PHYS 262

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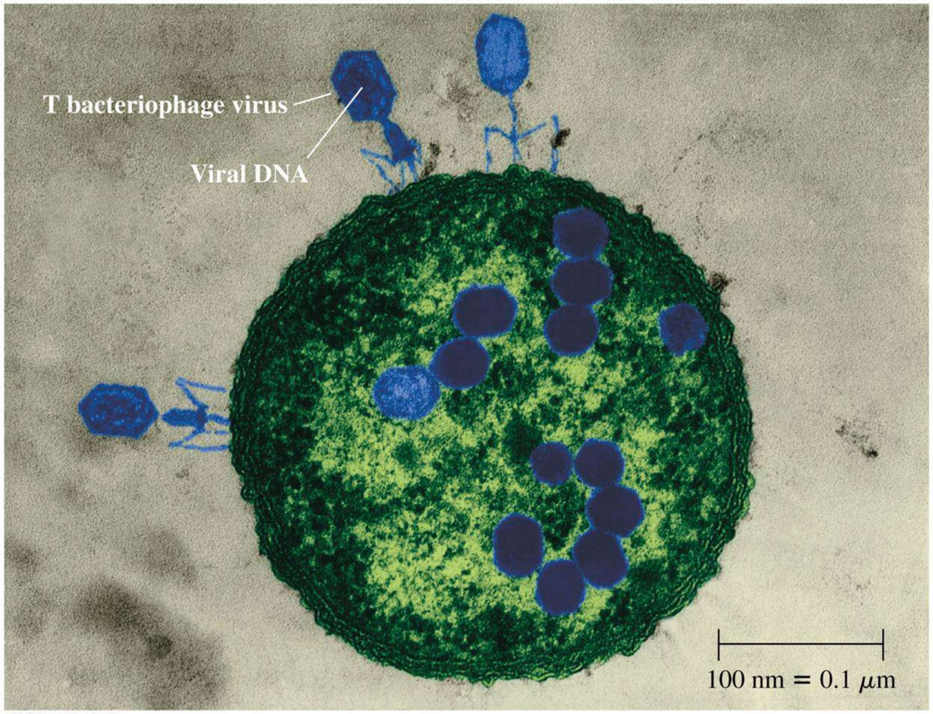
Chapter 39: Particles Behaving as Waves

- Matter Waves
- Atomic Line-Spectra and Energy Levels
- Bohr's Model of H-atom
- The Laser
- Continuous Spectra & Blackbody Radiation



Matter Waves

As we have seen, light has a *duality* of being a wave and a particle.

By a *symmetry* argument, de Broglie in 1924 proposed that *all form of matter* should also posses this duality.

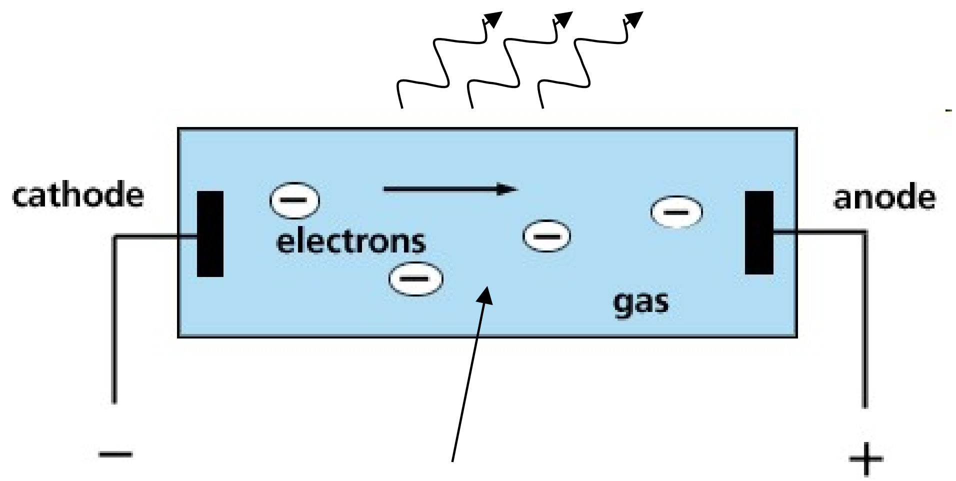
Recall for photons, we have: $\lambda = \dfrac{h}{p}$

For a massive particle with momentum $p = mv$ (or γmv) and total energy E, de Broglie proposed:

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv} \quad \left(= \frac{h}{\gamma mv} \right)} \qquad \boxed{f = \frac{E}{h}}$$

(de Broglie wavelength)

