Chapter 17 - Temperature and Heat

1. [17.79] You are making pesto for your pasta and have a cylindrical measuring cup 10.0 cm high made of ordinary glass [$\beta_c = 2.7 \times 10^{-5}$ $(\mathrm{C}^o)^{-1}$] that is filled with olive oil [$\beta_o = 6.8 \times 10^{-4}$ $(\mathrm{C}^o)^{-1}$] to a height of 2.00 mm below the top of the cup. Initially, the cup and oil are at room temperature (22.0^o C). You get a phone call and forget about the olive oil, which you inadvertently leave on the hot stove. The cup and oil heat up slowly and have a common temperature at all times. At what temperature will the olive oil start to spill out of the cup?

   - We know: $T_0 = 22.0^o\,\mathrm{C} = 295.16$ K, $h_c(T_0) = 10$ cm, $h_o(T_0) = h_c(T_0) - \Delta h(T_0) = 9.8$ cm. Assuming that the cup is very thin, the bottom cross-section area $A(T)$ of the cup and oil are the same at every temperature T. Then:

   $$V_c(T) = h_c(T)A(T) \quad , \quad V_o(T) = h_o(T)A(T)$$

   $$V_c(T) - V_c(T_0) = \beta_c V_c(T_0)(T - T_0) \quad , \quad V_o(T) - V_o(T_0) = \beta_o V_o(T_0)(T - T_0)$$

   The oil spills when $h_c(T) = h_0(T)$:

   $$h_c(T) = h_o(T) \quad \Rightarrow \quad \frac{V_c(T)}{A(T)} = \frac{V_o(T)}{A(T)} \quad \Rightarrow \quad V_c(T) = V_o(T)$$

   $$V_c(T_0)\Big[\beta_c(T - T_0) + 1\Big] = V_o(T_0)\Big[\beta_o(T - T_0) + 1\Big]$$

   $$h_c(T_0)\Big[\beta_c(T - T_0) + 1\Big] = h_o(T_0)\Big[\beta_o(T - T_0) + 1\Big]$$

   $$T = \frac{h_c(T_0)(T_0\beta_c - 1) - h_o(T_0)(T_0\beta_o - 1)}{h_c(T_0)\beta_c - h_o(T_0)\beta_o} = T_0 - \frac{\Delta h(T_0)}{h_c(T_0)\beta_c - h_o(T_0)\beta_o}$$

   $$T = 53.3^o\,\mathrm{C}$$

2. [17.83] A metal rod that is 30.0 cm long expands by 0.0650 cm when its temperature is raised from 0.0^o C to 100^o C. A rod of a different metal and same length expands by 0.0350 cm for the same rise of temperature. A third rod, also 30 cm long, is made up of pieces of each of the above metals placed end to end and expands 0.0580 cm between 0.0^o C and 100^o cm. Find the length of each portion of the composite rod.

   - $L = 30\,\mathrm{cm}$:

   $$\Delta L_1 = \alpha_1 L \Delta T \quad \Rightarrow \quad \alpha_1 = \frac{\Delta L_1}{L\Delta T}$$

   $$\Delta L_2 = \alpha_2 L \Delta T \quad \Rightarrow \quad \alpha_2 = \frac{\Delta L_2}{L\Delta T}$$

   Let the pieces of the composite rod be L_1 and L_2 long:

   $$L_1 + L_2 = L$$

   $$\alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T = \Delta L_{12}$$

   Let $l_i = L_i/L$:

   $$l_1 + l_2 = 1$$

   $$l_1 \Delta L_1 + l_2 \Delta L_2 = \Delta L_{12}$$

Hence:

$$l_1 = \frac{\Delta L_2 - \Delta L_{12}}{\Delta L_2 - \Delta L_1} \quad , \quad l_2 = \frac{\Delta L_1 - \Delta L_{12}}{\Delta L_1 - \Delta L_2}$$

$$L_1 = \frac{\Delta L_2 - \Delta L_{12}}{\Delta L_2 - \Delta L_1} L = 23\,\text{cm} \quad , \quad L_2 = \frac{\Delta L_1 - \Delta L_{12}}{\Delta L_1 - \Delta L_2} L = 7\,\text{cm}$$

3. [17.88] A steel rod 0.450 m long and an aluminum rod 0.250 m long, both with the same diameter, are placed end to end between rigid supports with no initial stress in the rods. The temperature of the rods is now raised by 60.0 C^o. What is the stress σ in each rod?

