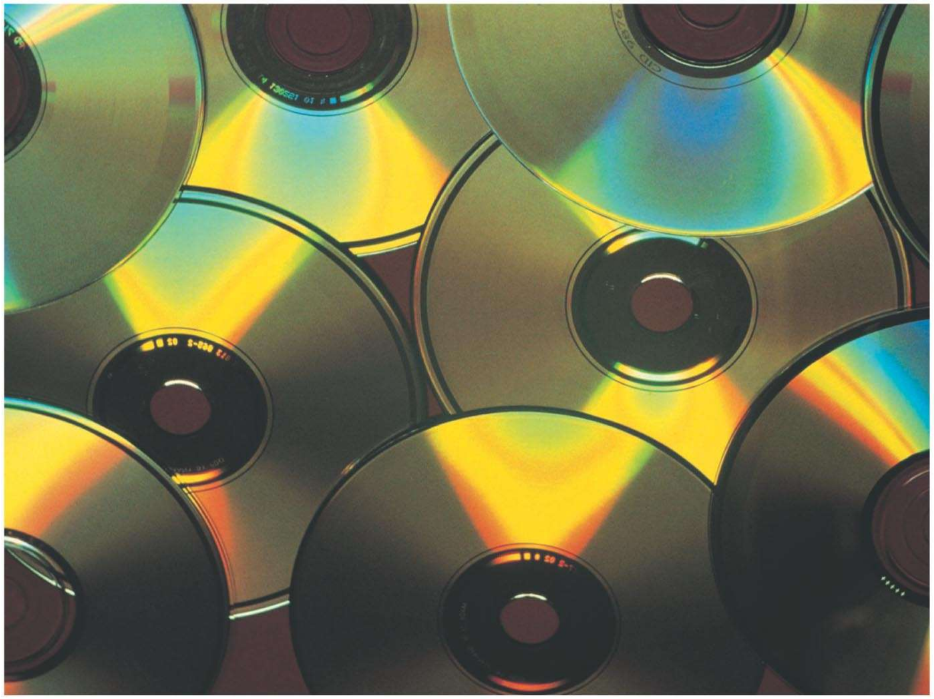
# PHYS 262

George Mason University

Prof. Paul So

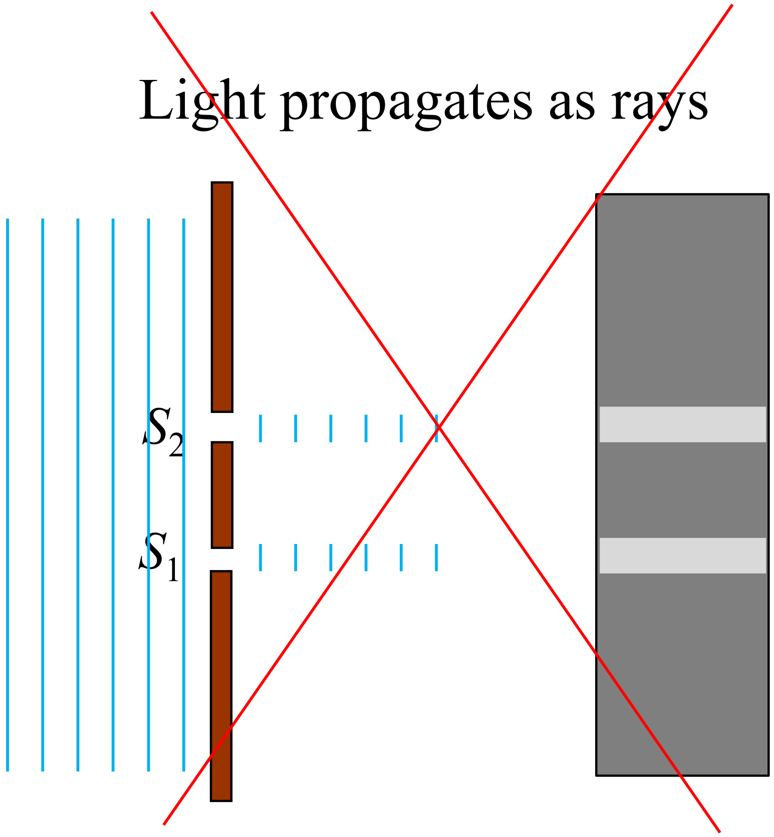
# Chapter 36: Diffraction

- Diffraction and Huygens' Principle
- Diffraction from a Single Slit
- Intensity in the Single-Slit Pattern
- Double-Slit Diffraction
- Diffraction Grating
- x-Ray Diffraction
- Resolving Power