Electric Discharge Tube with Diluted Gas



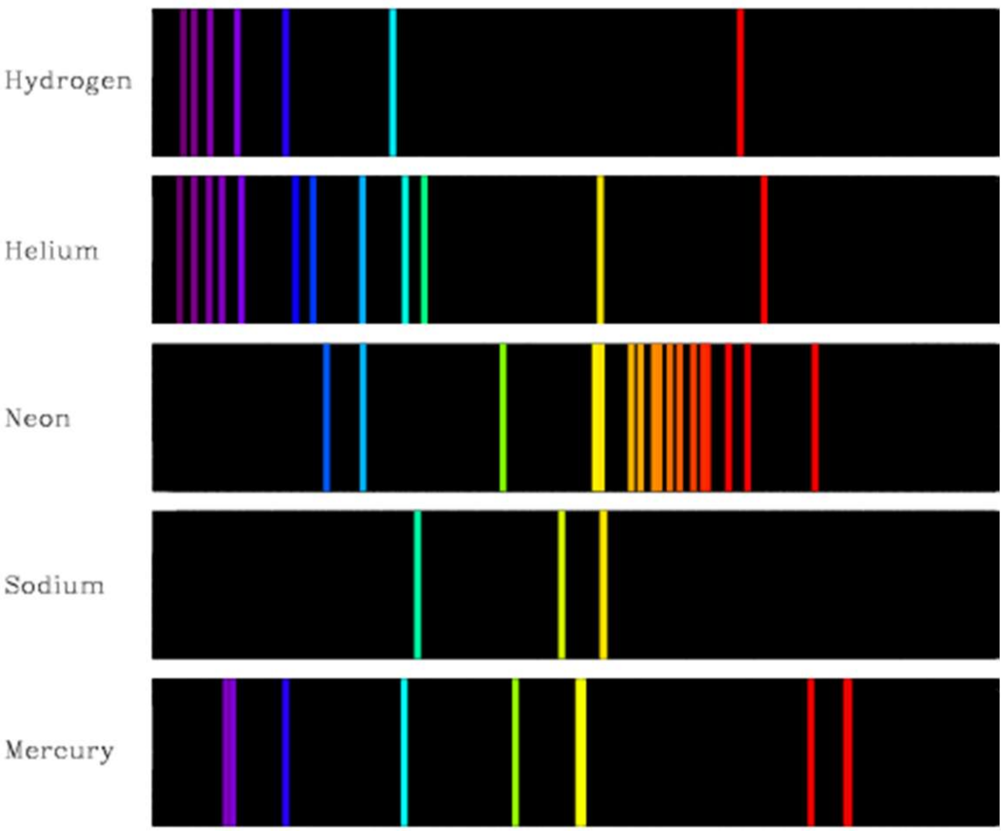
Diluted gas containing trace elements such as H, He, Na, Hg, …

Observation: Energetic electrons from cathode excite gaseous atoms in the tube, light can be emitted.

Gas is *diluted* so that the emission process is by *individual* atoms. The spectrum of light emitted are sets of unique lines characteristic of the specific type of atoms in the gas.

$D_{\text{discharge tube}}$

Emission Line Spectra

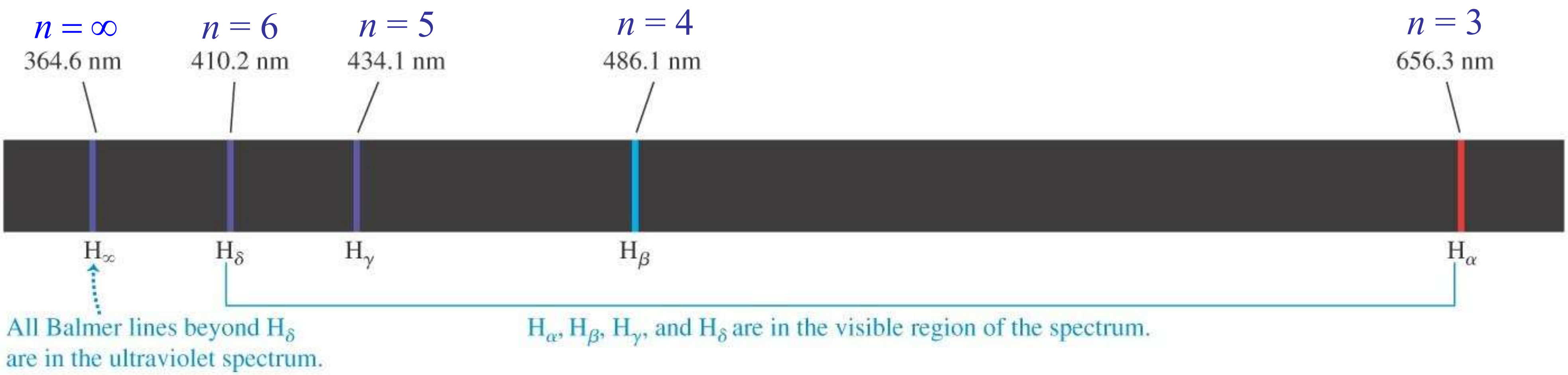


To understand the reason why the spectrum is a set of discreet lines vs a continuous spectrum,

Will require our understanding of:

- Light behaving like particles
- Electron behaving like waves

The Hydrogen Spectrum



In 1885, Johann Balmer first analyzed its H spectrum and derived an *empirical* relation to accurately describe the wavelengths in the spectrum.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5, \cdots$$

$R = 1.097 \times 10^7\, m^{-1}$ is called the **Rydberg constant** which was experimentally found to match with observed data.

