Video Tutor Solutions

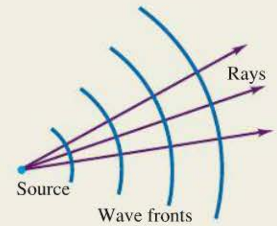
Light and its properties: Light is an electromagnetic wave. When emitted or absorbed, it also shows particle properties. It is emitted by accelerated electric charges.

A wave front is a surface of constant phase; wave fronts move with a speed equal to the propagation speed of the wave. A ray is a line along the direction of propagation, perpendicular to the wave fronts.

When light is transmitted from one material to another, the frequency of the light is unchanged, but the wavelength and wave speed can change. The index of refraction n of a material is the ratio of the speed of light in vacuum c to the speed v in the material. If λ_0 is the wavelength in vacuum, the same wave has a shorter wavelength λ in a medium with index of refraction n. (See Example 33.2.)

$$n = \frac{c}{v} \quad (33.1)$$

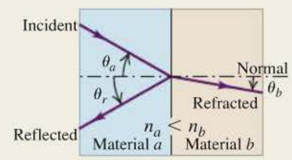
$$\lambda = \frac{\lambda_0}{n} \quad (33.5)$$



Reflection and refraction: At a smooth interface between two optical materials, the incident, reflected, and refracted rays and the normal to the interface all lie in a single plane called the plane of incidence. The law of reflection states that the angles of incidence and reflection are equal. The law of refraction relates the angles of incidence and refraction to the indexes of refraction of the materials. (See Examples 33.1 and 33.3.)

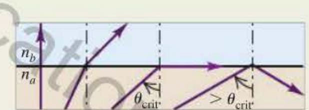
$$\theta_r = \theta_a \quad (33.2)$$
(law of reflection)

$$n_a \sin\theta_a = n_b \sin\theta_b \quad (33.4)$$
(law of refraction)



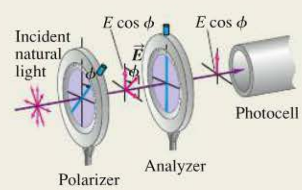
Total internal reflection: When a ray travels in a material of greater index of refraction n_a toward a material of smaller index n_b, total internal reflection occurs at the interface when the angle of incidence exceeds a critical angle θ_{crit}. (See Example 33.4.)

$$\sin\theta_{crit} = \frac{n_b}{n_a} \quad (33.6)$$



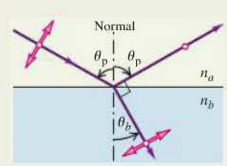
Polarization of light: The direction of polarization of a linearly polarized electromagnetic wave is the direction of the $\vec{E}$ field. A polarizing filter passes waves that are linearly polarized along its polarizing axis and blocks waves polarized perpendicularly to that axis. When polarized light of intensity I_{max} is incident on a polarizing filter used as an analyzer, the intensity I of the light transmitted through the analyzer depends on the angle ϕ between the polarization direction of the incident light and the polarizing axis of the analyzer. (See Example 33.5.)

$$I = I_{max}\cos^2\phi \quad (33.7)$$
(Malus's law)

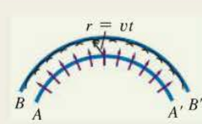


Polarization by reflection: When unpolarized light strikes an interface between two materials, Brewster's law states that the reflected light is completely polarized perpendicular to the plane of incidence (parallel to the interface) if the angle of incidence equals the polarizing angle θ_p. (See Example 33.6.)

$$\tan\theta_p = \frac{n_b}{n_a} \quad (33.8)$$
(Brewster's law)



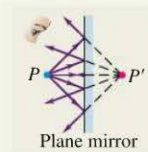
Huygens's principle: Huygens's principle states that if the position of a wave front at one instant is known, then the position of the front at a later time can be constructed by imagining the front as a source of secondary wavelets. Huygens's principle can be used to derive the laws of reflection and refraction.



CHAPTER 34 SUMMARY

Video Tutor Solutions

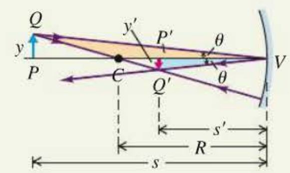
Reflection or refraction at a plane surface: When rays diverge from an object point P and are reflected or refracted, the directions of the outgoing rays are the same as though they had diverged from a point P' called the image point. If they actually converge at P' and diverge again beyond it, P' is a real image of P; if they only appear to have diverged from P', it is a virtual image. Images can be either erect or inverted.



