Chapter 18 - Thermal Properties of Matter

1. [18.55] A hot-air baloon stays aloft because hot air at atmospheric pressure is less dense than the cooler air at the same pressure. If the volume of the baloon is 500 m^3 and the surrounding air is at 15^{o}C, what must the temperature of the air in the baloon be for it to lift a total load of 290 kg (in addition to the mass of the hot air)? The density of air at 15^{o}C and atmospheric pressure ($p_0 = 1.01325 \times 10^5$ Pa) is 1.23 kg/m^3.

   - $V = 500$ m^3, $T_0 = 15^o$C, $m_L = 290$ kg, $\rho_0 = 1.23$ kg/m^3. The force of buoyancy lifts the baloon:

   $$F_b = \rho_o V g = (m_L + m)g$$

   The mass m of the hot-air in the ballon is related to its pressure p_0 (atmospheric) and temperature $T > T_0$ (the molar mass of air is $M = 29$ g/mol, and the gas constant is $R = 8.314$ J/mol):

   $$p_0 V = nRT \quad , \quad m = nM$$

   $$T = \frac{p_0 V}{nR} = \frac{p_0 V M}{mR} = \frac{p_0 V M}{(\rho_0 V - m_L)R} = 544 \text{ K} = 271^o C$$

2. [18.57] A cylinder 1.00 m tall with inside diameter 0.120 m is used to hold propane gas (molar mass 44.1 g/mol) for use in a barbecue. It is initially filled with gas until the gauge pressure is 1.3×10^6 Pa and the temperature is 22^{o}C. The temperature of the gas remains constant as it is partially emptied out of the tank until the gauge pressure is 2.50×10^5 Pa. Calculate the mass of the propane that has been used.

   - $h = 1.00$ m, $d = 0.120$ m, $M = 44.1$ g/mol, $p_0 = 1.3 \times 10^6$ Pa, $T = 22^o$C $=$ const, $p_1 = 2.50 \times 10^5$ Pa.

   $$p_0 V = n_0 RT \quad , \quad p_1 V = n_1 RT \quad , \quad m = (n_0 - n_1)M \quad , \quad V = \frac{\pi d^2 h}{4}$$

   $$m = \frac{p_0 - p_1}{RT}\frac{\pi d^2 h}{4} M = 213 \text{ g}$$

3. [18.61] An automobile tire has a volume of 0.0150 m^3 on a cold day when the temperature of the air in the tire is 5.0^{o}C and atmospheric pressure is 1.02 atm. Under these conditions, the gauge pressure is measured to be 1.70 atm (about 25 lb/in^2). After the car is driven on a highay for 30 min, the temperature of the air in the tire has risen to 45.0^{o}C and the volume has risen to 0.0159 m^3. What then is the gauge pressure?

   - $p_0 = 1.02$ atm, $T_1 = 5.0^o$C $= 278.16$ K, $V_1 = 0.0150$ m^3, $p_1 = p_0 + 1.70$ atm $= 2.72$ atm, $T_2 = 45.0^o$C $= 318.16$ K, $V_2 = 0.0159$ m^3. The elastic force of the inflated tire effectively produces pressure $\delta p = p_1 - p_0$, which we may assume to stay constant under the small tire volume change.

   $$p_1 V_1 = nRT_1 \quad , \quad p_2 V_2 = nRT_2$$

   $$p_2 = \frac{nRT_2}{V_2} = \frac{V_1 T_2}{V_2 T_1} p_1 = 2.94 \text{ atm}$$

   $$p_2 - p_0 = 1.92 \text{ atm}$$

4. [18.65] A large tank of water has a hose connected to it. The tank is sealed at the top and has compressed air between the water surface and the top. When the water height h is 3.5 m, the absolute pressure p of the compressed air is 4.20×10^5 Pa. Assume that the air above water expands at constant temperature, and take the atmospheric pressure to be 1.00×10^5 Pa. (a) What is the speed with which water flows out of the hose when $h = 3.50$ m (the hose opening is 1.00 m above ground and the tank's height is 4.00 m)? (b) As water flows out of the tank, h decreases. Calculate the flow speed for $h = 3.00$ m and $h = 2.00$ m. (c) At what value of h does the flow stop?