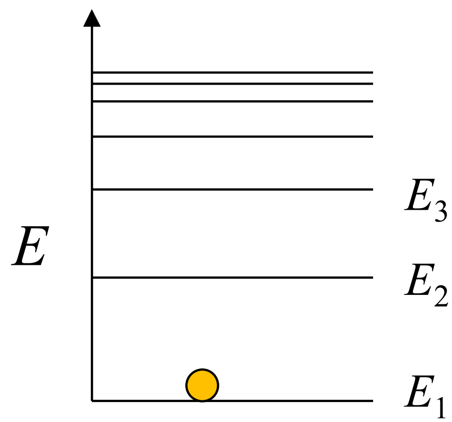
Photon Emission by Atoms

In order to explain the observed *discrete* spectra lines from atomic emissions, Niels Bohr in 1913 combined the following two central ideas in his model:

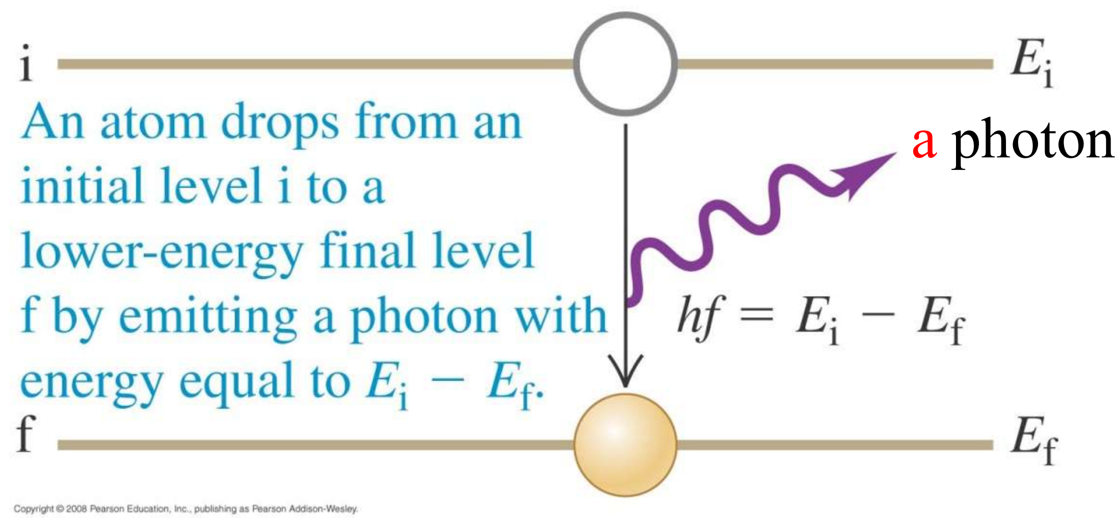
electrons as waves & **light as packets of energy**

(energy levels in atoms) ***(photons)***

Energy Levels in a typical atom



Each atom has a *specific set* of possible internal energy states. An atom can possess any one of these levels but cannot take on any *intermediate* values.



$$hf = \frac{hc}{\lambda} = E_i - E_f$$

The Hydrogen Spectrum

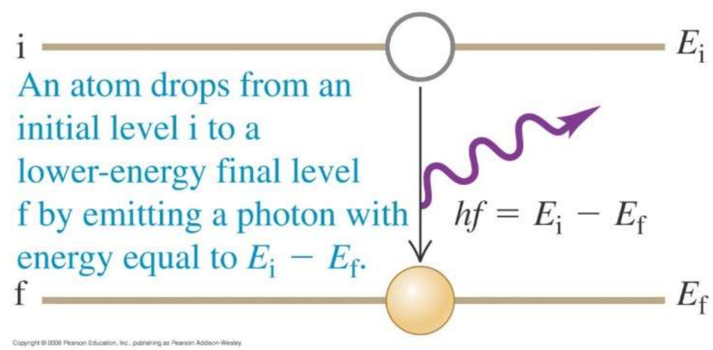
Multiplying Balmer's empirical equation by *hc*, we have

$$\Delta E = hc\frac{1}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad n = 3, 4, \cdots$$

Identifying the RHS as the difference between the energy associated to two energy levels, i.e., *i* and *f*, we can write

$$E_i = -\frac{hcR}{n^2} \qquad E_f = -\frac{hcR}{2^2}$$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f$$