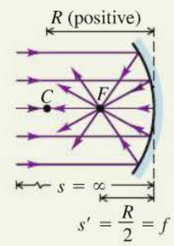
Plane mirror

Lateral magnification: The lateral magnification m in any reflecting or refracting situation is defined as the ratio of image height y' to object height y. When m is positive, the image is erect; when m is negative, the image is inverted.

$$m = \frac{y'}{y} \quad (34.2)$$



Focal point and focal length: The focal point of a mirror is the point where parallel rays converge after reflection from a concave mirror, or the point from which they appear to diverge after reflection from a convex mirror. Rays diverging from the focal point of a concave mirror are parallel after reflection; rays converging toward the focal point of a convex mirror are parallel after reflection. The distance from the focal point to the vertex is called the focal length, denoted as f. The focal points of a lens are defined similarly.



Relating object and image distances: The formulas for object distance s and image distance s' for plane and spherical mirrors and single refracting surfaces are summarized in the table. The equation for a plane surface can be obtained from the corresponding equation for a spherical surface by setting $R = \infty$. (See Examples 34.1–34.7.)

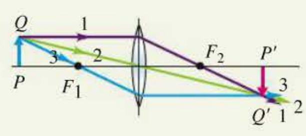
	Plane Mirror	Spherical Mirror	Plane Refracting Surface	Spherical Refracting Surface
Object and image distances	$\frac{1}{s} + \frac{1}{s'} = 0$	$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$	$\frac{n_a}{s} + \frac{n_b}{s'} = 0$	$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$
Lateral magnification	$m = -\frac{s'}{s} = 1$	$m = -\frac{s'}{s}$	$m = -\frac{n_a s'}{n_b s} = 1$	$m = -\frac{n_a s'}{n_b s}$

Object–image relationships derived in this chapter are valid only for rays close to and nearly parallel to the optic axis; these are called paraxial rays. Nonparaxial rays do not converge precisely to an image point. This effect is called spherical aberration.

Thin lenses: The object–image relationship, given by Eq. (34.16), is the same for a thin lens as for a spherical mirror. Equation (34.19), the lensmaker's equation, relates the focal length of a lens to its index of refraction and the radii of curvature of its surfaces. (See Examples 34.8–34.11.)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (34.16)$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (34.19)$$



Sign rules: The following sign rules are used with all plane and spherical reflecting and refracting surfaces.

- $s > 0$ when the object is on the incoming side of the surface (a real object); $s < 0$ otherwise.
- $s' > 0$ when the image is on the outgoing side of the surface (a real image); $s' < 0$ otherwise.
- $R > 0$ when the center of curvature is on the outgoing side of the surface; $R < 0$ otherwise.
- $m > 0$ when the image is erect; $m < 0$ when inverted.

CHAPTER 35 SUMMARY

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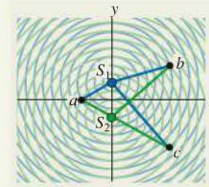
Interference and coherent sources: Monochromatic light is light with a single frequency. Coherence is a definite, unchanging phase relationship between two waves. The overlap of waves from two coherent sources of monochromatic light forms an interference pattern. The principle of superposition states that the total wave disturbance at any point is the sum of the disturbances from the separate waves.

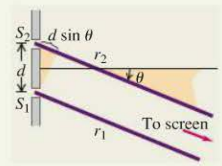


Two-source interference of light: When two sources are in phase, constructive interference occurs where the difference in path length from the two sources is zero or an integer number of wavelengths; destructive interference occurs where the path difference is a half-integer number of wavelengths. If two sources separated by a distance d are both very far from a point P, and the line from the sources to P makes an angle θ with the line perpendicular to the line of the sources, then the condition for constructive interference at P is Eq. (35.4). The condition for destructive interference is Eq. (35.5). When θ is very small, the position y_m of the mth bright fringe on a screen located a distance R from the sources is given by Eq. (35.6). (See Examples 35.1 and 35.2.)