- $p_0 = 1.00 \times 10^5$ Pa, $h_0 = 1.00$ m, $H = 4.00$ m, $h_1 = 3.5$ m, $p_1 = 4.20 \times 10^5$ Pa

$$p(h)(H-h)A = nRT \quad \Rightarrow \quad p(h) = \frac{nRT}{A(H-h)} = \frac{H-h_1}{H-h}p_1$$

$$p(h)+\rho_w g(h-h_0) = \frac{\rho_w v^2}{2}+p_0 \quad \Rightarrow \quad v = \sqrt{\frac{2(p(h)-p_0)}{\rho_w} + 2g(h-h_0)} = \sqrt{\frac{2}{\rho_w}\left(\frac{H-h_1}{H-h}p_1 - p_0\right) + 2g(h-h_0)}$$

$$v(h = 3.5 \text{ m}) = 26.30 \text{ m/s}$$
$$v(h = 3 \text{ m}) = 16.10 \text{ m/s}$$
$$v(h = 2 \text{ m}) = 5.44 \text{ m/s}$$

$$v = 0 \quad \Rightarrow \quad p(h) - p_0 = \frac{H-h_1}{H-h}p_1 - p_0 = \rho_w g(h_0 - h)$$

$$(H-h_1)\frac{p_1}{\rho_w g} - (H-h)\frac{p_0}{\rho_w g} = (h_0-h)(H-h) = h^2 - (H+h_0)h + h_0 H$$

$$h^2 - \left(H + h_0 + \frac{p_0}{\rho_w g}\right)h + \left(h_0 + \frac{p_0 - p_1}{\rho_w g}\right)H + \frac{h_1 p_1}{\rho_w g} = 0$$

$$h_\pm = \frac{1}{2}\left\{H + h_0 + \frac{p_0}{\rho_w g} \pm \sqrt{\left(H + h_0 + \frac{p_0}{\rho_w g}\right)^2 - 4\left[\left(h_0 + \frac{p_0-p_1}{\rho_w g}\right)H + \frac{h_1 p_1}{\rho_w g}\right]}\right\}$$

$$h_- = 1.74 \text{ m} \quad , \quad h_+ = 13.46 \text{ m} > H$$

5. [18.71] You blow up a spherical baloon to a diameter of 50.0 cm until the absolute pressure inside is 1.25 atm and the temperature is 22.0°C. Assume that all gas is N_2 with molar mass 28.0 g/mol. (a) Find the mass of a single N_2 molecule. (b) How much translational kinetic energy does an average N_2 molecule have? (c) How many N_2 molecules are in this baloon? (d) What is the total translational kinetic energy of all the molecules in the baloon?

- $d = 50.0$ cm, $p = 1.25$ atm, $T = 22.0^o\text{C} = 295.16$ K, $M = 28.0$ g/mol, $k_B = 1.38 \times 10^{-23}$ J/K

$$m_1 = \frac{M}{N_A} = 4.65 \times 10^{-26} \text{ kg}$$

$$\langle E_1^{\text{kin}} \rangle = \frac{3k_B T}{2} = 6.11 \times 10^{-21} \text{ J}$$

$$pV = Nk_B T \quad \Rightarrow \quad N = \frac{pV}{k_B T} = \frac{p \times \frac{\pi}{6}d^3}{k_B T} = 2.01 \times 10^{24}$$

$$\langle E^{\text{kin}} \rangle = N\langle E_1^{\text{kin}} \rangle = 12.3 \text{ kJ}$$

6. [18.77] Show that a projective with mass m can escape from the surface of a planet if it is launched vertically upward with a kinetic energy greater than mgR_p, where R_p is the planet's radius and g is the planet's gravitational acceleration at the surface. Ignore air resistance. (a) If the planet in question is the Earth, at what temperature does the average translational kinetic energy of a nitrogen molecule (molar mass 28.0 g/mol) equal that required to escape? (b) What about a hydrogen molecule (molar mass 2.02 g/mol)? (c) Repeat part (b) for the Moon, for which $g = 1.63$ m/s^2 and $R_p = 1640$ km. (d) While the Earth and the Moon have similar average surface temperatures, the Moon has essentially no atmosphere. Use your results of (b) and (c) to explain why.

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$$mg = G\frac{mM}{R_p^2} \quad \Rightarrow \quad G\frac{mM}{R_p} = mgR_p$$

$$\frac{mv^2}{2} - G\frac{mM}{R_p} > 0 \quad \Rightarrow \quad E^{\text{kin}} > G\frac{mM}{R_p} = mgR_p$$

- (a) Earth's average radius is $R_p = 6371$ km. For nitrogen:

$$\langle E_1^{\text{kin}} \rangle = \frac{3k_BT}{2} = mgR_p \quad \Rightarrow \quad T = \frac{2mgR_p}{3k_B} = \frac{2MgR_p}{3k_BN_A} = 1.4 \times 10^5 \text{ K}$$

- (b) For hydrogen on Earth:

$$T = 1.01 \times 10^4 \text{ K}$$

- (c) For hydrogen on Moon:

$$T = 433 \text{ K} = 160^o\text{C}$$