PHYS 262

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Chapter 38: Light Waves Behaving as Particles

- Photoelectric Effects
- X-ray Production
- Compton Scattering & Pair Production
- Heisenberg Uncertainty Principle
- Wave/Particle Duality

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Quantum Nature of Light

Two seemingly paradoxical nature of light (EM waves):

→ By the end of the 19^{th} century, most physicists (Maxwell, Hertz, and others) have firmly established that electromagnetic waves are waves which exhibit interference and diffraction (Ch. 35-36).

→ But newer experiments on the emissions and absorptions of EM waves have shown behaviors which CANNOT be explained with light being a wave… It requires a radical new thinking of light as *quantized packets of energy* called **photons** (as particles).

Max Planck's Hypothesis of Quantized Light (Photon)

Light is quantized! Each packet of light (a photon) has energy depending on its frequency:

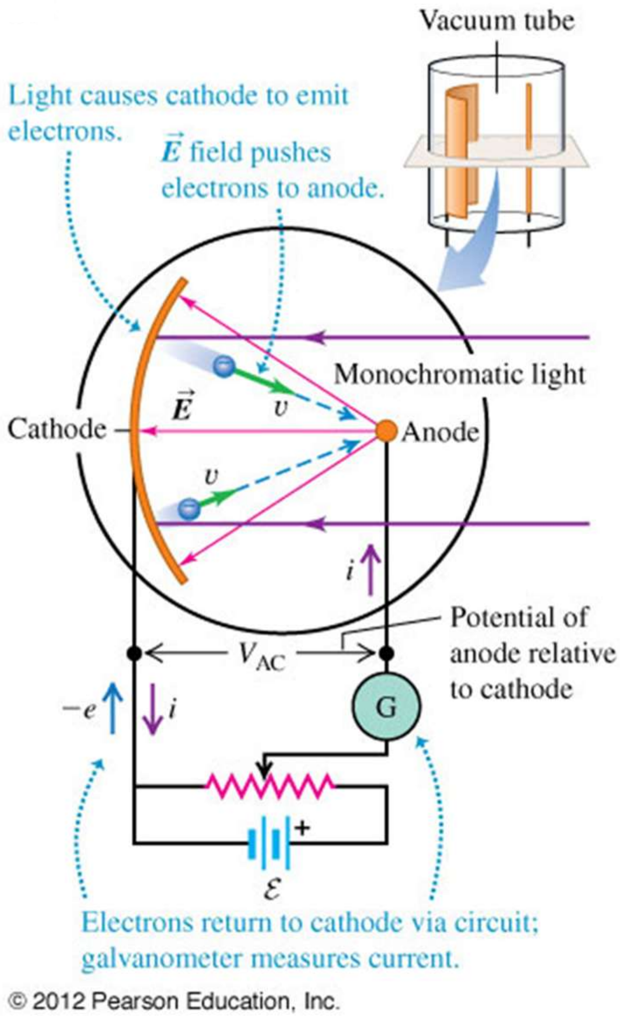
$$E = hf = \frac{hc}{\lambda}$$

where $h = 6.626 \times 10^{-34}\, J \cdot s$ is a universal constant called **Planck's Constant**.

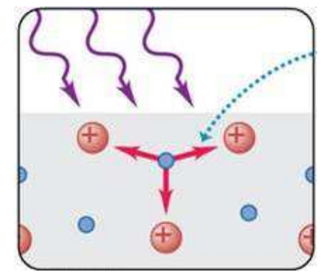
Note the smallness of this number.

The Photoelectric Effect

An experimental demonstration of the *particle* nature of light.

