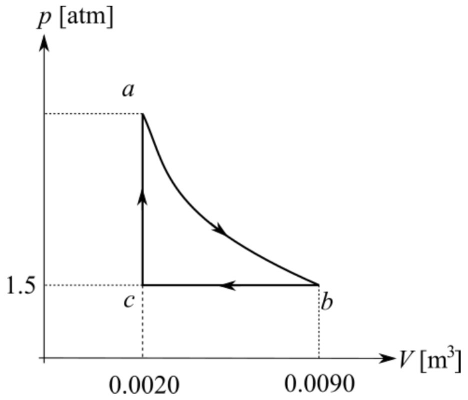
Chapter 20 - The Second Law of Thermodynamics

1. [20.5] The pV-diagram in the figure shows a cycle of a heat engine that uses 0.250 mol of an ideal gas having $\gamma = 1.40$. The curved part ab of the cycle is adiabatic. (a) Find the pressure of the gas at point a. (b) How much heat enters this gas per cycle, and where does it happen? (c) How much heat leaves this gas per cycle, and where does it occur? (d) How much work does the engine do in a cycle? (e) What is the thermal efficiency of the engine?



- $p_1 = 1.5$ atm, $V_1 = 0.0020$ m^3, $V_2 = 0.0090$ m^3, $n = 0.250$ mol
- (a)

$$p_1 V_1 = nRT_c \quad \Rightarrow \quad T_c = \frac{p_1 V_1}{nR} = 146.25 \text{ K}$$

$$p_1 V_2 = nRT_b \quad \Rightarrow \quad T_b = \frac{p_1 V_2}{nR} = 658.11 \text{ K}$$

$$p_1 V_2^\gamma = p_a V_1^\gamma \quad \Rightarrow \quad p_a = \left(\frac{V_2}{V_1}\right)^\gamma p_1 = 12.3 \text{ atm}$$

$$p_a V_1 = nRT_a \quad \Rightarrow \quad T_a = \frac{p_a V_1}{nR} = 1199.22 \text{ K}$$

- (b,c,d)

$$C_p - C_V = R \quad , \quad \frac{C_p}{C_V} = \gamma \quad \Rightarrow \quad C_V = \frac{R}{\gamma - 1} \quad , \quad C_p = \frac{\gamma R}{\gamma - 1}$$

$$Q_{\text{intake}} = Q_{ca} = nC_V(T_a - T_c) = 5.47 \text{ kJ}$$

$$Q_{\text{outtake}} = |Q_{bc}| = nC_p(T_b - T_c) = 3.72 \text{ kJ}$$

$$W = Q_{\text{intake}} - Q_{\text{outtake}} = 1.75 \text{ kJ}$$

- (e)

$$\eta = \frac{W}{Q_{\text{intake}}} = 31.9 \text{ \%}$$

2. [20.37] A certain heat engine operating on the Carnot cycle absorbs 150 J of heat per cycle at its hot reservoir at 130^{o}C and has a thermal efficiency of 22.0%. (a) How much work does this engine do per cycle? (b) How much heat does the engine waste each cycle? (c) What is the temperature of the cold reservoir? (d) By how much does the engine change the entropy of the world each cycle? (e) What mass of water could this engine pump per cycle from a 35.0 m deep well?

- $Q_{\text{in}} = 150$ J, $T_H = 130^o\text{C} = 403.16$ K, $\eta = 0.220$

- (a-c)

$$\eta = \frac{W}{Q_{\text{in}}} \quad \Rightarrow \quad W = \eta Q_{\text{in}} = 33 \text{ J}$$

$$Q_{\text{out}} = Q_{\text{in}} - W = 117 \text{ J}$$

$$\eta = 1 - \frac{T_C}{T_H} \quad \Rightarrow \quad T_C = (1-\eta)T_H = 314.46 \text{ K} = 41.3^o\text{C}$$

- (d)

$$\Delta S = \frac{Q_{\text{out}}}{T_C} - \frac{Q_{\text{in}}}{T_H} = 0$$

- (e) h=35.0 m

$$W = mgh \quad \Rightarrow \quad m = \frac{W}{gh} = 96.1 \text{ g}$$

3. [20.45] An experimental power plant at the Natural Energy Laboratory of Hawaii generates electricity from the temperature gradient of the ocean. The surface and deep water temperatures are 27^{o}C and 6^{o}C respectively. (a) What is the maximum theoretical effiiency of this power plant? (b) If the power plant is to produce 210 kW of power, at what rate must heat be extracted from the warm water? At what rate must heat be absorbed by cold water? Assume the maximum theoretical efficiency. (c) The cold water that enters the plant leaves it at temperature 10^{o}C. What must be the flow rate of cold water through the system? Give your answer in kg/h and L/h.