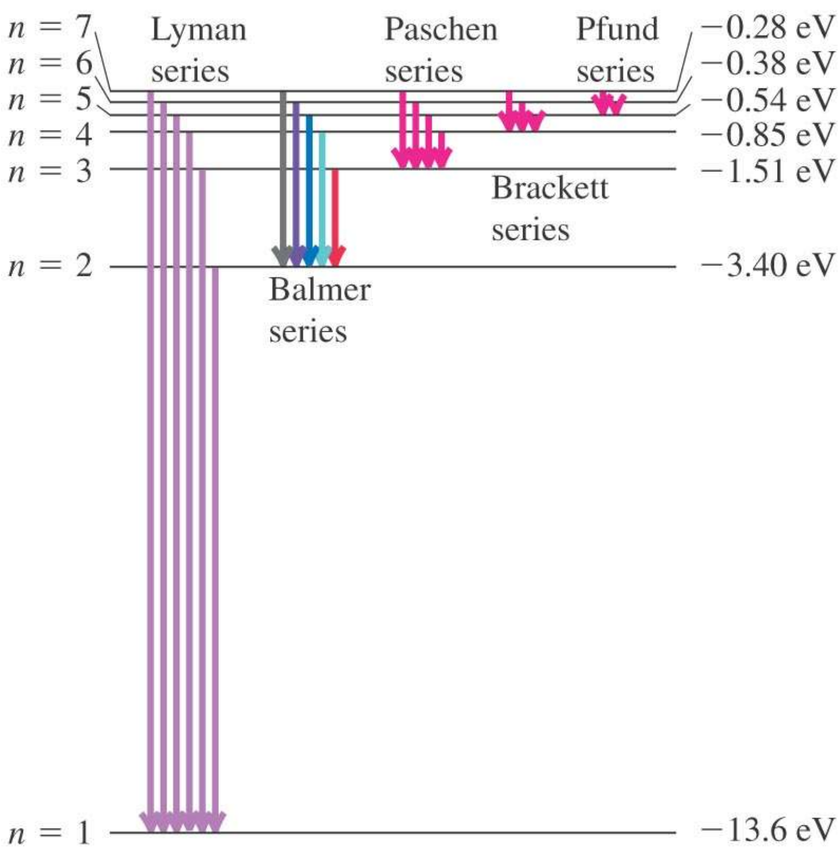


$$E_i = -\frac{hcR}{n^2}$$

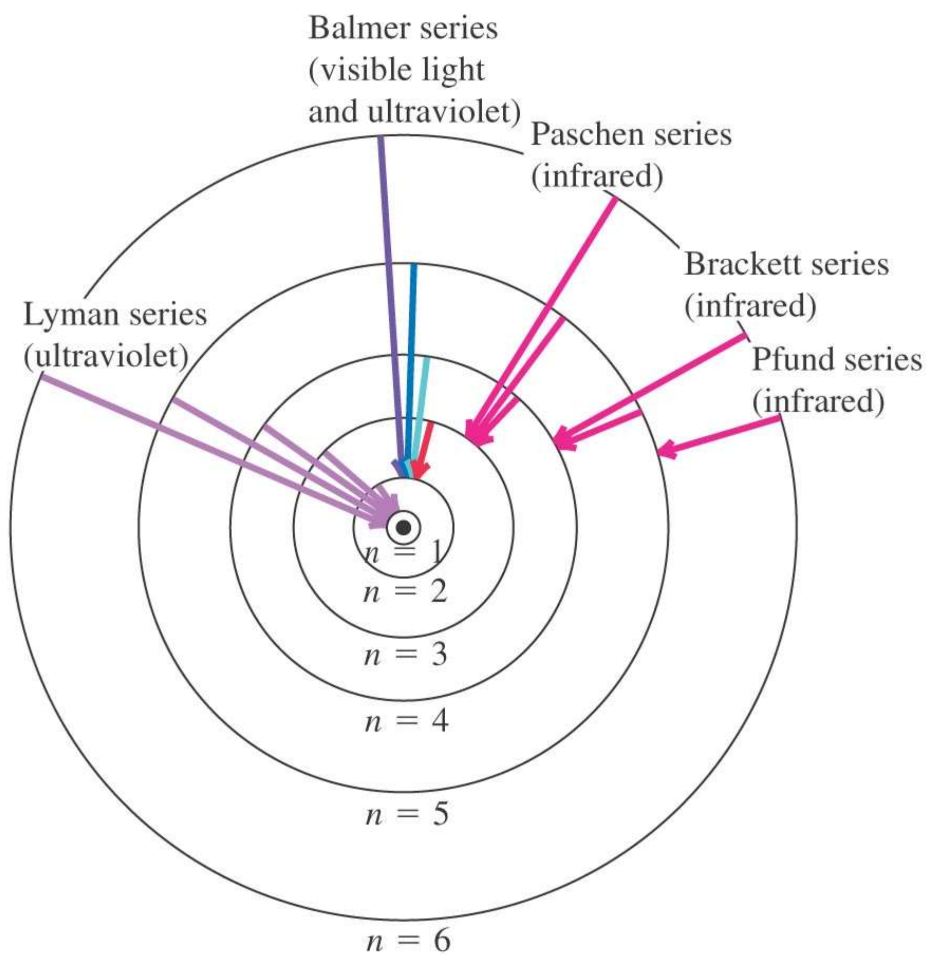
$$E_f = -\frac{hcR}{2^2}$$

The Hydrogen Spectrum

Energy-level diagram for hydrogen, showing some transitions corresponding to the various series

