$ in the lab frame. (a) What is the kinetic energy of each particle? (b) The e^+ and e^- meet head-on and annihilate. What is the energy of each photon that is produced? (c) What is the wavelength of each photon? How does the wavelength compare to the photon wavelength when the initial kinetic energy of the e^+ and e^- is negligibly small?

(a) $v = 0.5000c$, $m_e c^2 = 511$ keV

$$E_e = \gamma m_e c^2 = \frac{m_e c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = 590 \text{ keV}$$

$$T_e = (\gamma - 1) m_e c^2 = 79.05 \text{ keV}$$

(b) Two photons are produced to conserve both energy and momentum. The total momentum of the electron and the positron was zero, so the two photons must have the same momenta and propagate in opposide directions. Hence, they must also have the same energy:

$$E_\gamma = E_e = 590 \text{ keV}$$

(c) ...

$$E_\gamma = \frac{hc}{\lambda} \quad \Rightarrow \quad \lambda = \frac{hc}{E_\gamma} = 2.101 \times 10^{-12} \text{ m}$$

5. [38.13] When ultraviolet light with a wavelength of 254 nm falls on a clean copper surface, the stopping potential necessary to stop emission of photoelectrons is 0.188 V. (a) What is the photoelectric threshold wavelength for this coper surface? (b) What is the work function for this surface?

   (a) $\lambda = 254\,\text{nm}$, $V_s = 0.181\,\text{V}$ ... Find the work function ϕ for copper in a table, or solve first the part (b):

   $$T_e = E_\gamma - \phi = \frac{hc}{\lambda} - \phi > 0 \quad \Rightarrow \quad \lambda < \frac{hc}{\phi} = 264\,\text{nm}$$

   (b) ...

   $$T_e = E_\gamma - \phi = \frac{hc}{\lambda} - \phi = eV_s \quad \Rightarrow \quad \phi = \frac{hc}{\lambda} - eV_s = 4.70 \text{ eV}$$