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \dots)$$
(constructive interference) (35.4)

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$
$$(m = 0, \pm 1, \pm 2, \dots)$$ (35.5)
(destructive interference)

$$y_m = R\frac{m\lambda}{d}$$ (35.6)
(bright fringes)

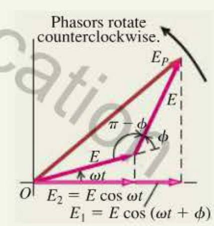


Intensity in interference patterns: When two sinusoidal waves with equal amplitude E and phase difference ϕ are superimposed, the resultant amplitude E_P and intensity I are given by Eqs. (35.7) and (35.10), respectively. If the two sources emit in phase, the phase difference ϕ at a point P (located a distance r_1 from source 1 and a distance r_2 from source 2) is directly proportional to the difference in path length $r_2 - r_1$. (See Example 35.3.)

$$E_P = 2E\left|\cos\frac{\phi}{2}\right|$$ (35.7)

$$I = I_0 \cos^2\frac{\phi}{2}$$ (35.10)

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1)$$ (35.11)



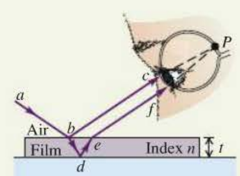
Interference in thin films: When light is reflected from both sides of a thin film of thickness t and no phase shift occurs at either surface, constructive interference between the reflected waves occurs when $2t$ is equal to an integral number of wavelengths. If a half-cycle phase shift occurs at one surface, this is the condition for destructive interference. A half-cycle phase shift occurs during reflection whenever the index of refraction in the second material is greater than that in the first. (See Examples 35.4–35.7.)

$$2t = m\lambda \quad (m = 0, 1, 2, \dots)$$
(constructive reflection from thin film, no relative phase shift) (35.17a)

$$2t = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \dots)$$
(destructive reflection from thin film, no relative phase shift) (35.17b)

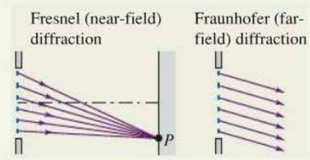
$$2t = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \dots)$$
(constructive reflection from thin film, half-cycle relative phase shift) (35.18a)

$$2t = m\lambda \quad (m = 0, 1, 2, \dots)$$
(destructive reflection from thin film, half-cycle relative phase shift) (35.18b)



Video Tutor Solutions

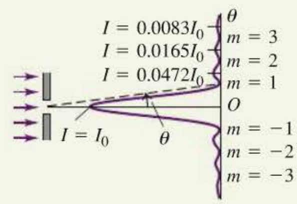
Fresnel and Fraunhofer diffraction: Diffraction occurs when light passes through an aperture or around an edge. When the source and the observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called Fraunhofer diffraction. When the source or the observer is relatively close to the obstructing surface, it is Fresnel diffraction.



Single-slit diffraction: Monochromatic light sent through a narrow slit of width a produces a diffraction pattern on a distant screen. Equation (36.2) gives the condition for destructive interference (a dark fringe) at a point P in the pattern at angle θ. Equation (36.7) gives the intensity in the pattern as a function of θ. (See Examples 36.1–36.3.)

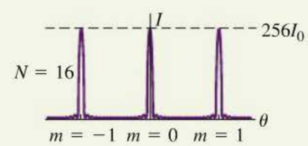
$$\sin\theta = \frac{m\lambda}{a} \qquad (m = \pm1, \pm2, \dots) \tag{36.2}$$

$$I = I_0\left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2 \tag{36.7}$$



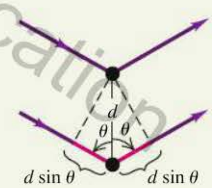
Diffraction gratings: A diffraction grating consists of a large number of thin parallel slits, spaced a distance d apart. The condition for maximum intensity in the interference pattern is the same as for the two-source pattern, but the maxima for the grating are very sharp and narrow. (See Example 36.4.)