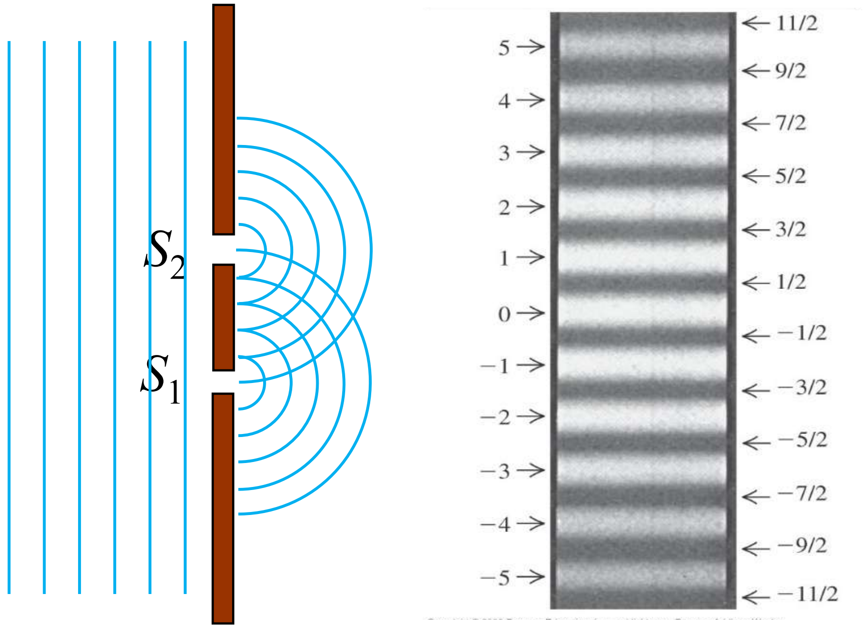


# Wave Nature of Light: Diffraction & Interference from Two Slits

~~Light propagates as rays~~

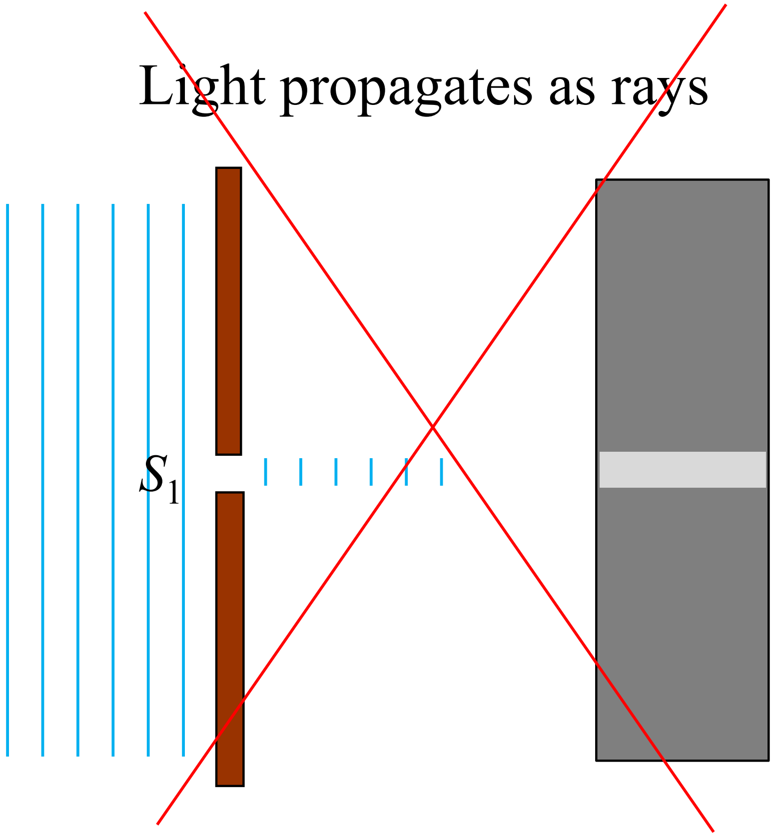


Light propagates as waves

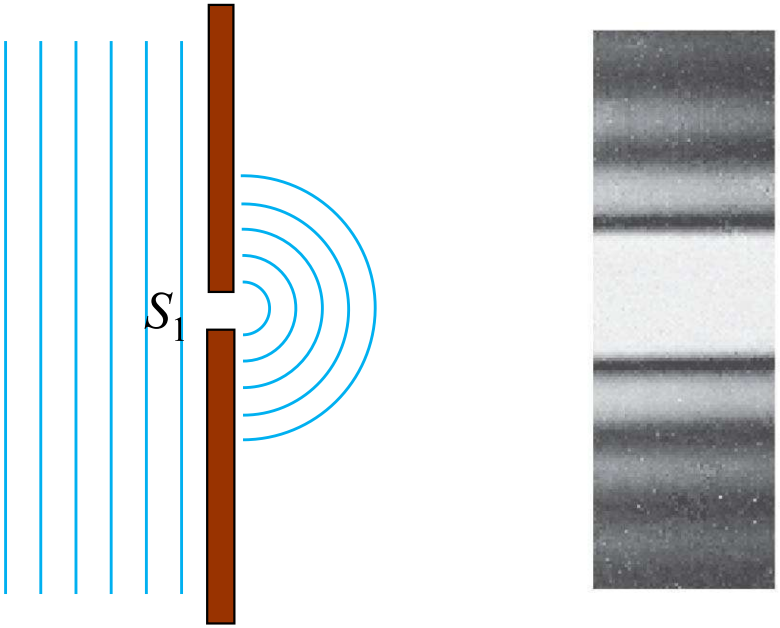


# Wave Nature of Light: Diffraction of a Single Slit

Light propagates as rays

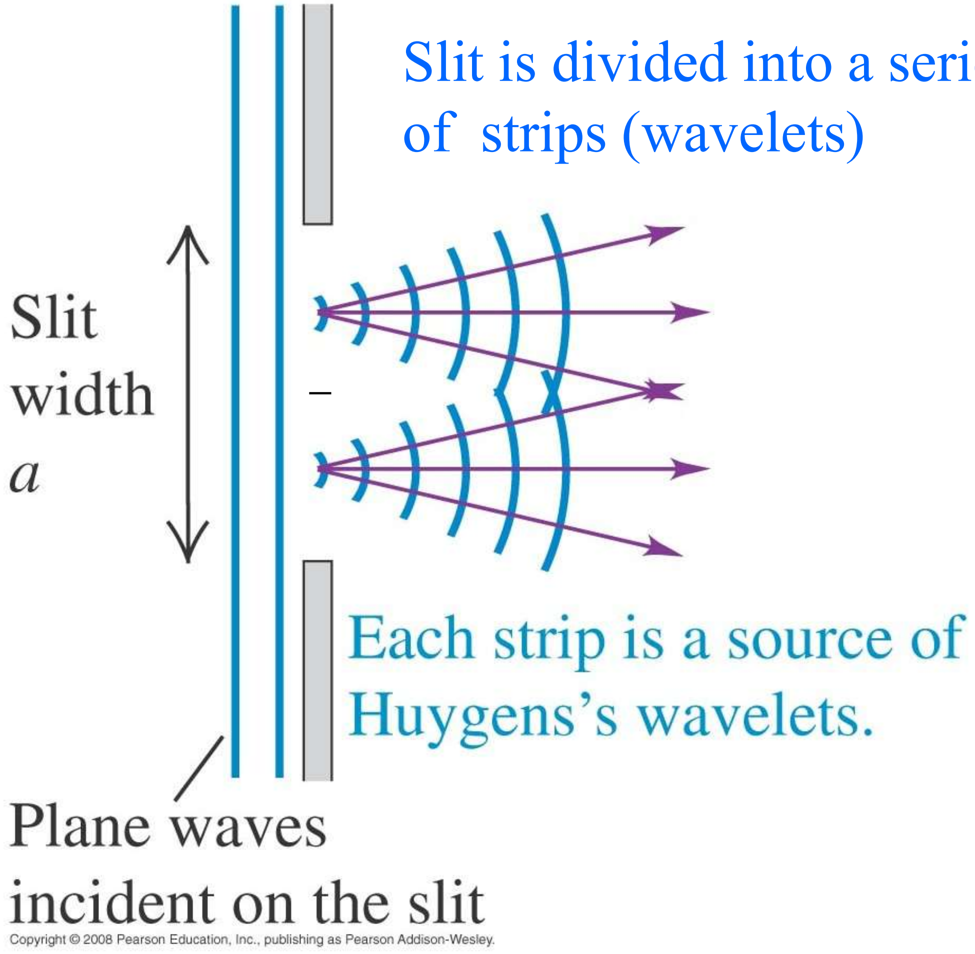


Light propagates as waves



# Diffraction and Huygen's Principle

Consider a simpler case: a single slit



Slit is divided into a series of strips (wavelets)

