Chapter 35 - Interference

1. [35.37] The radius of curvature of the convex surface of a planoconvex lens is 68.4 cm. The lens is placed convex side down on a perfectly flat glass plate that is illuminated from above with red light having wavelength of 580 nm. Find the diameter of the second bright ring in the interference pattern.

   - We are given $R = 68.4$ cm, $\lambda = 580$ nm. There is an air gap between the lens and the plate whose vertial thickness is $t(r)$ as a function of the horizontal radial distance from the center of the lens. We know that $t(0) = 0$ because the lens is convex and touches the plate at its center. Then:

   $$\Big[R - t(r)\Big]^2 + r^2 = R^2 \quad \Rightarrow \quad t(r) = R - \sqrt{R^2 - r^2}$$

   $$h^2 + r_0^2 = R^2 \quad \Rightarrow \quad h = \sqrt{R^2 - r_0^2}$$

   For nearly normal incidence, the condition for constructive interference between the beams that reflect from the glass plate and the bottom surface of the lens is:

   $$2t(r) = \left(m + \frac{1}{2}\right)\lambda$$

   The 1/2 apepars because the reflection in air from glass introduces a π-phase shift. The second bright ring is at $m = 1$ (the first one is at $m = 0$):

   $$t(r) = \frac{3\lambda}{4}$$

   $$R - \sqrt{R^2 - r^2} = \frac{3\lambda}{4}$$

   $$r = \sqrt{R^2 - \left(R - \frac{3\lambda}{4}\right)^2} = \sqrt{\frac{3\lambda}{4}\left(2R - \frac{3\lambda}{4}\right)} = 0.77 \text{ mm}$$

   $$d = 2r = 1.54 \text{ mm}$$

2. [35.45] Two speakers, 2.50 m apart, are driven by the same audio oscillator so that each one produces a sound consisting of two distinct frequencies, 0.900 kHz and 1.20 kHz. The speed of sound is 344 m/s. Find all the angles relative to the usual centerline in front of (and far from) the speakers at which both frequencies interfere constructively.

   - $d = 2.50$ m, $f_1 = 0.900$ kHz, $f_2 = 1.20$ kHz, $c = 344$ m/s. Far away from the speakers, constructive interference is at angles θ_i such that:

   $$d \sin\theta_i = m_i \lambda_i = m_i \frac{c}{f_i} \quad , \quad m_i \text{ is integer}$$

   We want both frequencies to interfere constructively at the same angle, $\theta_1 = \theta_2$:

   $$\sin\theta_i = \frac{m_1 c}{f_1 d} = \frac{m_2 c}{f_2 d} \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{f_1}{f_2} = \frac{3}{4}$$

   This is possible because f_1/f_2 is a rational number, m_1 and m_2 must be proportional to 3 and 4 respectively (and both integer):

   $$m_1 = 3m \quad , \quad m_2 = 4m \quad , \quad m \text{ is integer}$$

This gives us the sought angles:

$$\theta_i = \arcsin\left(\frac{3c}{f_1 d}m\right) = \arcsin\left(\frac{4c}{f_2 d}m\right) \in \{0, \pm 27.3^o, \pm 66.5^o\}$$

for $m \in \{0, \pm 1, \pm 2\}$.

3. [35.49] Two speakers A and B are 3.50 m apart, and each one is emitting a frequency of 444 Hz. However, because of signal delays in the cables, speaker A is one-fourth of a period ahead of speaker B. For points far from the speakers, find all angles relative to the centerline at which the sound from these speakers cancels. Include angles on both sides of the centerline. The speed of sound is 340 m/s.

4. [35.51] A thin uniform film of refractive index 1.750 is placed on a sheet of glass of refractive index 1.50. At room temperature 20^oC this film is just thick enough for light with wavelength 582.4 nm reflected off the top of the film to be cancelled by light reflected from the top of the glass. After the glass is placed in the oven and slowly heated to 170^oC, you find that the film cancels reflected light with wavelength 588.5 nm. What is the coefficient of linear expansion of the film. Ignore the temperature dependence of the refractive index.