$$d\sin\theta = m\lambda \qquad (m = 0, \pm1, \pm2, \pm3, \dots) \tag{36.13}$$



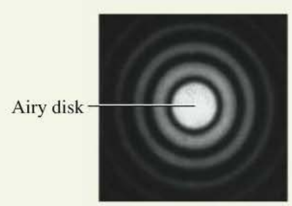
X-ray diffraction: A crystal serves as a three-dimensional diffraction grating for x rays with wavelengths of the same order of magnitude as the spacing between atoms in the crystal. For a set of crystal planes spaced a distance d apart, constructive interference occurs when the angles of incidence and scattering (measured from the crystal planes) are equal and when the Bragg condition [Eq. (36.16)] is satisfied. (See Example 36.5.)

$$2d\sin\theta = m\lambda \qquad (m = 1, 2, 3, \dots) \tag{36.16}$$



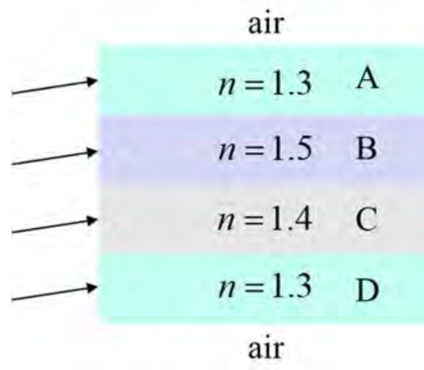
Circular apertures and resolving power: The diffraction pattern from a circular aperture of diameter D consists of a central bright spot, called the Airy disk, and a series of concentric dark and bright rings. Equation (36.17) gives the angular radius θ_1 of the first dark ring, equal to the angular size of the Airy disk. Diffraction sets the ultimate limit on resolution (image sharpness) of optical instruments. According to Rayleigh's criterion, two point objects are just barely resolved when their angular separation θ is given by Eq. (36.17). (See Example 36.6.)

$$\sin\theta_1 = 1.22\frac{\lambda}{D} \tag{36.17}$$

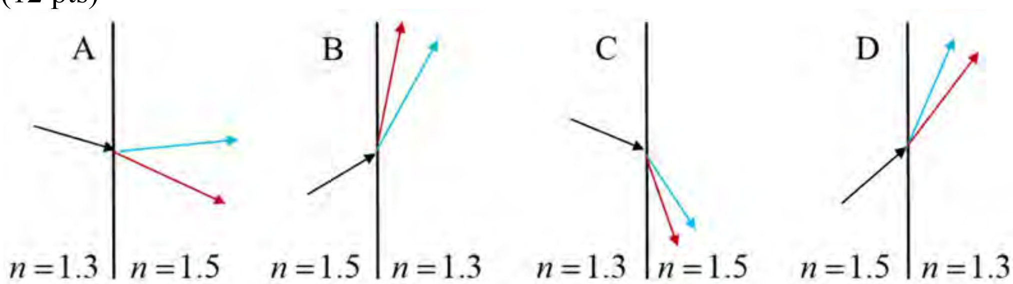


1. (40 pts) Answer the following questions. Justify your answers. (Use additional white paper if you need.)

a) (8 pts) Four light rays enter four long channels of optical materials from the left as show. The index of refraction for the four materials are as indicated. Air is on top of and below the four stacked optical channels. Through which channel (A, B, C, or D) could a light ray be most likely trapped totally within it after many internal reflections as it propagates toward the right?

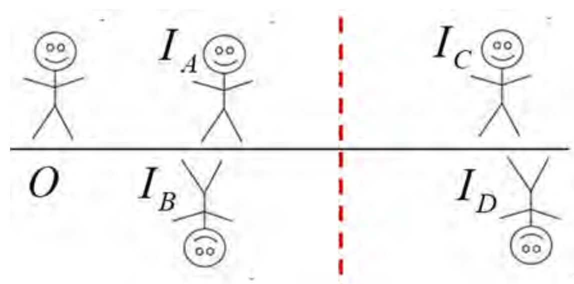


b) (12 pts)



The four panels above show four different situations when an incident ray (black arrow) composes of two colors (blue and red) refracts through an interface between two different materials. The approximate index of refraction for each material is indicated in the graph. Which panel(s) (A, B, C, and D) is (are) physically possible?

c) (12 pts) A stick figure (O) stands in front of a spherical mirror at a location indicated by the red dotted line. The optical axis of the spherical mirror is indicated by the horizontal line. The other four stick figures labeled by I_A, I_B, I_C, I_D suggest the general locations where the images might be produced by the mirror. (Note: The stick figures are qualitative representation of the situations only. The size of the figures and the distances from the mirror are not drawn to scale.)

