Chapter 19 - The First Law of Thermodynamics

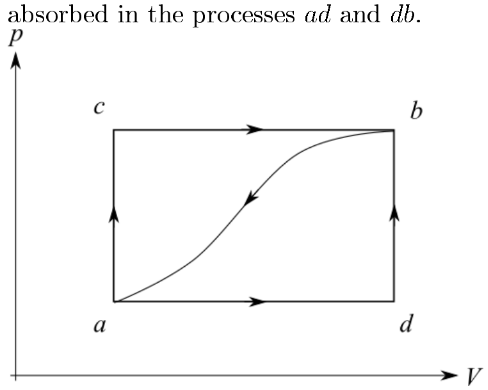
1. [19.37] One mole of ideal gas is slowly compressed to one third of its original volume. In this compression, the work done by the gas is 600 J. For the gas, $C_p = \frac{7}{2}R$. (a) If the process is isothermal, what is the heat flow for the gas? Does heat flow into or out of the gas? (b) If the process is isobaric, what is the change of the internal energy of the gas? Does the internal energy increase or decrease?

2. [19.39] A quantity of air is taken from state a to state b along a path that is a straight line in the $p-V$ diagram. Note $(p_a, V_a) < (p_b, V_b)$. (a) In this process, does the temperature of the gas increase, decrease or stay the same (explain)? (b) If $V_a = 0.0700$ m^3, $V_b = 0.1100$ m^3, $p_a = 1.00 \times 10^5$ Pa and $p_b = 1.40 \times 10^5$ Pa, what is the work W done by the gas in this process? Assume that the gas may be treated as ideal.

   - (a) temperature increases
   - (b)

$$W = \int_a^b p dV = \int_{V_a}^{V_b} dV \left[p_a + \frac{p_b - p_a}{V_b - V_a}(V - V_a) \right] = p_a(V_b - V_a) + \frac{1}{2}(p_b - p_a)(V_b - V_a) = \frac{p_a + p_b}{2}(V_b - V_a) = 4.8 \text{ kJ}$$

3. [19.41] When a system is taken from state a to state b in the figure along the path acb, 90.0 J of heat flows into the system and 60.0 J of work is done by the system. (a) How much heat flows into the system along the path adb if the work done by the system is 15.0 J? (b) When the system is returned from b to a along the curved path, the absolute value of the work done by the system is 35.0 J. Does the system absorb or liberate heat? How much heat? (c) If $U_a = 0$ and $U_d = 8.0$ J, find the heat absorbed in the processes ad and db.



   - $Q_{acb} = 90.0$ J, $W_{acb} = 60.0$ J, $W_{adb} = 15.0$ J, $W_{ba} = -35.0$ J, $U_a = 0$ and $U_d = 8.0$ J

$$\Delta U_{ab} = U_b - U_a = Q_{acb} - W_{acb} = Q_{abd} - W_{adb}$$
$$Q_{adb} = Q_{acb} - W_{acb} + W_{adb} = 45.0 \text{ J}$$
$$C_p = C_V + R$$
$$-\Delta U_{ab} = Q_{ba} - W_{ba} \quad \Rightarrow \quad Q_{ba} = -(Q_{acb} - W_{acb}) + W_{ba} = -65 \text{ J}$$
$$Q_{ad} = U_d - U_a + W_{ad} = U_d - U_a + W_{adb} = 23 \text{ J}$$
$$Q_{bd} = Q_{adb} - Q_{ad} = 22 \text{ J}$$