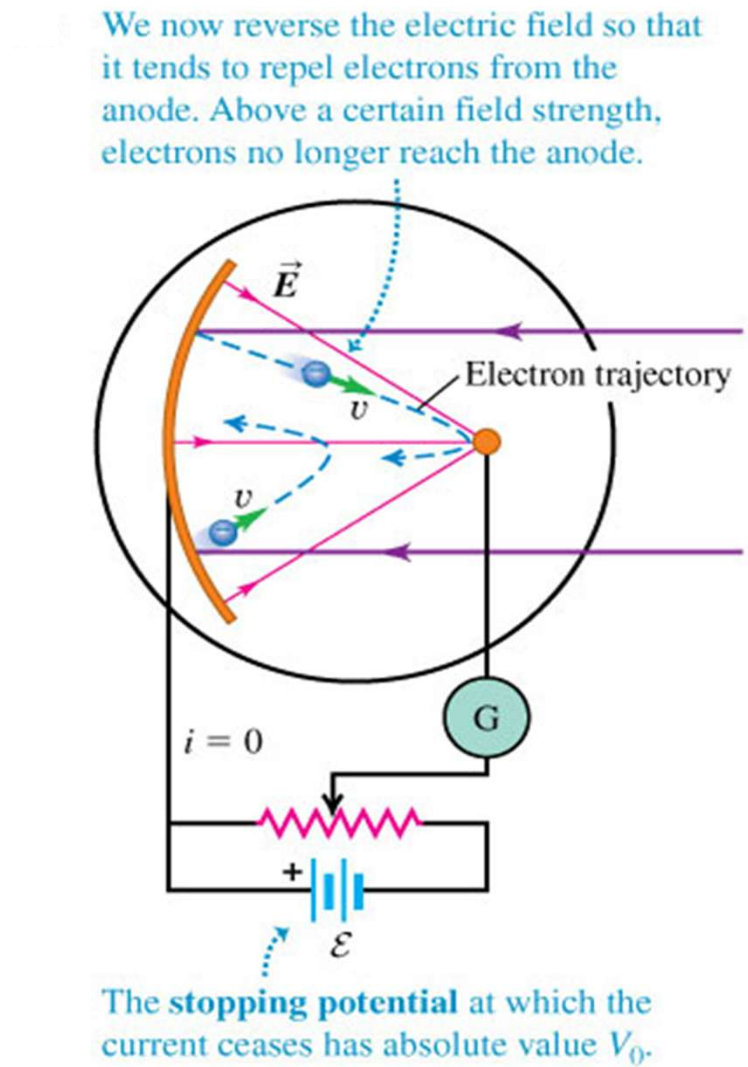


Electrons on the metal surface (cathode) are normally bounded to the positive ions on the surface.



The potential energy which an e needs to escape the surface is call the **work function** ϕ

The Photoelectric Effect



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When V_{AC}(reversed) $\rightarrow -V_0$

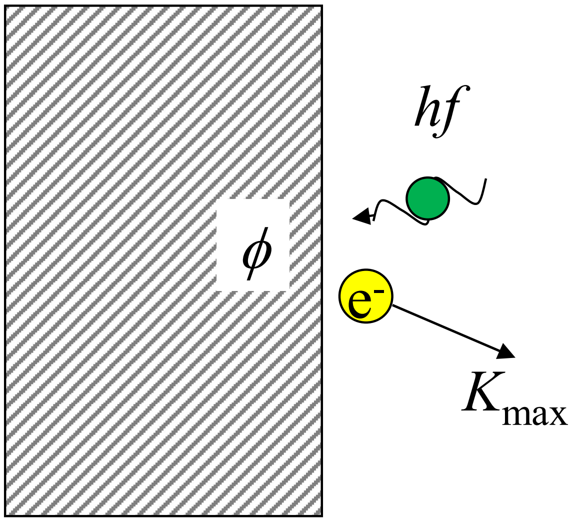
Above a certain potential strength V_0 , NO e- can reach the anode!

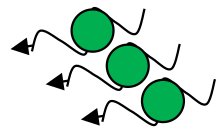
The minimum V_0 needed to stop all e- getting across to the anode is called the **stopping potential** V_0 and

V_0 is basically a direct measurement of the maximum KE (K_{max}) of these electrons and they are related by,

$$K_{max} = eV_0$$

Einstein's Photon Explanation





Incident light as a collection of photons (particles)

- Each photon has energy hf
- Intensity of light ~ # of photons

The interaction is an *all-or-none* process. **Electrons bounded to the surface of the metal can absorb a single photon at a time or none at all.** If hf is large enough to overcome ϕ, an electron will be ejected with kinetic energy $K_{\max}$.

By energy conservation, we have: $K_{\max} = hf - \phi$ $\quad$ ϕ depends on the metal surface

$$eV_0 = hf - \phi$$

Notes

- Convenient Energy Units:

1eV = energy required to move one unit of charge across an electric potential of 1 V.

$$1\text{eV} = \left(1.602\times10^{-19}C\right)\left(1V\right) = 1.602\times10^{-19}J$$

$$h = 6.626\times10^{-34}J\cdot s\left(\frac{1eV}{1.602\times10^{-19}J}\right) = 4.136\times10^{-15}eV\cdot s$$

$$hc = 4.136\times10^{-15}eV\cdot s\left(3.00\times10^{8}m/s\right) = 1.241\times10^{-6}eV\cdot m = 1241eV\cdot nm$$

- Energy and Momentum of a Photon:

$$\boxed{P = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}}$$ (momentum of a photon)

- The duality of light (wave & particle) applied to the entire EM spectrum !