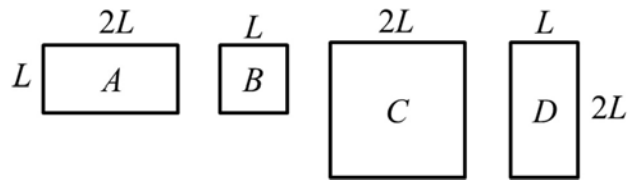


i. Which of the image(s) (I_A, I_B, I_C, I_D) is (are) physically *impossible* for either a concave or convex mirror?

ii. Which of the image(s) (I_A, I_B, I_C, I_D) could be produced by a concave mirror?

iii. Which of the image(s) (I_A, I_B, I_C, I_D) could be virtual?

d) (8 pts) Four small rectangular openings with sides as shown to the right. The length L of the openings is three times the wavelength λ of the light passing through them, i.e., $L = 3\lambda$.

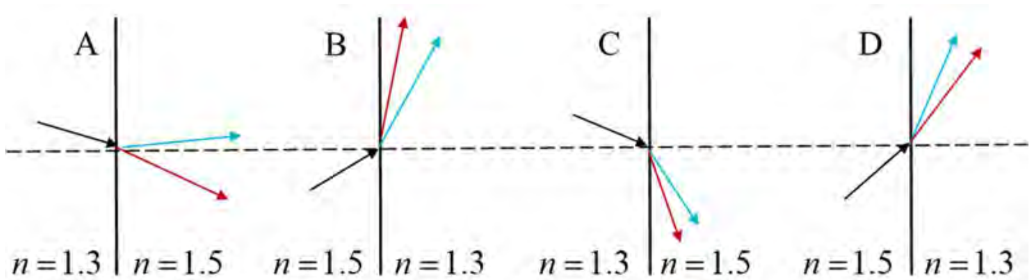


i. Through which rectangular opening(s) will one observe the most spreading of the light in the up-down direction due to diffraction?
ii. Through which rectangular opening(s) will one observe the most spreading of the light in the left-right direction due to diffraction?

Solutions:

a) **Channel B**: To ensure that a light ray will be trapped totally within a particular optical channel after many internal reflections as it propagates toward the right, the optical channel must act like a fiber optic cable allowing total internal reflection to occur with it. For a particular channel, total internal reflection is possible only if the index of refraction of the material on top of and below it is lower than its own index of refraction. Among the four channels, only Channel B satisfies this condition with the index of refraction for both materials on the top and bottom less than n=1.5.

b)



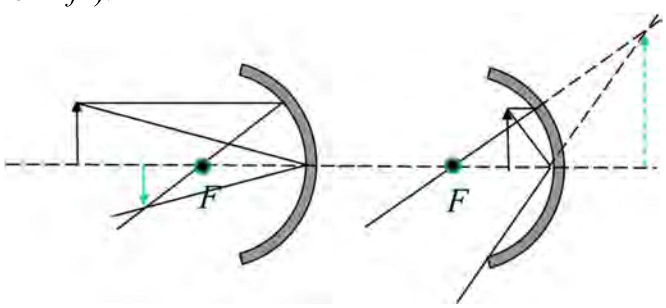
Panel B is the only physically possible: The incident ray refracting across the interface must satisfy Snell's law. The normal to the interface is added to aid the analysis.

- Panel A is not possible: Different colors from the same incident light will disperse with different angle of refraction but they all must refract to the SAME SIDE with respect to the normal. Here, the blue and red ray diffract to different side of the normal.
- Panel C is not possible: According to Snell's law $n_L \sin\theta_L = n_R \sin\theta_R$, if $n_R > n_L$, the angle of refraction and the angle of incidence must have reverse relationship, i.e., $\theta_R < \theta_L$. Both refracted colored rays as compared with the incident ray have the opposite relationship in Panel C.