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$$p_1V_1 = nRT_a \quad , \quad p_2V_1 = nRT_c \quad , \quad p_1V_2 = nRT_d \quad , \quad p_2V_2 = nRT_b$$

$$W_{acb} = W_{cb} = p_2(V_2 - V_1)$$

$$W_{adb} = W_{ad} = p_1(V_2 - V_1)$$

$$\frac{p_2}{p_1} = \frac{W_{acb}}{W_{adb}}$$

$$\begin{aligned}
Q_{acb} &= nC_V(T_c - T_a) + nC_p(T_b - T_c) = \frac{C_V}{R}(p_2V_1 - p_1V_1) + \frac{C_V + R}{R}(p_2V_2 - p_2V_1) \\
&= \frac{C_V}{R}(p_2 - p_1)V_1 + \frac{C_V + R}{R}p_2(V_2 - V_1) = \frac{C_V}{R}(p_2 - p_1)V_1 + \left(1 + \frac{C_V}{R}\right)W_{acb} \\
&= \frac{C_V}{R}p_1V_1\left(\frac{W_{acb}}{W_{adb}} - 1\right) + \left(1 + \frac{C_V}{R}\right)W_{acb}
\end{aligned}$$

$$\begin{aligned}
Q_{adb} &= nC_p(T_d - T_a) + nC_V(T_b - T_d) = \frac{C_p}{R}(p_1V_2 - p_1V_1) + \frac{C_V + R}{R}(p_2V_2 - p_1V_2) \\
&= \frac{C_V + R}{R}p_1(V_2 - V_1) + \frac{C_V}{R}(p_2 - p_1)V_2 = \frac{C_V}{R}(p_2 - p_1)V_2 + \left(1 + \frac{C_V}{R}\right)W_{adb} \\
&= \frac{C_V}{R}p_1V_2\left(\frac{W_{acb}}{W_{adb}} - 1\right) + \left(1 + \frac{C_V}{R}\right)W_{adb}
\end{aligned}$$

4. [19.51] Starting with 2.50 mol of N_2 gas (assumed to be ideal) in a cylinder at 1.00 atm and 20.0^{o}C, a chemist first heats the gas at constant volume, adding 1.52×10^4 J of heat, then continues heating and allows the gas to expand at constant pressure to twice its original volume. (a) Calculate the final temperature of the gas. (b) Calculate the amount of work done by the gas. (c) calculate the amount of heat added to the gas while it was expanding. (d) Calculate the change in internal energy of the gas for the whole process.

   - $n = 2.50$ mol, $p_1 = 1$ atm $= 1.01325 \times 10^5$ Pa, $T_1 = 20.0^o$C $= 293.16$ K, $V_1 = V_2 \equiv V_{12}$, $Q_{12} = 1.52 \times 10^4$ J, $p_2 = p_3 \equiv p_{23}$, $V_3 = 2V_2$

$$\Delta U_{12} = Q_{12} = nC_V(T_2 - T_1) = \frac{5}{2}nR(T_2 - T_1) \quad \Rightarrow \quad T_2 = T_1 + \frac{Q_{12}}{\frac{5}{2}nR}$$

$$p_1V_1 = nRT_1 \quad , \quad p_2V_2 = nRT_2 \quad \Rightarrow \quad V_1 = V_2 = \frac{nRT_1}{p_1} \quad \Rightarrow \quad p_2 = p_1\frac{T_2}{T_1} = p_1\left(1 + \frac{Q_{12}}{\frac{5}{2}nRT_1}\right) \quad , \quad W_{12} = 0$$

$$p_3 = p_2 \quad , \quad V_3 = 2V_2 \quad \Rightarrow \quad W_{23} = p_2(V_3 - V_2) = p_2V_2 = nRT_1 + \frac{2}{5}Q_{12}$$

$$p_3V_3 = nRT_3 \quad \Rightarrow \quad T_3 = \frac{p_3V_3}{nR} = \frac{2p_2V_2}{nR} = 2T_1 + \frac{4}{5}\frac{Q_{12}}{nR} = 1171.4\text{ K} = 898.2^o\text{C}$$

$$W = W_{12} + W_{13} = W_{23} = 1.22 \times 10^4\text{ J}$$

$$Q_{23} = nC_p(T_3 - T_2) = \frac{7}{2}nR(T_3 - T_2) = \frac{7}{2}nR\left(T_1 + \frac{2}{5}\frac{Q_{12}}{nR}\right) = \frac{7}{2}nRT_1 + \frac{7}{5}Q_{12} = 4.26 \times 10^4\text{ J}$$

$$\Delta U = Q_{12} + Q_{23} - W = 4.56 \times 10^4\text{ J}$$