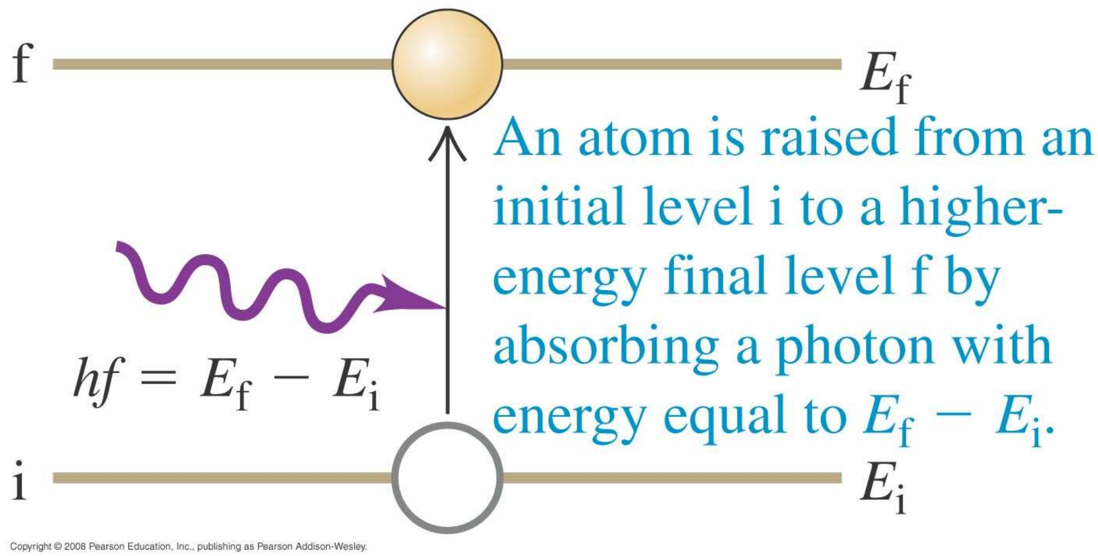


"Permitted" orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.



Photon Absorption by Atoms

In general, a photon, *emitted* when an *excited* atom makes a transition from a higher level to a lower level, can also be *absorbed* by a similar atom that is initially in the lower level.



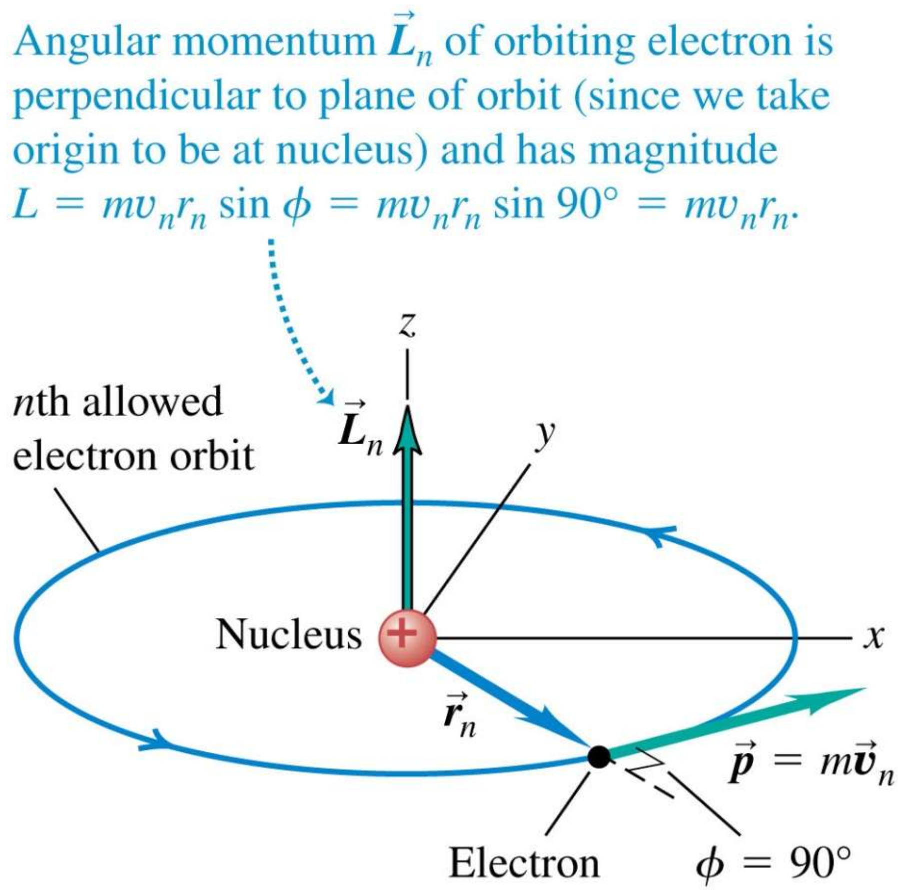
After the atom has been excited by absorbing the photon, it typically relax back to the lower energy levels within a short **lifetime** characteristic of the excited level. An excited level is called **metastable** if it has a relatively long lifetime.