   - $n_f = 1.750$, $n_g = 1.50$, $T_0 = 20^o$C, $\lambda_0 = 582.4$ nm, $T_1 = 170^o$C, $\lambda_1 = 588.5$ nm. Destructive interference occurs when the film thickness d is:

     $$n_f > n_g \quad \Rightarrow \quad 2d = \left(m + \frac{1}{2}\right)\lambda \quad , \quad m \in \mathbb{Z}$$

     At $T = T_0$, we have ($m = 0$, the film is "just thick enough"):

     $$d_0 = \frac{\lambda_0}{4}$$

     and at $T = T_1$:

     $$d_1 = \frac{\lambda_1}{4}$$

     Hence, the expansion coefficient α is:

     $$\frac{\Delta d}{d} = \frac{d_1 - d_0}{d_0} = \alpha \Delta T = \alpha(T_1 - T_0) \quad \Rightarrow \quad \alpha = \left(\frac{d_1}{d_0} - 1\right)\frac{1}{T_1 - T_0} = \left(\frac{\lambda_1}{\lambda_0} - 1\right)\frac{1}{T_1 - T_0} = 6.98 \times 10^{-5}\ \mathrm{K}^{-1}$$

5. [35.55] A source S of monochromatic light and a detector D are both located in air a distance h above a horizontal plane sheet of glass and are separated by a horizontal distance x. Waves reaching D directly from S interfere with the waves that reflect off the glass. The distance x is small compared to h ($x \ll h$) so that reflection is close to normal incidence. Show that the condition for constructive interference is:

   $$\sqrt{x^2 + 4h^2} - x = \left(m + \frac{1}{2}\right)\lambda$$

   and the destructive interference:

   $$\sqrt{x^2 + 4h^2} - x = m\lambda$$

   - Note that there is a π-phase shift in the reflection from the glass plate (glass has a larger refractive index than air). The distance traveled by light directly from S to D is $r_2 = x$. The total distance traveled on the path that reflects from the glass sheet is:

     $$r_1 = 2\sqrt{h^2 + \left(\frac{x}{2}\right)^2} = \sqrt{4h^2 + x^2}$$

If there weren't a π-phase shift on reflection, the constructive interference would occur when:

$$r_1 - r_2 = m\lambda$$

With the π-shift, this is where the destructive interference occurs, while we have constructive when:

$$r_1 - r_2 = \sqrt{x^2 + 4h^2} - x = \left(m + \frac{1}{2}\right)\lambda$$

6. [35.59] In a Young's two-slit experiment a piece of glass with an index of refraction n and thickness L is placed in front of the upper slit. Derive the expression for the intensity I of the light at points on the screen as a function of n, L, θ, where θ is the usual angle measured from the centerline between the two slits. Where are the interference pattern maxima (at which θ)?

- The electric field of a linearly-polarized electromagnetic wave propagates as:

$$E(x,t) = E_0 \cos(\omega t - kx)$$

Suppose that the electric field a distance L in front of the slits (where we temporarily set the origin, $x = 0$) is $E = E_0 \cos(\omega t)$. The light that goes through air toward the uncovered slit has the wavenumber k set by the speed of light in air

$$\omega = ck \quad , \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

so that the field at the uncovered slit is:

$$E_1 = E_0 \cos(\omega t - kL)$$

The light that goes through glass has a different wavenumber $k' = \omega/c'$ because it has a different light speed $c' = c/n$. Hence, the electric field at this slit is:

$$E_1 = E_0 \cos(\omega t - k'L) = E_0 \cos(\omega t - nkL)$$

We used:

$$k' = \frac{\omega}{c'} = \frac{\omega}{c/n} = n\frac{\omega}{c} = nk$$

We see that the phase shift introduced by glass is:

$$\Delta\phi_0 = (n-1)kL = 2\pi\frac{(n-1)L}{\lambda}$$

where λ is the light's wavelength in air. Past the slits, the beams travel additional distances r_1 and r_2 that differ by $d\sin\theta$. At the screen, the electric fields belonging to the two beams are:

$$E_1 = \cos\Big(\omega t - kL - k(r_2 + d\sin\theta)\Big) \quad , \quad E_2 = \cos(\omega t - nkL - kr_2)$$

The total phase difference between the two beams is:

$$\Delta\phi = \frac{2\pi}{\lambda}\Big[(n-1)L + d\sin\theta\Big]$$

and we can write:

$$E_1 = \cos(\alpha) \quad , \quad E_2 = \cos(\alpha + \Delta\phi)$$

$$E_1 + E_2 = 2\cos\left(\frac{\Delta\phi}{2}\right)\cos\left(\alpha + \frac{\Delta\Phi}{2}\right)$$

Here, α contains ωt, so the second cosine factor oscillates in time, while the first cosine defines the amplitude of oscillations. The light intensity is proportional to E^2, so that:

$$I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right) = I_0 \cos^2\left\{\frac{\pi}{\lambda}\left[(n-1)L + d\sin\theta\right]\right\}$$

The interference pattern maxima are found at angles:

$$\frac{\pi}{\lambda}\left[(n-1)L + d\sin\theta\right] = m\pi \quad , \quad m \in \mathbb{Z}$$

$$\theta = \arcsin\left[\frac{m\lambda - (n-1)L}{d}\right]$$