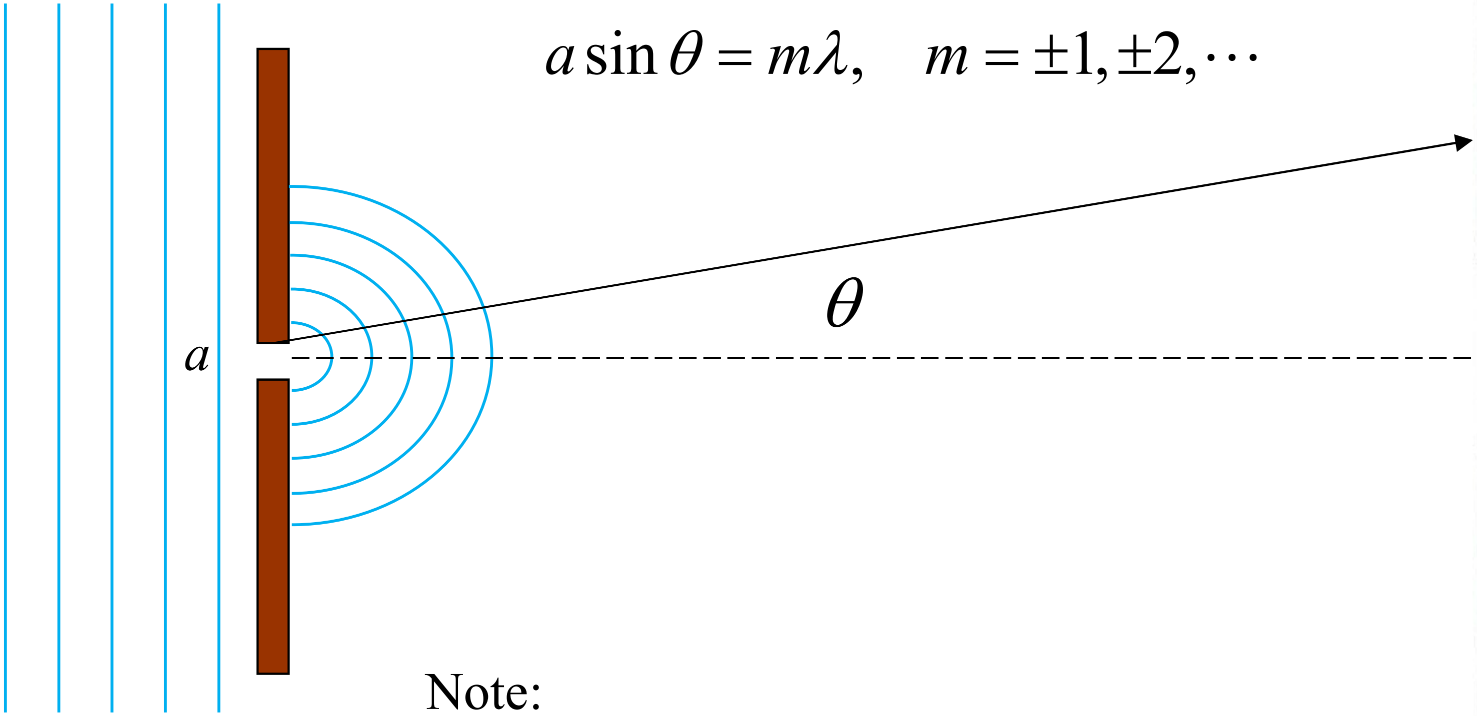
Waves spread out from each strips as wavelets creating interference patterns beyond and around sharp edges.

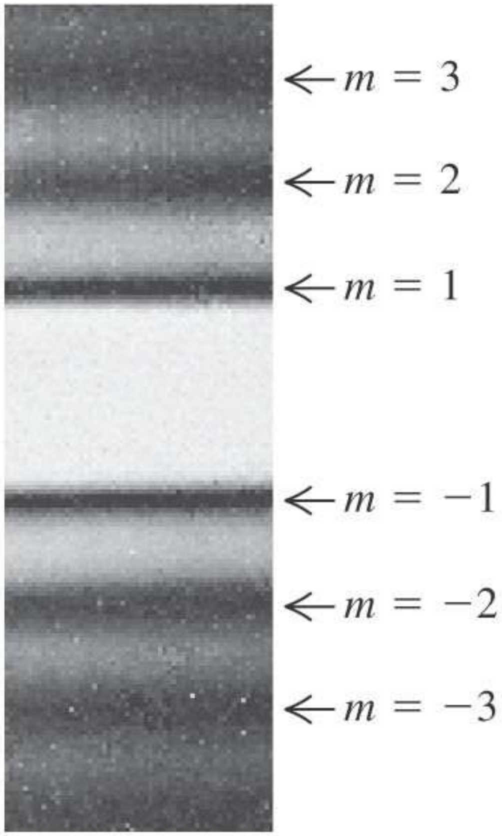
Similar to the two-source interference pattern, these wavelets interfere as they spread out and create the diffraction pattern.

# Single-Slit Diffraction: Dark Fringes

General formula for the dark fringes:

$$a\sin\theta = m\lambda, \quad m = \pm 1, \pm 2, \cdots$$

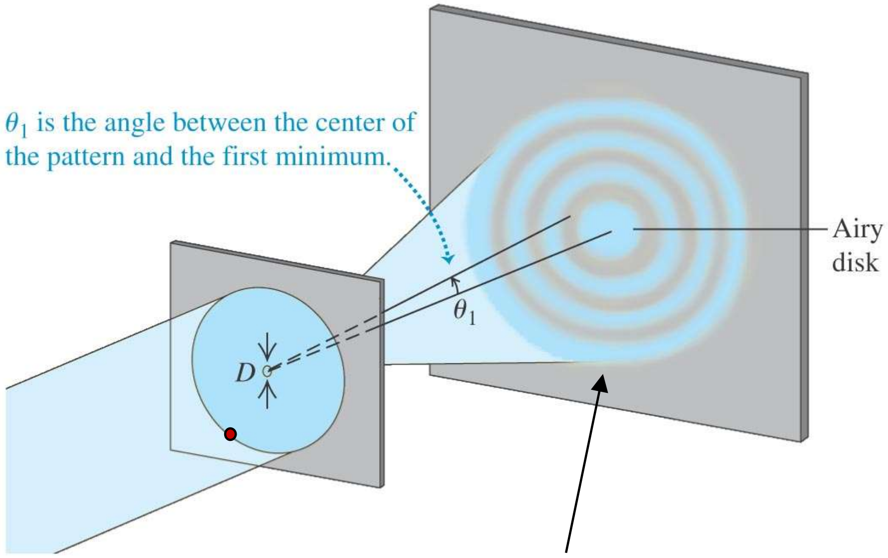




Note:

1. $m = 0$ is *not* the first minimum !
   In fact, it is the location for the central max.
2. Secondary maximum occurs *near* $3\lambda/2$, $5\lambda/2$, etc.
   but not exactly.

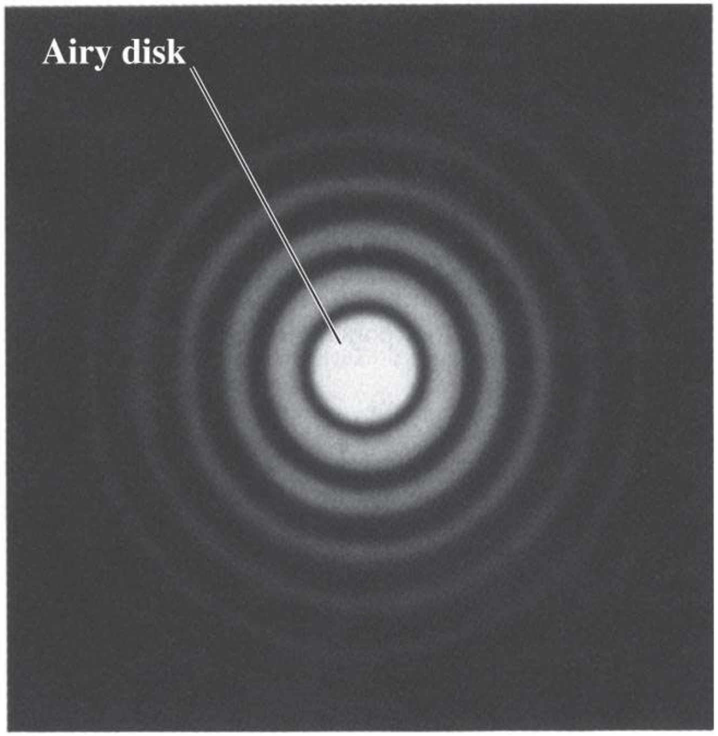
# Diffraction from a Circular Aperture



$\theta_1$ is the angle between the center of the pattern and the first minimum.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