The Bohr Model

It is a mixture of classical and new (quantum) ideas in trying to theoretically calculate the energy levels of a hydrogen or hydrogen-like atom.

Assumptions of the model:

1. e^- moves in **stable** circular orbits around the nucleus under the influence of Coulomb Force.
2. Atom *only* radiates when e^- jumps from a higher energy orbit to a lower one, i.e., $hf = E_i - E_f$.
3. Only certain circular orbits are allowed: angular momentum of e^- around nucleus are **quantized**, i.e., $|\vec{\mathbf{L}}|$ must be multiples of $\hbar = h/2\pi$.



Electron Waves and the Bohr's Model

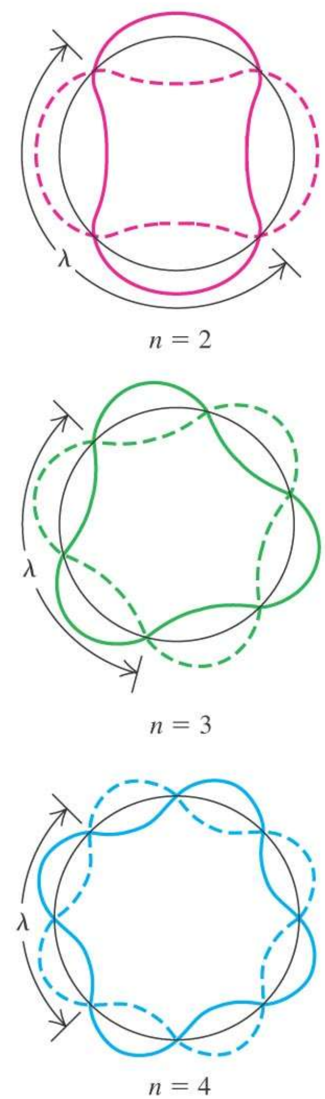
In the Bohr's model, angular momentum of the electron in a particular Bohr's orbit is **quantized**.

$$L_n = mv_n r_n = n\frac{h}{2\pi} = n\hbar, \quad n = 1, 2, 3, \cdots$$

n is called the **principal quantum number** for the orbit.

Quantized means that waves must match along orbit.

Using the de Broglie wave hypothesis, one can imagine an electron as a **standing wave** in a given energy state *n*.



Bohr's Model (mathematical details)

$n = 1$ gives the smallest orbit for the Bohr atom and it is called the **Bohr's radius**,

$$a_0 = r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2} = 5.29 \times 10^{-11} m$$

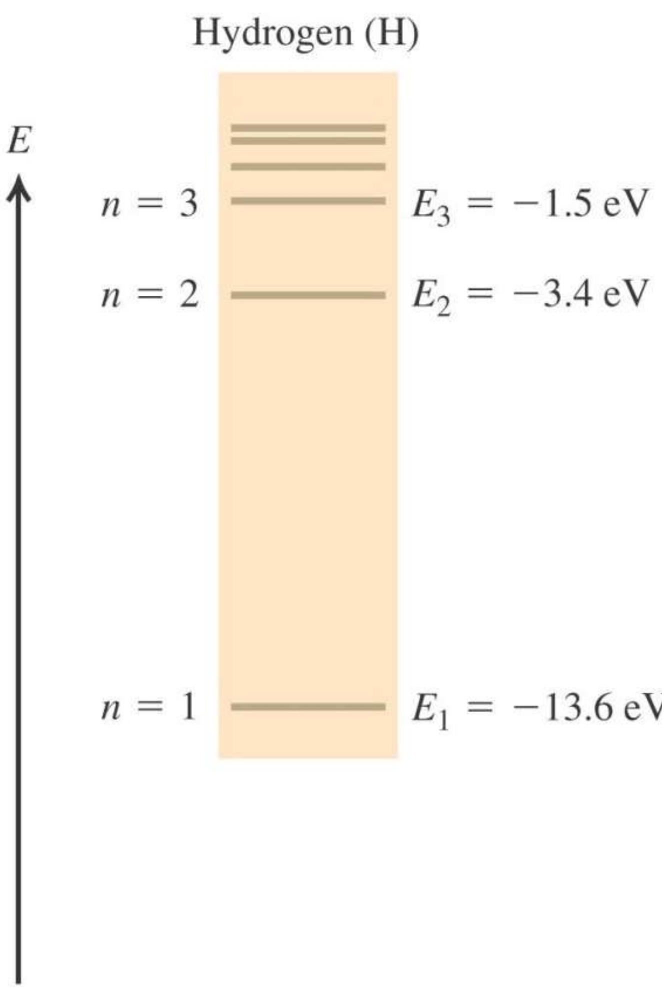
With this fundamental length scale for an atom, the other radii can be written as,

$$r_n = a_0 n^2, \quad n = 1, 2, 3, \cdots$$

Energy levels are also *quantized* as a consequence of the electron behaving as a wave in the atom (angular momentum quantization),

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = \frac{-13.6eV}{n^2}$$

Bohr's Model



- The **ground state energy** of the H-atom is given by E_1 = -13.6 eV when n = 1