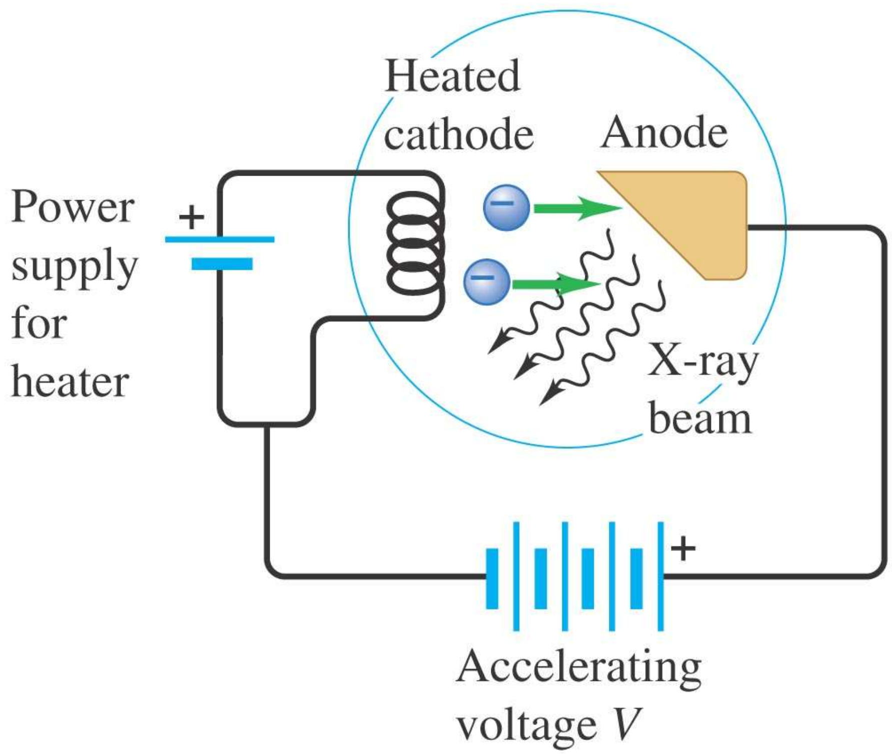
X-Ray Production

X-rays are produced when rapidly moving electrons that have been accelerated through a *large* potential difference (10^3 to 10^6 V) strike a metal target.

X-rays emission is the *inverse* of the photoelectric effect.

Photoelectric: $hf \rightarrow K$ of e
X-Ray prod: K of $e \rightarrow hf$

Electrons are emitted thermionically from the heated cathode and are accelerated toward the anode; when they strike it, x rays are produced.



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X-Ray Production: Discrete Set of λs

Classical EM → *deceleration* of high energy e^- should produce EM waves in a *broad* range of f.

BUT, the observation is that …

1. Independent of target material: *bremsstrahlung* (braking radiation) → gives maximum f (energy) or minimum λ directly proportional to the energy of the accelerated eletrons.