diffraction pattern



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

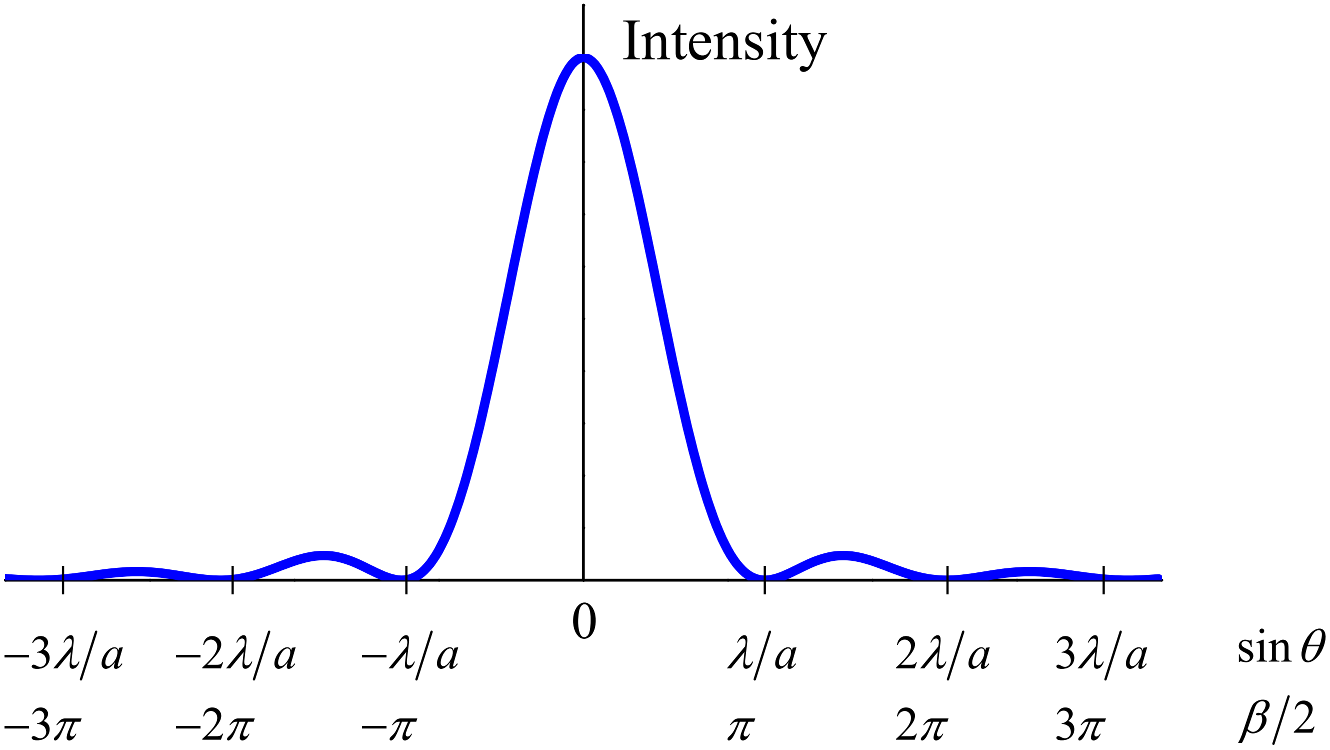
Similar to a single slit, light spreads out after passing thru circular apertures and produces a diffraction pattern on wall.

$$\sin\theta_1 = 1.22\frac{\lambda}{D}$$ (“1.22” is a geometric factor)

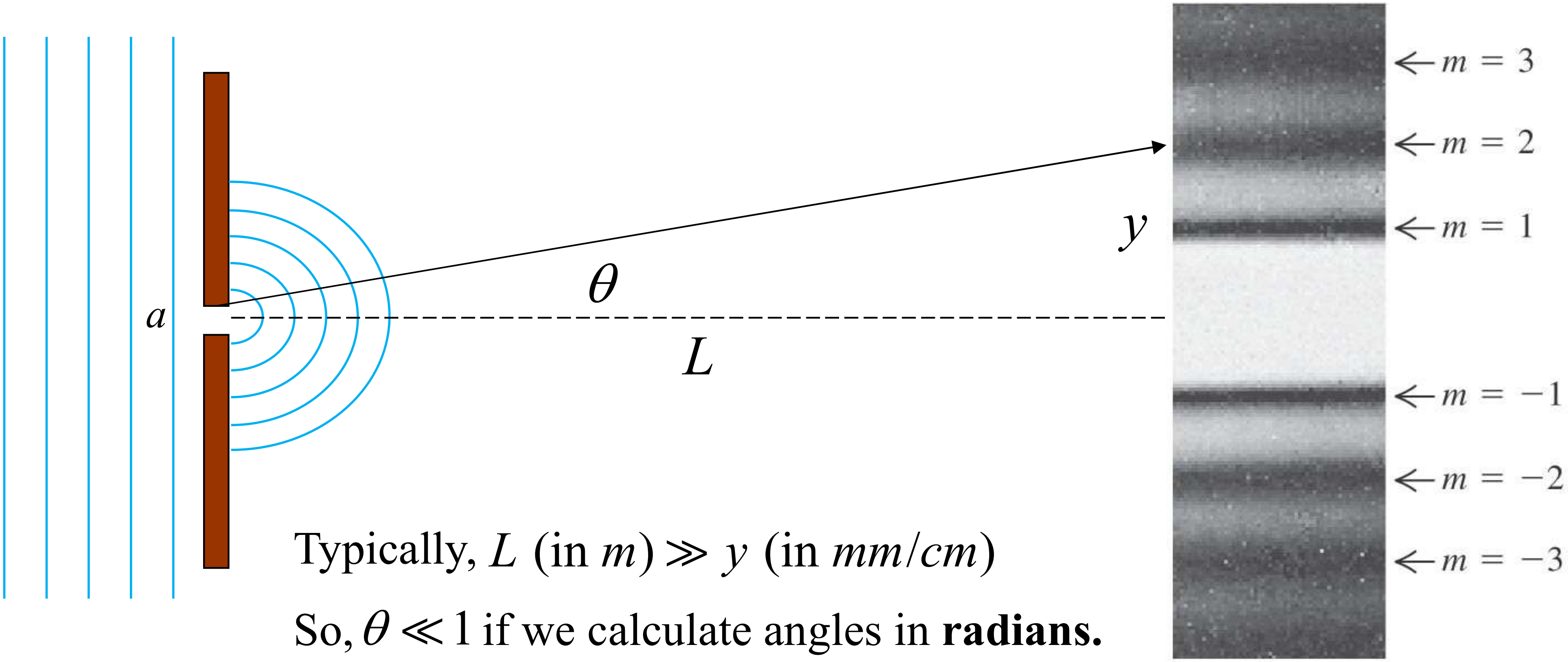
# Intensity in Single-Slit Pattern

The intensity of the pattern as a function of $\theta$ is,

$$I = I_0 \left[ \frac{\sin\left(\pi a \sin\theta/\lambda\right)}{\pi a \sin\theta/\lambda} \right]^2$$