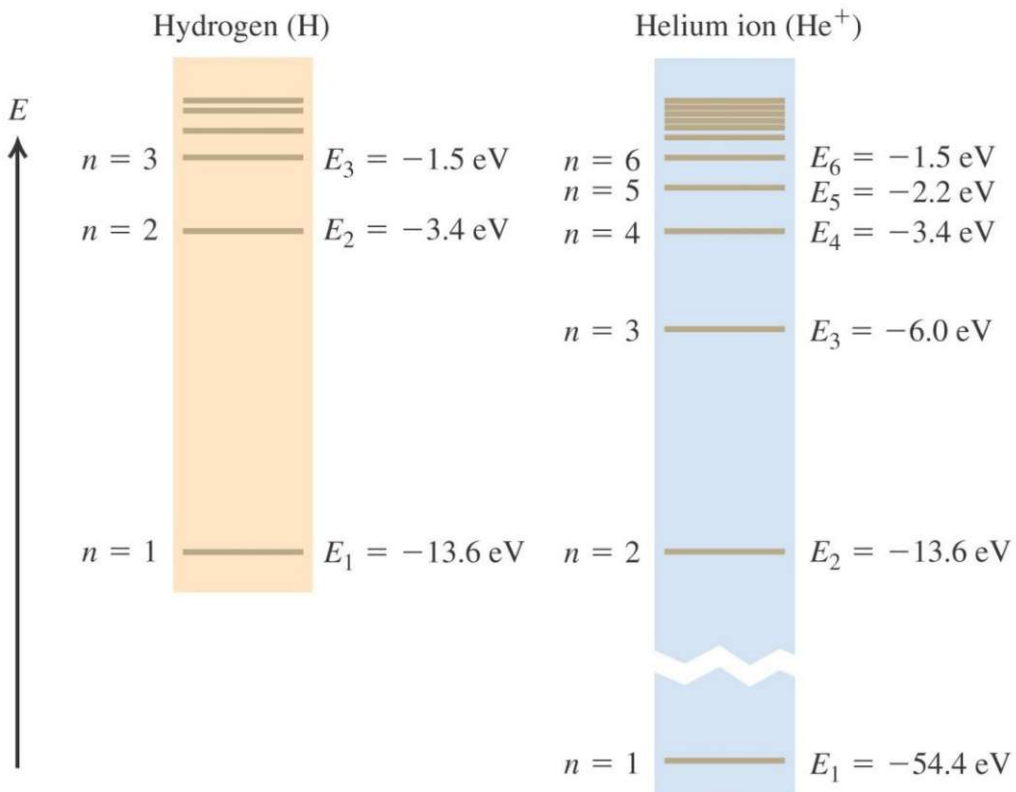
- The energy required to remove an electron completely is given by the transition from

$$n = 1 \quad to \quad n = \infty$$

and it is called the **ionization energy**

$$E_{ionization} = E_\infty - E_1 = 13.6eV$$

Hydrogen-like Atoms



Singly ionized helium (He+), doubly ionized lithium (Li2+) are examples of hydrogen-like atoms with a *single* electron around the nucleus.

For hydrogen-like atoms, e^2 in all equations from previous analysis is replaced by Ze^2, where Z is the *atomic number* of the element.

$$r_n \rightarrow r_n / Z$$

$$E_n \rightarrow Z^2 E_n$$

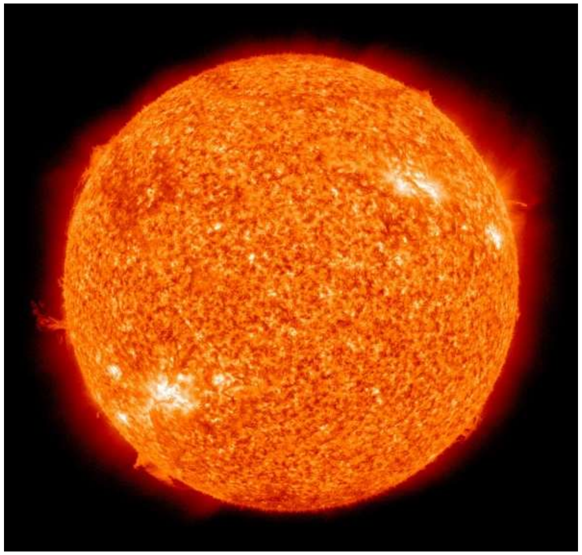
Blackbody Radiation

The total intensity (per unit surface area) of emitted light by a blackbody at absolute temperature T is given by the **Stefan-Boltzmann Law**:

$$I = \sigma T^4$$

where σ is call the **Stefan-Boltzmann constant** and it has the following value:

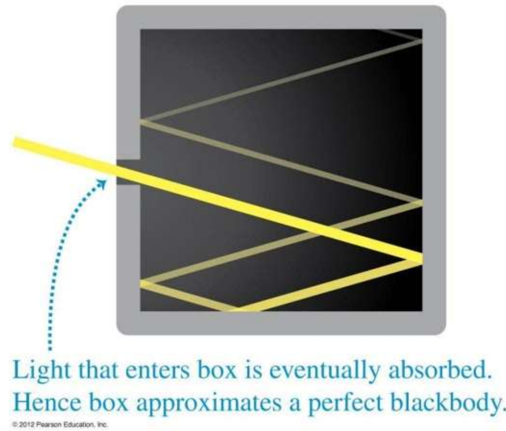
$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$$

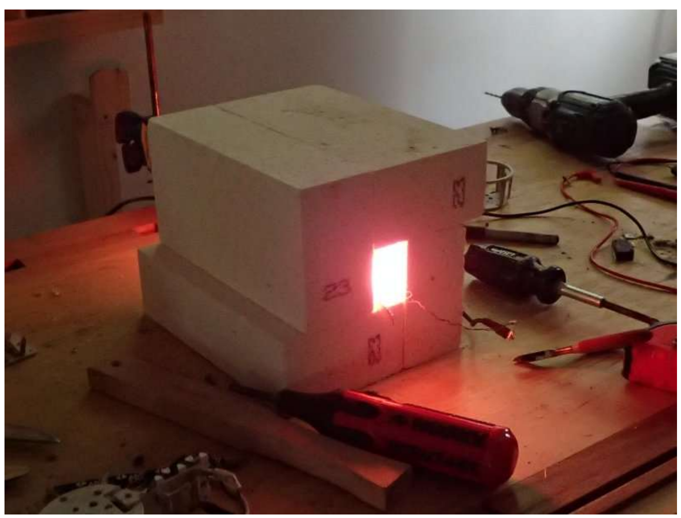


The Sun

Blackbody Radiation: Cavity Radiation

An example of a “blackbody” is a cavity with a small opening !

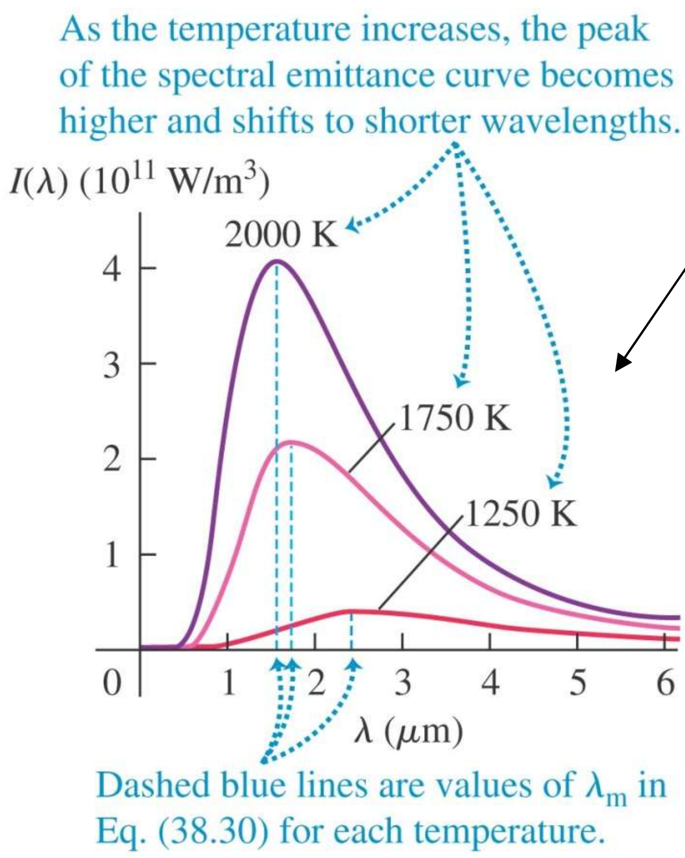




Electric Forge

Blackbody Radiation: Spectral Emittance

The intensity is not uniformly distributed over all wavelengths. The intensity distribution for a given range of wavelength is called the *spectral emittance* $I(\lambda)$.



(Experimental observed spectral emittance for different T's.)

Note: as expected, $I = \int_0^\infty I(\lambda)d\lambda$

Planck's Blackbody Radiation Law

$$I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}$$

Heisenberg's Uncertainty Principle

In nature, space-time coordinates are linked to its dynamical counterparts as **conjugate** variable pairs in physics.

$$(p_x, x) \quad (p_y, y) \quad (p_z, z) \quad \text{and} \quad (E, t)$$

And, most importantly, the Heisenberg's Uncertainty Principle enforces an *inverse* proportional relation on the two *conjugate* pairs of dynamics variables:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}, \quad \Delta y \Delta p_y \geq \frac{\hbar}{2}, \quad \Delta z \Delta p_z \geq \frac{\hbar}{2}, \quad \Delta E \Delta t \geq \frac{\hbar}{2}$$

By decreasing the uncertainty in one of the variables (x or t), its corresponding **conjugate** variable$(p_x \text{ or } E)$ must increase accordingly !

But, there are **no** restrictions for *unconjugated* variables: $\Delta x \Delta p_y \;\; or \;\; \Delta x \Delta y, etc.$