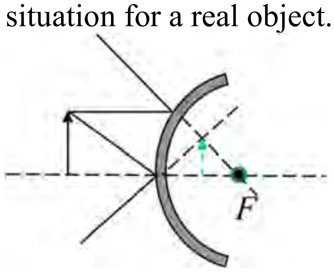
- Panel D is also not possible: According to dispersion, the index of refraction of a material varies with the color of light with blue light having a slightly larger index of refraction than a red light in the same material. Then, from Snell's Law, the red light with the smaller index of refraction will have a larger deflection as it refracts (bend away more from the normal) as compared with the blue light. The ordering of the refracted beams in Panel D is wrong.

Panel B is the situation that does not violate Snell's law, $\theta_{refraction} > \theta_{incident}$ with $n_{right} < n_{left}$ and has the right dispersion order for the red and blue light.

c) For a real object, here are the two possible cases for a concave mirror ($O > f$ and $O < f$):



For a convex mirror, since F is behind the mirror, this is the only possible situation for a real object.



These are all the possible images for spherical mirrors (concave or convex) with a real object. So,

i. Images $\mathbf{I_A}$ and $\mathbf{I_D}$ are NOT POSSIBLE.

ii. Images $\mathbf{I_B}$ and $\mathbf{I_C}$ can be produced by a concave mirror.

iii. Image $\mathbf{I_C}$ is virtual.

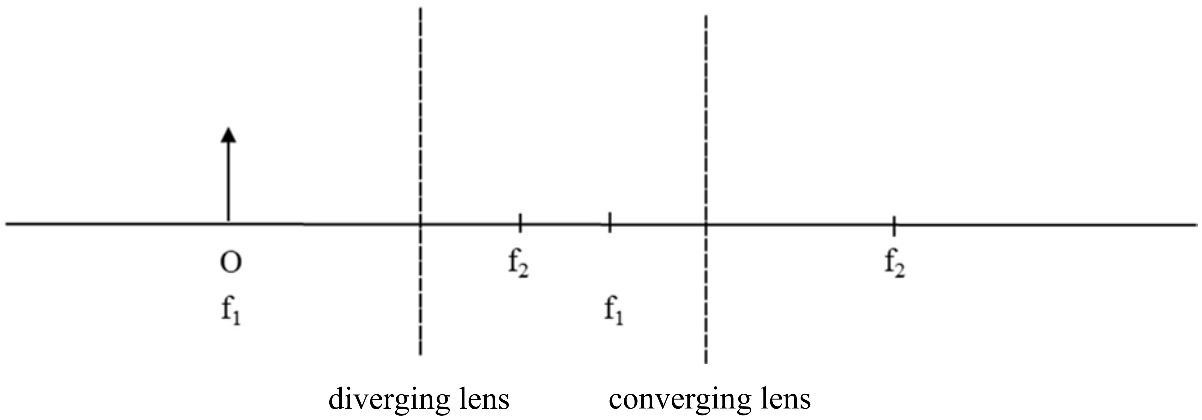
d) For a 2D opening, the spreading of light due to diffraction along either the x or y direction is determined by the width of the open along that direction. So, the spreading of light along the up and down direction will be determined by the y dimension of the opening and the spreading along the left and right direction will be determined by the x dimension of the opening. From an analysis of single slit diffraction patterns, the degree of spreading, determined by the location of the first diffraction minimum from the central peak, is governed by $a \sin\theta_1 = \lambda$, where θ_1 is where the 1st diffraction minimum is located and a is the width of the opening in either the x or y direction.

i. **Openings A and B**: Along the vertical direction, both A and B have the smallest width *L* so light passing through them will spread more in the up and down direction than the other two cases.