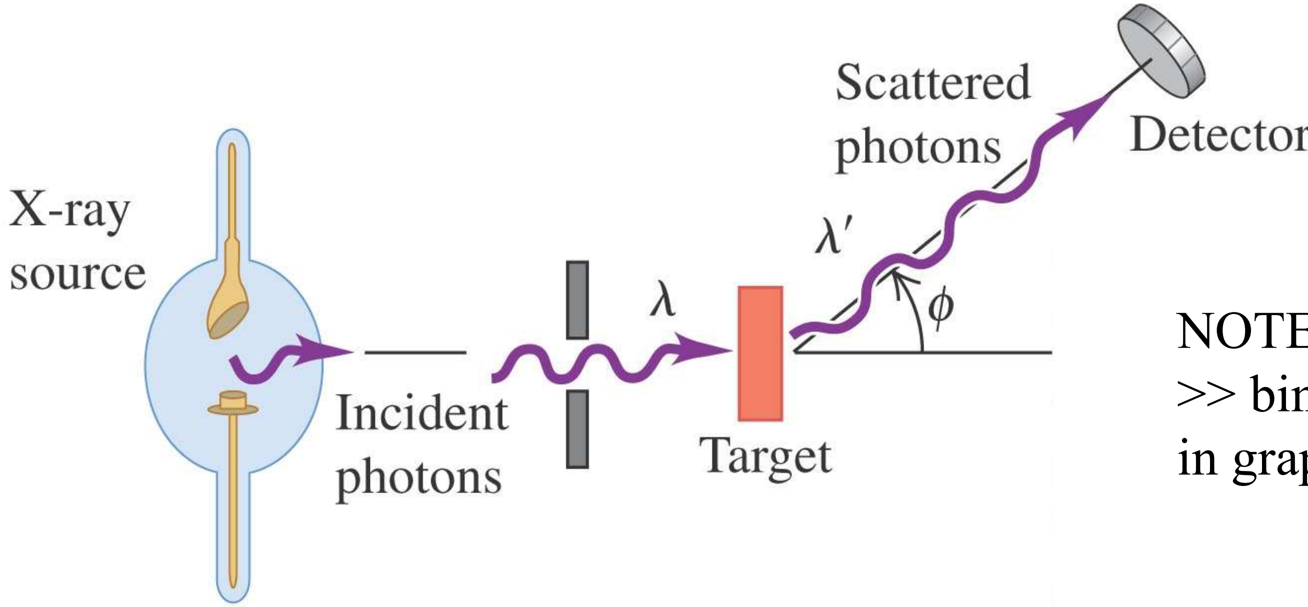
$$eV_{AC} = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

max KE of accelerated e

2. Dependent on target material, a *characteristic* spectrum of X-Rays will be emitted → electrons with sufficient KE can excite atoms in the target material. When they decay back to their ground state, light (X-Rays in this case) will be emitted.

Compton Scattering

In 1923, Arthur H. Compton provided an additional direct conformation on the quantum nature of x-rays.



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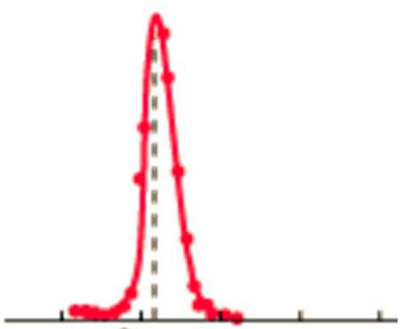
NOTE: Energy of photon >> binding energy of e^- in graphite.

- X-rays of well-defined λ are made to fall on a graphite target
- For various scattering angle ϕ, intensities of scattered x-rays are measured as a function of the wavelength.