# Notes on Length Scales and Angles



Typically, $L$ (in $m$) $\gg$ $y$ (in $mm/cm$)

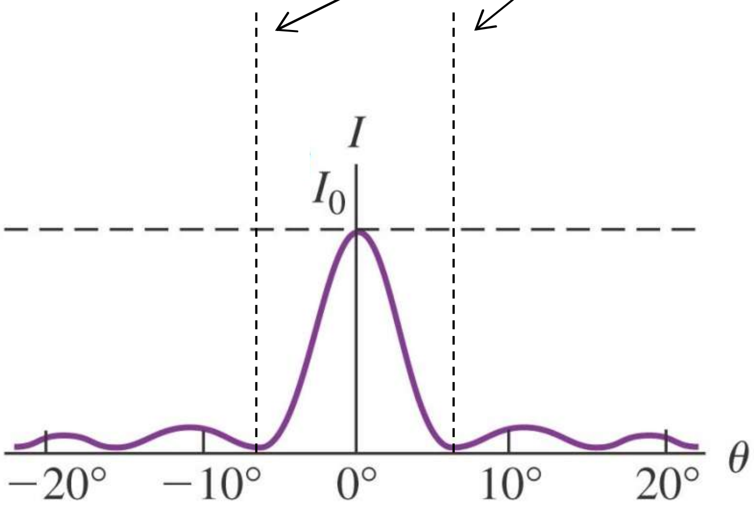
So, $\theta \ll 1$ if we calculate angles in **radians.**

Then, we can approximate $\theta \approx \sin\theta \approx \tan\theta$

# Width of a Diffraction Pattern

One can estimate the width of central max using the locations of 1st min on both left and right sides of the central max:

$$\sin\theta_1 = \pm\frac{\lambda}{a}$$



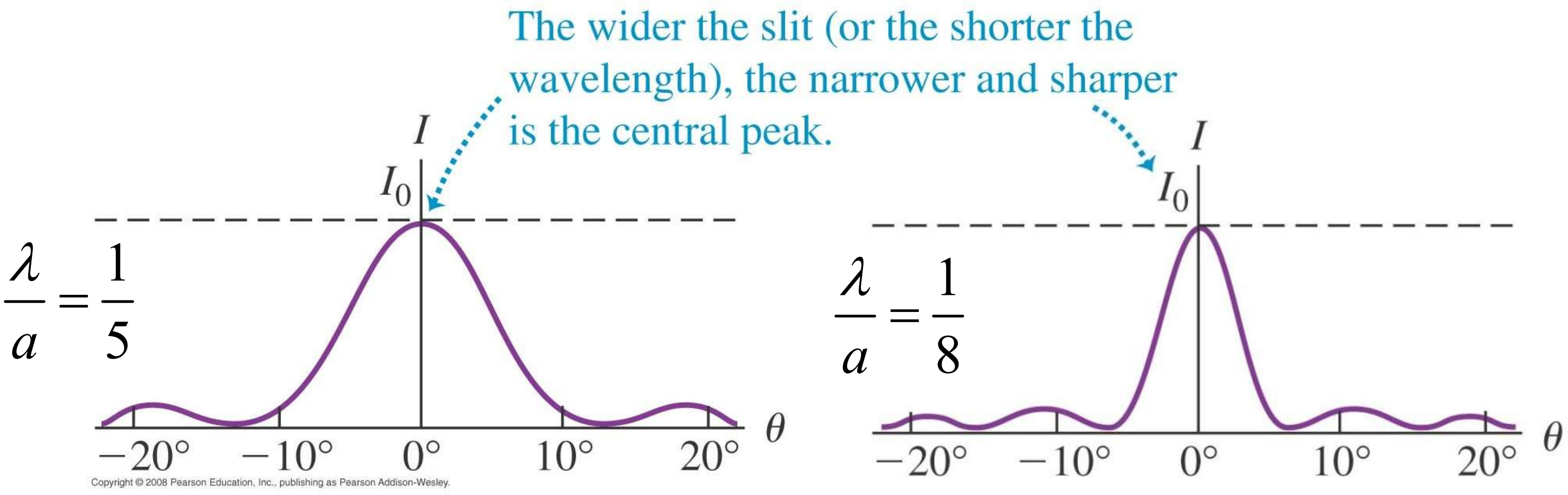
# Width of a Diffraction Pattern

One can estimate the width of central max using the locations of 1st min on both left and right sides of the central max:

$$\sin\theta_1 = \pm\frac{\lambda}{a}$$

On the other hand, if $\lambda < a$ and as $\left|\frac{\lambda}{a}\right| \downarrow$

➡ 1st min moves closer (peak sharper)!