- $L_s = 0.450\,\text{m}$, $L_a = 0.250\,\text{m}$, $\Delta T = 60\,\text{C}^o$. Each rod tries to thermally expand by:

$$\Delta L_{0i} = \alpha_i L_i \Delta T$$

but this is partially compensated by a contraction due to the stress σ (the same in both rods in mechanical equilibrium):

$$\Delta L_{\sigma i} = -L_i \frac{\sigma}{Y_i}$$

The full expansion of each rod is:

$$\Delta L_i = \Delta L_{0i} + \Delta L_{\sigma i} = \left(\alpha_i \Delta T - \frac{\sigma}{Y_i}\right) L_i$$

and we must also have:

$$\Delta L_s + \Delta L_a = 0$$

Hence:

$$\left(\alpha_s \Delta T - \frac{\sigma}{Y_s}\right) L_s + \left(\alpha_a \Delta T - \frac{\sigma}{Y_a}\right) L_a = 0$$

$$\sigma = \frac{\alpha_s L_s + \alpha_a L_a}{L_s Y_s^{-1} + L_a Y_a^{-1}} \Delta T$$

4. [17.95] At very low temperatures the molar heat capacity of rock salt varies with temperature according to Debye's T^3 law:

$$C(T) = k \frac{T^3}{\Theta^3}$$

where $k = 1940\,\text{J/mol K}$ and $\Theta = 281\,\text{K}$. (a) How much heat is required to raise the temperature of 1.50 mol of rock salt from 10.0 K to 40.0 K? (b) What is the average molar heat capacity in this range? (c) What is the true molar heat capacity at 40.0 K?

- (a)

$$dQ = nC(T)dT$$

$$Q = n \int_{T_1}^{T_2} dT\, C(T) = n \int_{T_1}^{T_2} dT\, k \frac{T^3}{\Theta^3} = \frac{nk}{4\Theta^3}(T_2^4 - T_1^4) = 83.6\,\text{J}$$

- (b)

$$\bar{C} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} dT\, C(T) = \frac{Q}{n(T_2 - T_1)} = \frac{k}{4\Theta^3} \frac{T_2^4 - T_1^4}{T_2 - T_1} = 1.9\,\text{J/mol K}$$

- (c)

$$C(T) = k\frac{T^3}{\Theta^3} = 5.6\,\mathrm{J/mol\,K}$$

5. [17.99] For your cabin in the wilderness, you decide to build a primitive refrigerator out of styrofoam (with thermal conductivity $k = 0.027\,\mathrm{W/mK}$), planning to keep the interior cool with a block of ice that has an initial mass of 24.0 kg. The box has dimensions of 0.500 m $\times$ 0.800 m $\times$ 0.500 m. Water from melting ice collects at the bottom of the box. Suppose the ice block is at 0.00^o C and the outside temperature is 21^o C. If the top of the empty box is never opened and you want the interior of the box to remain at 5^o C for exactly one week, until all the ice melts, what must be the thickness of the styrofoam?

   - $m_i = 24.0\,\mathrm{kg}$, $V_i = 0.2\,\mathrm{m}^3$, $A_i = 2.1\,\mathrm{m}^2$, $T_i = 0^o\,\mathrm{C}$, $T_f = 5^o\,\mathrm{C}$, $T_o = 21^o\,\mathrm{C}$, $t = 604800\,\mathrm{s}$. The rate of heat intake through styrofoam walls of thickness d is:

$$H = \frac{dQ}{dt} = \frac{kA_i}{d}(T_o - T_f)$$

   If the fridge temperature is not to change, all this heat must be absorbed by the ice to melt it, and by the collected water to warm it up to the fridge temperature T_f:

$$Q = m_i L_i + m_i c_w (T_f - T_i)$$

   where the latent heat of fusion for ice is: $L_i = 334 \times 10^3$ J/kg and specific heat of water is $c_w = 4190$ J/kg K. So:

$$Q = m_i\Big[L_i + c_w(T_f - T_i)\Big] = \frac{kA_i}{d}(T_o - T_f)t$$

$$d = \frac{kA_i(T_o - T_f)t}{m_i\Big[L_i + c_w(T_f - T_i)\Big]} = 6.4\,\mathrm{cm}$$

$$??? \quad d = 2.5\,\mathrm{cm}$$

6. [17.101] A copper calorimeter can ($c_c = 390$ J/kg K) with mass 0.446 kg contains 0.0950 kg of ice. The system is initially at 0^o C. (a) If 0.0350 kg of steam at 100.0^o C and 1.0 atm pressure is added to the can, what is the final temperature of the system? (b) At the final temperature, how many kilograms are there of ice, liquid water and steam?

   - Ice melts when steam is added, steam cools and partially condenses. If there is any ice left at the end, the final equilibrium temperature is $T_0 = 0^o$ C. If there is any steam left, the final equilibrium temperature is $T_s = 100.0^o$ C. Assume that all ice melts and all steam condenses. Then, the can gets:

$$Q_c = m_c c_c (T - T_i)$$

   heat. The conversion of steam to water at temperature T releases energy:

$$Q_s = m_s\Big[L_s + c_w(T_s - T)\Big]$$

   Ice melting takes energy:

$$Q_i = m_i\Big[L_i + c_w(T - T_i)\Big]$$

The latent heats and specific heat of water are $L_i = 334 \times 10^3$ J/kg, $L_s = 2256 \times 10^3$ J/kg, $c_w = 4190$ J/kg K. So:

$$Q_s = Q_i + Q_c$$

$$m_s\Big[L_s + c_w(T_s - T)\Big] = m_i\Big[L_i + c_w(T - T_i)\Big] + m_c c_c(T - T_i)$$

$$T = \frac{m_s(L_s + c_w T_s) - m_i(L_i - c_w T_i) + m_c c_c T_i}{m_c c_c + (m_i + m_s)c_w} = 86.1^o\,\mathrm{C}$$