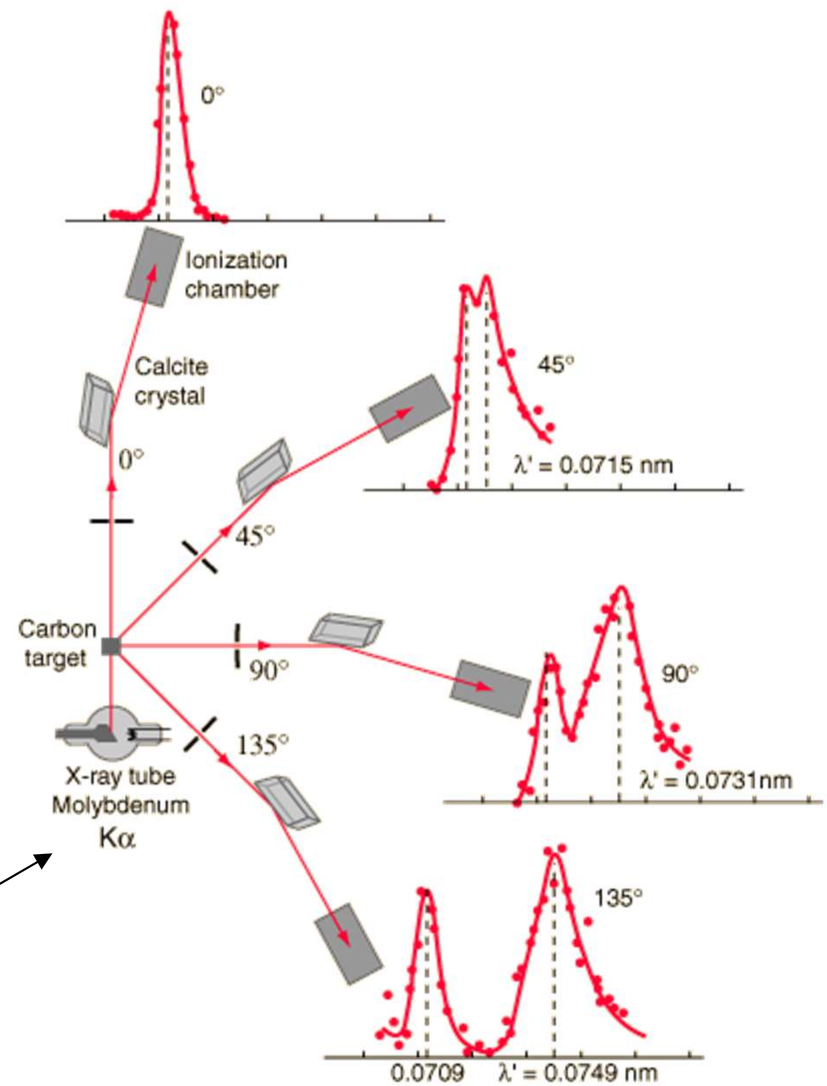
Compton Scattering

Classical prediction:

Since diff. electrons will have diff. velocity, the intensity profile for the scattered x-rays is expected to be a single peak with a spread around λ_0.

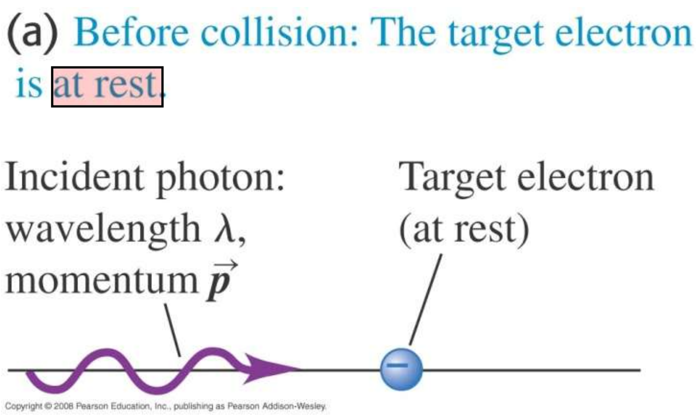


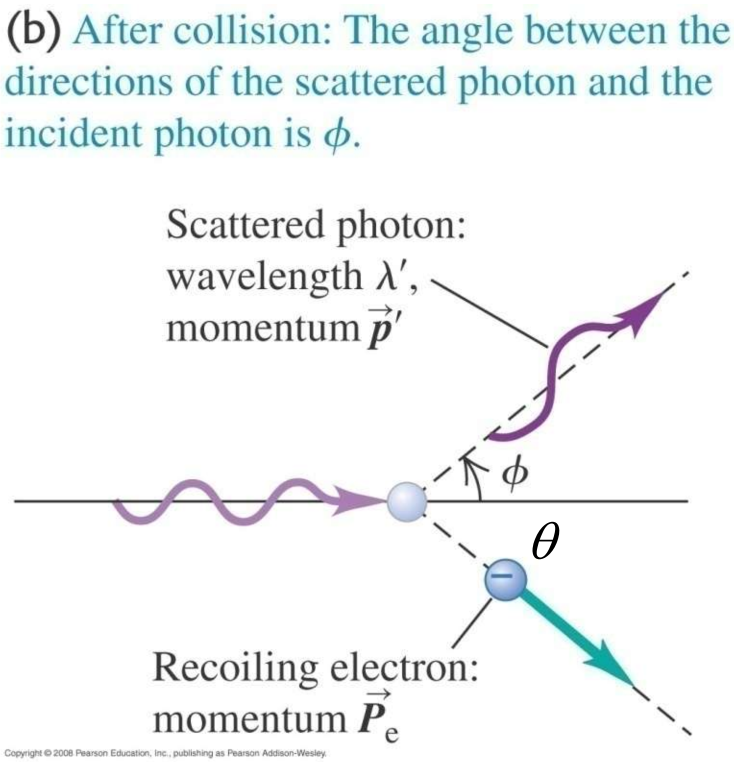
But, the actual experiment gives *two peaks* with a *Compton Shift* which depends on ϕ. Classical physics can't explain this !