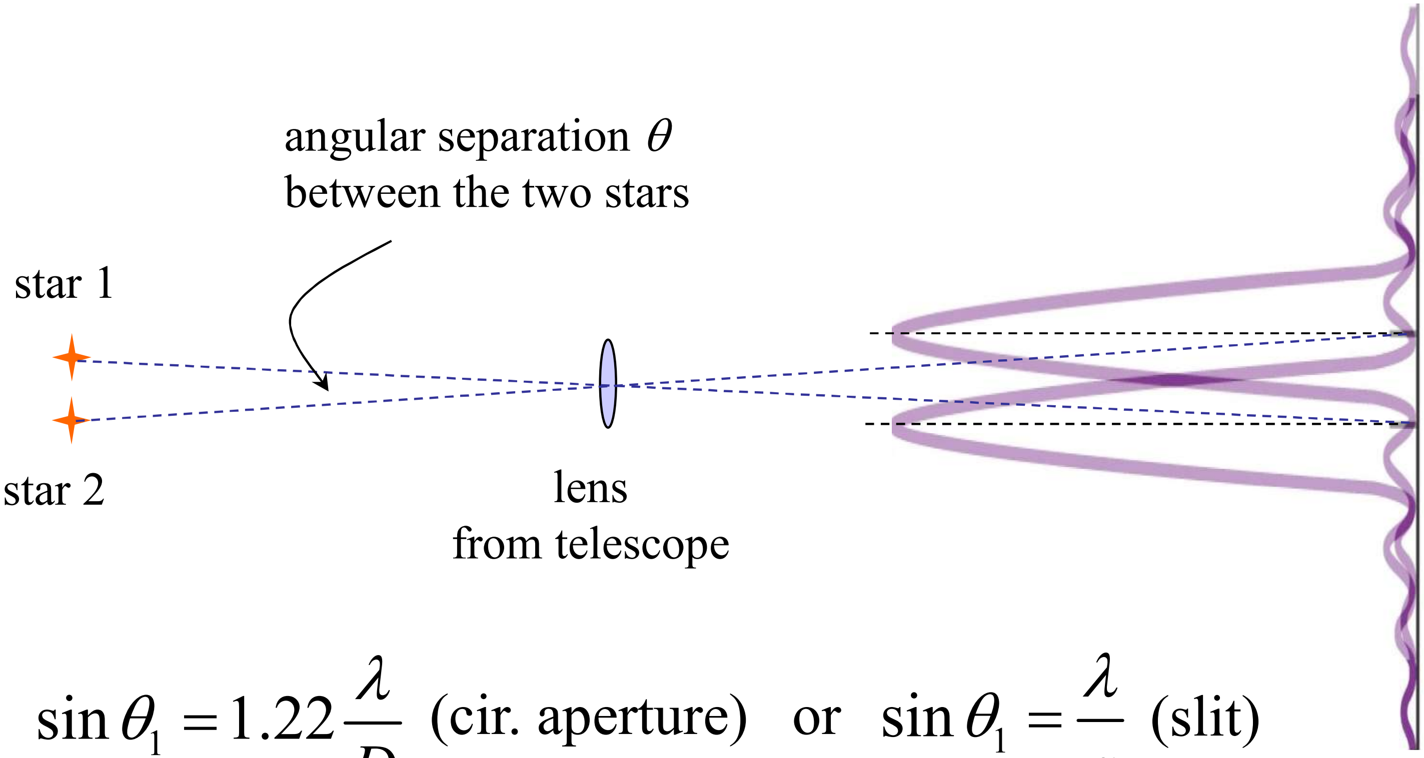

The wider the slit (or the shorter the wavelength), the narrower and sharper is the central peak.

$\frac{\lambda}{a} = \frac{1}{5}$

$\frac{\lambda}{a} = \frac{1}{8}$

# Resolving Power for Circular Apertures

Consider two *non-coherent* point sources (so that they don't interfere), i.e. two distant stars,



We will observe two diffraction patterns on top of each others.

$$\sin\theta_1 = 1.22\frac{\lambda}{D} \text{ (cir. aperture) or } \sin\theta_1 = \frac{\lambda}{a} \text{ (slit)}$$

# Resolving Power for Circular Apertures

The overlap of the two diffraction pattern might prevent one from discerning the two sources of light.

A workable criterion is called the **Rayleigh's Criterion** which is similar in spirit to our discussion for the resolving power for the diffraction grating:

> The two diffraction pattern can be resolvable if the central max from one pattern is at least as far as the 1st min of the other image.

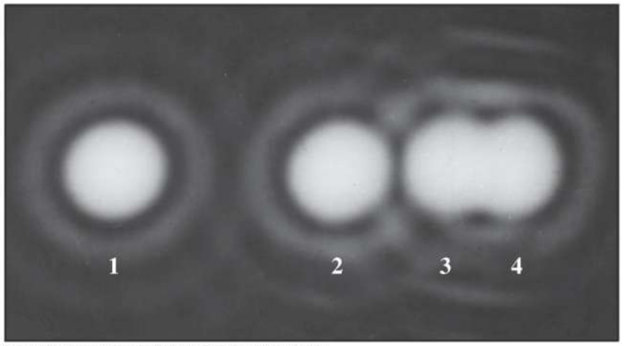
# Resolving Power for Circular Apertures

A optical device such as a telescope or microscope will have a *high* **Resolving Power** if its has a *small* **Limit of Resolution** so that nearby objects with a small angular separation can be resolved.

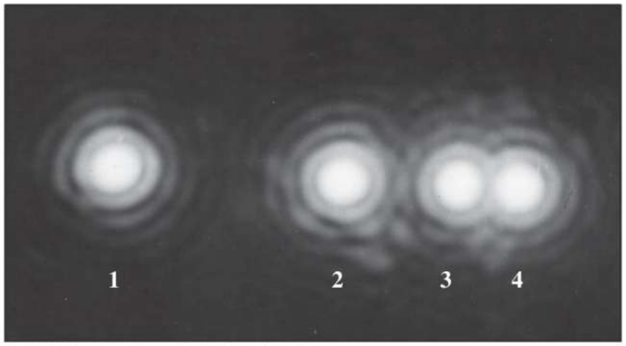
This gives the following ways to increase the Resolving Power:

- increase the diameter $D$ → use a bigger len/mirror in telescope
- decrease the wavelength $\lambda$ → use a shorter wavelength of light in chip production