   - (a) $T_s = 27^o\text{C} = 300.16$ K, $T_d = 6^o\text{C} = 279.16$ K

   $$\eta = \eta_{\text{max}} \equiv 1 - \frac{T_d}{T_s} = 0.07 = 7\%$$

   - (b) $P = 210$ kW

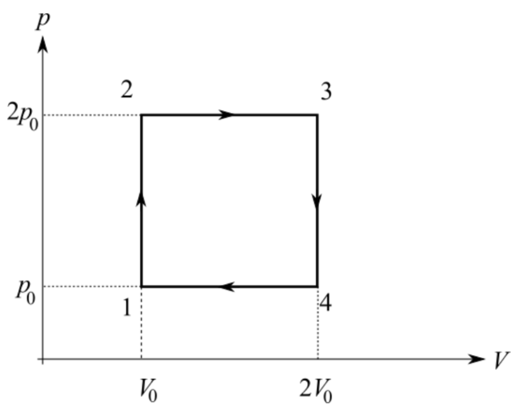
   $$P = \frac{dW}{dt} \quad , \quad \eta = \frac{dW}{dQ_{\text{in}}} \quad \Rightarrow \quad \frac{dQ_{\text{in}}}{dt} = \frac{dW}{\eta dt} = \frac{P}{\eta} \approx 3{\times}10^6 \text{ W} \quad \frac{dQ_{\text{out}}}{dt} = \frac{dQ_{\text{in}} - dW}{dt} = \left(\frac{1}{\eta} - 1\right) P \approx 2.7{\times}10^6 \text{ W}$$

   - (c) $T = 10^o\text{C} = 283.16$ K, $c_V = 4190$ J/kg K

   $$dQ_{\text{out}} = dm\, c_V(T - T_d)$$

   $$dm = \frac{dQ_{\text{out}}}{c_V(T - T_d)} \quad \Rightarrow \quad \frac{dm}{dt} = \frac{1}{c_V(T-T_d)} \frac{dQ_{\text{out}}}{dt} = 161.1 \text{ kg/s} \approx 5.8 \times 10^5 \text{ kg/h}$$

4. [20.46] What is the thermal efficiency of an engine that operates by taking n moles of diatomic ideal gas through the cycle $1 \to 2 \to 3 \to 4 \to 1$ shown in the figure.



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$$W = \oint p dV = (2p_0 - p_0)(2V_0 - V_0) = p_0 V_0$$

$$pV = nRT \quad \Rightarrow \quad T = \frac{pV}{nR}$$

$$C_V = \frac{5}{2}R \quad , \quad C_p = C_V + R = \frac{7}{2}R$$

$$Q_{1\to 2} = nC_V(T_2 - T_1) = \frac{C_V}{R}(2p_0V_0 - p_0V_0) = \frac{C_V}{R}p_0V_0 = \frac{5}{2}p_0V_0$$

$$Q_{2\to 3} = nC_p(T_3 - T_2) = \frac{C_p}{R}(2p_0 2V_0 - 2p_0V_0) = \frac{C_p}{R}2p_0V_0 = 7p_0V_0$$

$$Q_{3\to 4} = nC_V(T_4 - T_3) = \frac{C_V}{R}(p_0 2V_0 - 2p_0 2V_0) = -\frac{C_V}{R}2p_0V_0 = -5p_0V_0$$

$$Q_{4\to 1} = nC_p(T_1 - T_4) = \frac{C_p}{R}(p_0V_0 - p_0 2V_0) = -\frac{C_p}{R}p_0V_0 = -\frac{7}{2}p_0V_0$$

$$Q_{\text{in}} = Q_{1\to 2} + Q_{2\to 3} = \frac{19}{2}p_0V_0$$

$$\eta = \frac{W}{Q_{\text{in}}} = \frac{2}{19} \approx 10.5\%$$