Compton Scattering

Quantum Explanation: Treating scattering as a “billiard-like” collision





Recall:

$$E_{photon} = hf = \frac{hc}{\lambda}$$

$$\left|\vec{\mathbf{P}}_{photon}\right| = \frac{h}{\lambda}$$

$$E_{electron} = \gamma mc^2$$

$$\vec{\mathbf{P}}_{electron} = \gamma m\vec{\mathbf{v}}$$

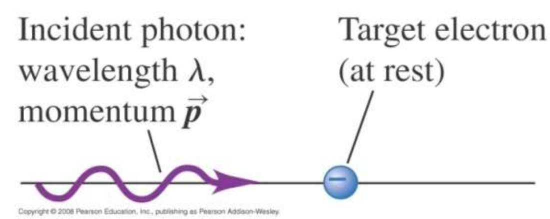
Compton Scattering

Conservation of Total Relativistic Energy:

Note: high energy collision → e's speed might be relativistic !

(before) (after)

$$\frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + \gamma mc^2$$

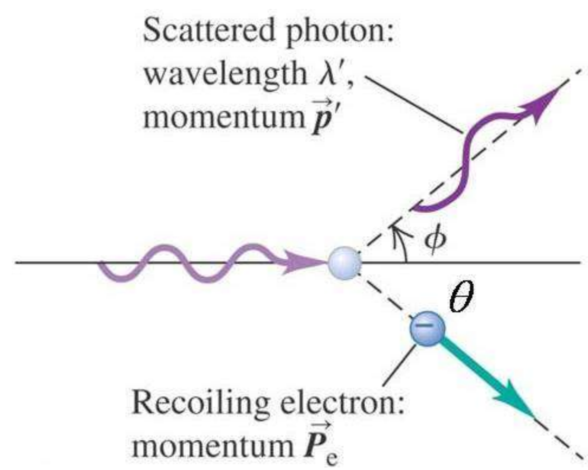


Conservation of Relativistic Momentum:

(before) (after)

$$x-dir: \quad \frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + \gamma mv\cos\theta$$

$$y-dir: \quad 0 = \frac{h}{\lambda'}\sin\phi - \gamma mv\sin\theta$$



(knowns → λ: given, ϕ: observation angle) 3 eqs. with 3 unknowns (θ, λ', v) !

Compton Scattering

Here is the result:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

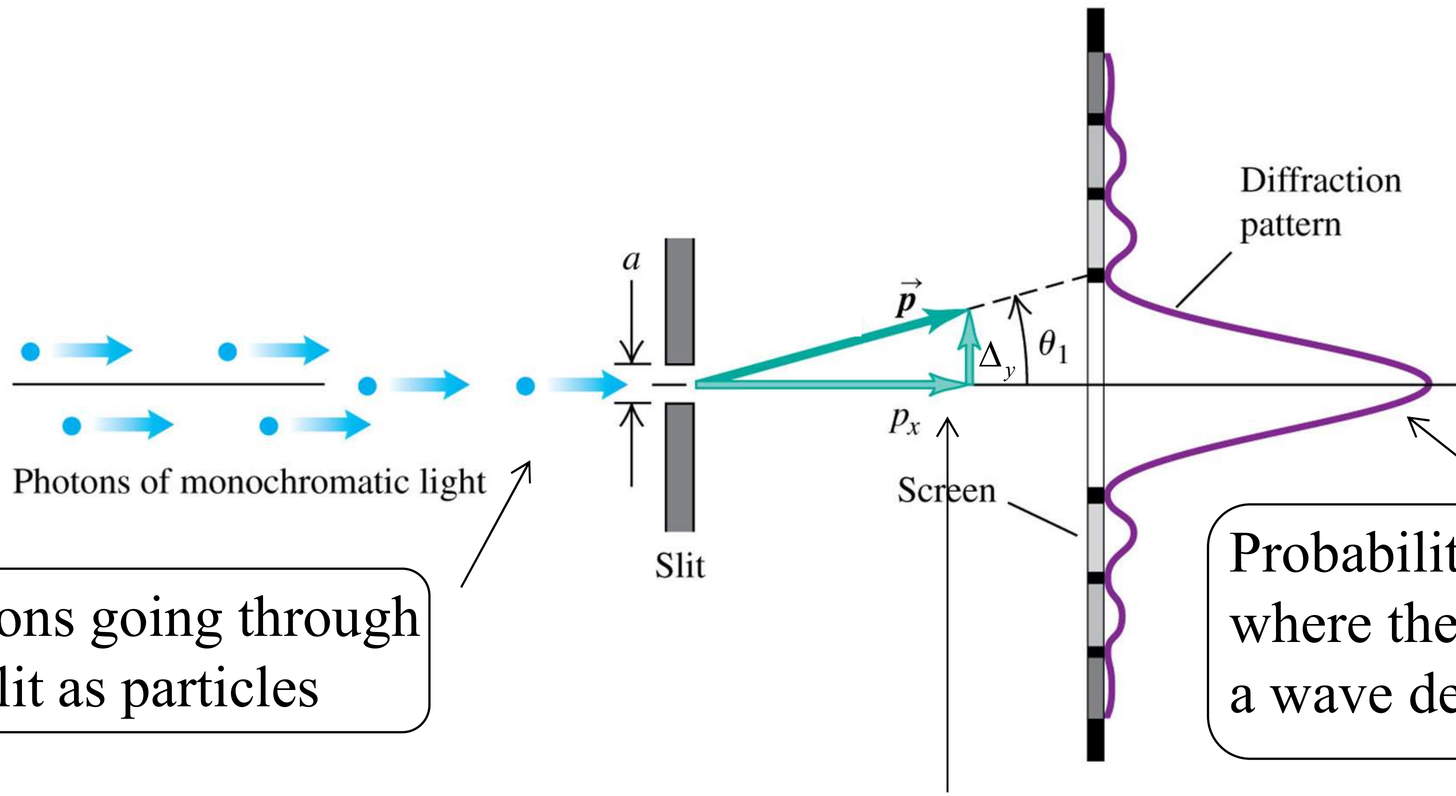
(Compton Shift Equation)