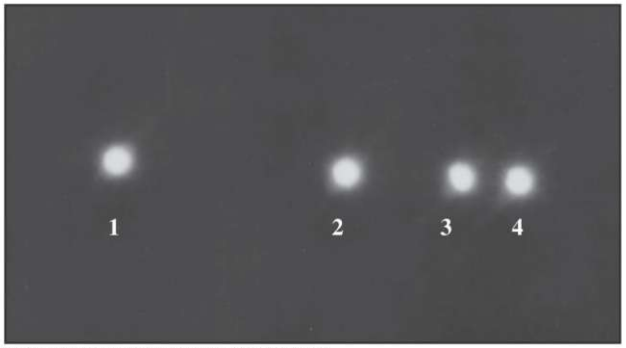
(a) Small aperture



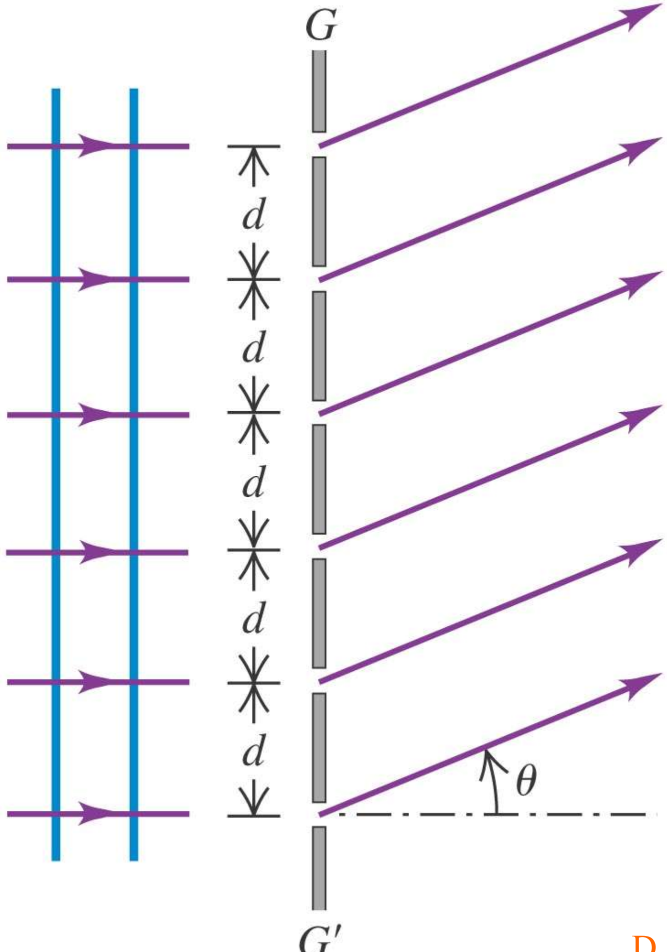
(b) Medium aperture



(c) Large aperture



# Diffraction Grating: Interference Patterns from Multiple Slits



$D_{\text{grating}}$

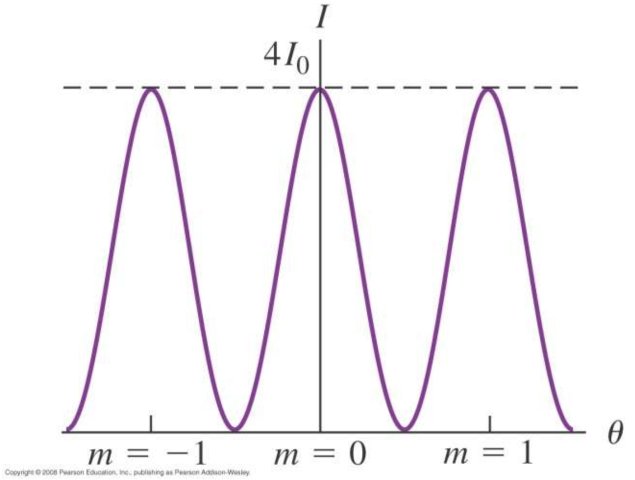
- an array of a large number of parallel slits
- all with the same width *a* and separation *d*
- slits in a grating is typically called **rulings** or **lines**.
- typically, a grating will have 1000s of lines per mm.

As in the discussion with our previous example, the diffraction pattern from this grating will have the condition for **maxima** similar to the two-slits patterns with the same grating spacing *d*,

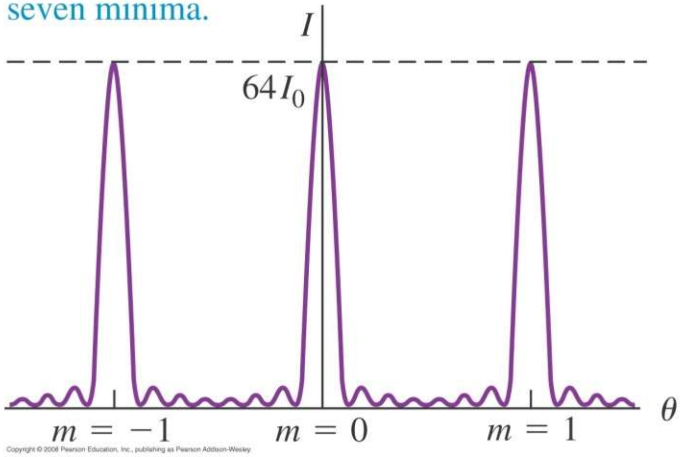
$$d \sin \theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \cdots$$

# Diffraction Grating: Interference Patterns from Multiple Slits

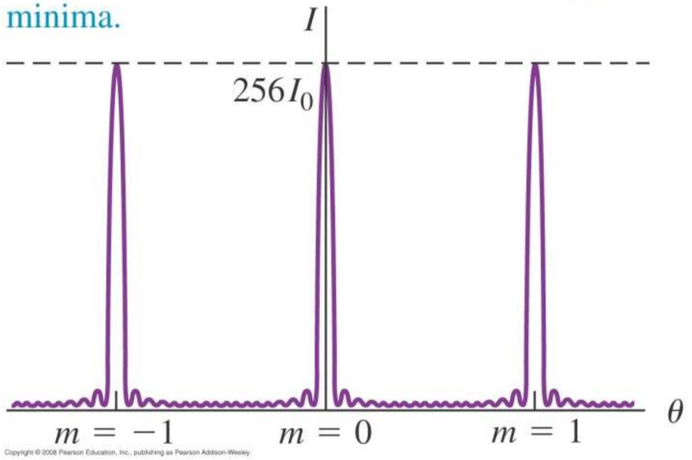
**(a)** $N = 2$: two slits produce one minimum between adjacent maxima.



**(b)** $N = 8$: eight slits produce taller, narrower maxima in the same locations, separated by seven minima.



**(c)** $N = 16$: with 16 slits, the maxima are even taller and narrower, with more intervening minima.



# Resolving Power of a Diffraction Grating

When measuring wavelengths, it is important to distinguish slightly different $\lambda$s. The ability of a grating to resolve the difference $\Delta\lambda$ in wavelengths is given by the ratio called the Chromatic Resolving Power $R$,

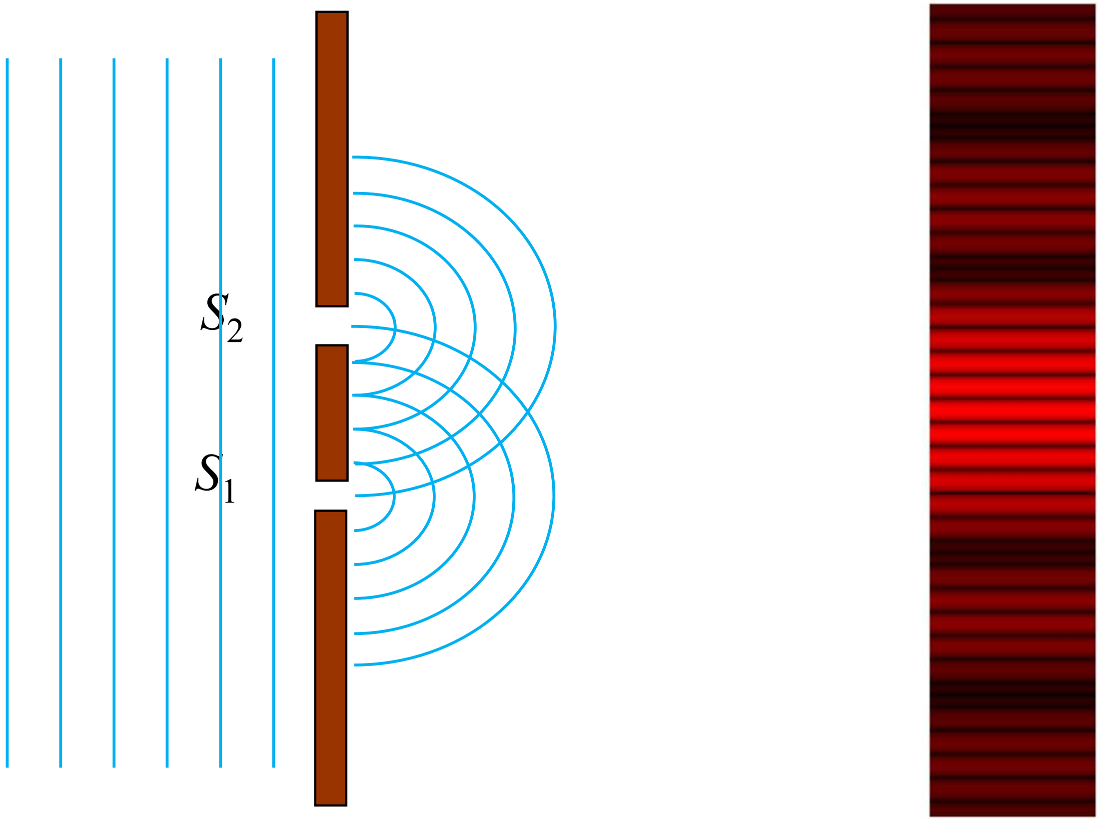
$$R = \frac{\lambda}{\Delta\lambda} = Nm$$ 1/$R$ is the relative error

So, the resolving power of a diffraction grating increases by

- using a grading with  a larger number $N$ of lines (or rulings)
- measuring the spectra line at a higher order $m$

# Intensity of Two-Slits Diffraction Patterns

With two slits, we have *diffraction* from the individual slits **and** *interference* from the two slits.