The factor h/mc has the units of length and

$$\lambda_C = \frac{h}{mc} = 0.002426nm$$

is called the **Compton Wavelength**.

Heisenberg's Uncertainty Principle



Photons going through the slit as particles

Probability in finding where they land requires a wave description

THEN the "spreading-out" of the photons implies that the photons reaching the screen must have a small *uncertainty* Δp_y *as large as* Δ_y in the vertical ($\hat{\mathbf{y}}$) direction of its momentum as it exits the slit.

Heisenberg's Uncertainty Principle

In nature, space-time coordinates are linked to its dynamical counterparts as **conjugate** variable pairs in physics.

$$\left(p_x, x\right) \quad \left(p_y, y\right) \quad \left(p_z, z\right) \quad \text{and} \quad \left(E, t\right)$$

And, most importantly, the Heisenberg's Uncertainty Principle enforces an *inverse* proportional relation on the two *conjugate* pairs of dynamics variables:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}, \quad \Delta y \Delta p_y \geq \frac{\hbar}{2}, \quad \Delta z \Delta p_z \geq \frac{\hbar}{2}, \quad \Delta E \Delta t \geq \frac{\hbar}{2}$$

By decreasing the uncertainty in one of the variables (x or t), its corresponding **conjugate** variable $\left(p_x \text{ or } E\right)$ must increase accordingly !

But, there are **no** restrictions for *unconjugated* variables: $\Delta x \Delta p_y$ *or* $\Delta x \Delta y$, *etc.*

Correspondence Principle

So, when does light acts *classically* and *quantum mechanically*?

As we have seen, the basic energy scale for light (a photon) is $\Delta E = hf$

When light interacts with matter, the typical interaction energy scale E as compares to a single photon energy ΔE determines whether light acts as a wave (CM description) or a particle (QM description):

if f is *small* (λ is long), $\Delta E = hf \ll E$ then light appears as a continuum
EM → CM (light as a wave)

If f is *large* (λ is short), $\Delta E = hf \approx E$ then light appears as discrete packets
EM → QM (light as particles)

Correspondence Principle

Radio waves (low freq): behaves like *classical* waves with diffraction and interference easily observable.

Visible lights (mid freq):
have both wave (classical) & particle (quantum) behaviors.

diffraction photoelectric effect

X-Rays (high freq): mostly particle (quantum) behaviors.

X-Rays productions and Compton Scattering

γ-Rays (very high freq): particle (quantum) behaviors.

Pair Productions and e^--e^+ Pair Annihilations (reverse of PP)