# Intensity of Two-Slits Diffraction Patterns

With two slits, we have *diffraction* from the individual slits **and** *interference* from the two slits.

The *combined* intensity is the *superposition* of the two effects:

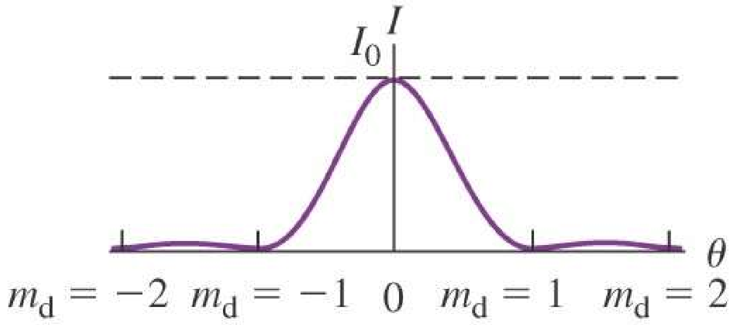
$$\boxed{I = I_0 \underbrace{\cos^2\left(\frac{\phi}{2}\right)}_{\text{interference factor}} \underbrace{\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2}_{\text{diffraction factor}}}$$

where, $\frac{\phi}{2} = \frac{\pi}{\lambda} d \sin\theta$ and $\frac{\beta}{2} = \frac{\pi}{\lambda} a \sin\theta$

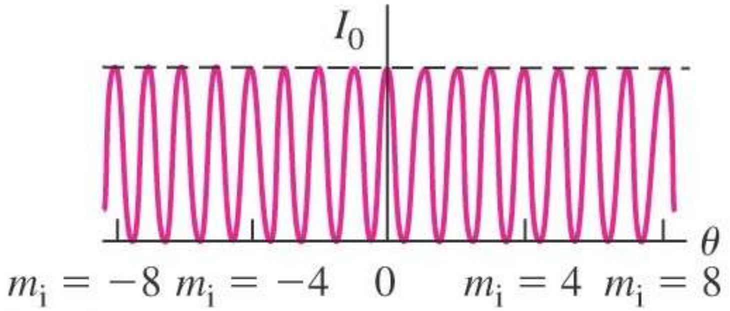
$d$ → separation bet. slits

$a$ → width of both slits

# Intensity of Two-Slits Diffraction Patterns



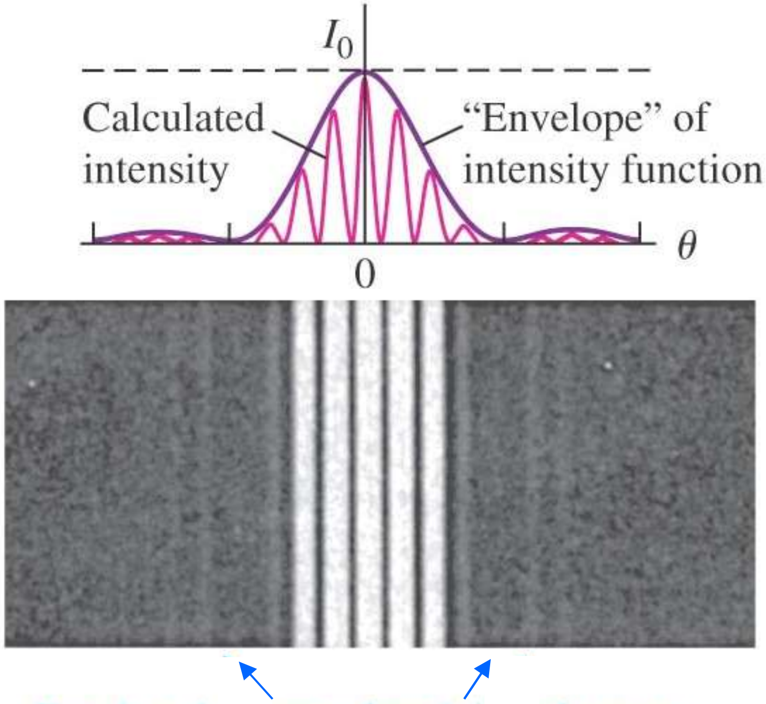
$$\otimes$$



$$d = 4a$$

$$=$$

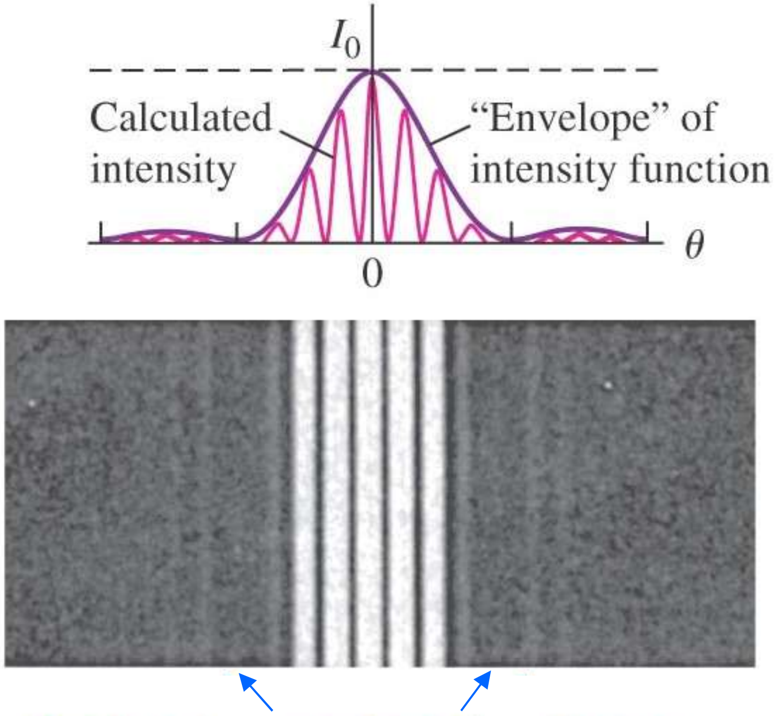
$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$



For $d = 4a$, every fourth interference maximum at the sides ($m_i = \pm 4, \pm 8, \ldots$) is missing,

$\text{D}_{\text{mathematica}}$ $\text{D}_{\text{double slit}}$

# Intensity of Two-Slits Diffraction Patterns



For $d = 4a$, every fourth interference maximum at the sides ($m_i = \pm 4, \pm 8, \ldots$) is missing,

$$d = 4a$$

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

Interfer max: $d \sin\theta = m\lambda$

diff min: $a \sin\theta' = m'\lambda$

When do they match?

$$\sin\theta = \sin\theta' \quad \Rightarrow \quad \frac{m}{m'} = \frac{d}{a} = 4$$