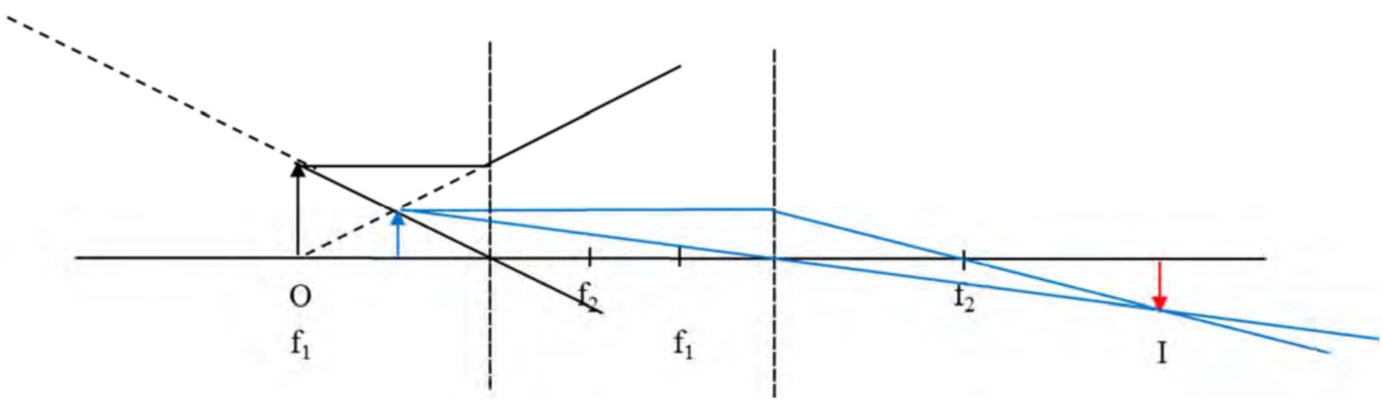
ii. **Openings B and D**: Along the horizontal direction, both B and D have the smallest width *L* and light passing through them will spread more in the left and right direction than the other two cases.

2. (25 pts) A thin diverging lens with a focal length $f_1 = -10.0cm$ on the left and a thin converging lens with a focal length $f_2 = 10.0cm$ on the right are separated by $15.0cm$. An object is placed at a distance $10.0cm$ to the left of the first lens.

i. Find the position of the final image.
ii. Draw the rays diagram for this situation.
iii. What is the magnification of the final image?
iv. Is the final image virtual or real?



Solution:



The black rays are from the original object and the blue rays are from the intermediate image (blue arrow). The final image is indicated by the red arrow.

For the lens #1, we have $\frac{1}{s'_1} = \frac{1}{f_1} - \frac{1}{s_1}$ and for lens #2, we have $\frac{1}{s'_2} = \frac{1}{f_2} - \frac{1}{s_2}$.
By convention, f_1 is negative (diverging) and f_2 is positive (converging). From the first lens equation, we have

$$\frac{1}{s'_1} = -\frac{1}{10} - \frac{1}{10} = -\frac{2}{10} \quad \rightarrow \quad s'_1 = -5.00cm\ .$$

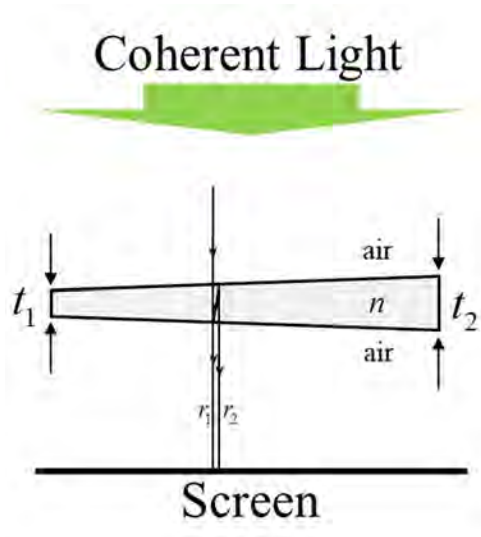
As indicated by the rays diagram, this image will be a virtual object for the diverging lens Then, from the diagram, we can see that $s_2 = 15cm - s'_1 = +20cm$. The intermediate image is a real object for len #2. Putting this into the equation for lens #2, we have

$$\frac{1}{s'_2} = \frac{1}{10} - \frac{1}{20} = \frac{2-1}{20} \quad \rightarrow \quad s'_2 = +20.0cm.$$

The final image is **real** as indicated and is located 20 cm behind len #2 (or 35cm behind lens #1).

$M = m_{div} m_{con} = -\frac{s'_1}{s_1}\left(-\frac{s'_2}{s_2}\right) = -\frac{-5}{10}\left(-\frac{20}{20}\right) = -0.5$. The final image is **real**, inverted, and smaller.

3. (25 pts) A wedge-shaped thin film with an index of refraction $n = 1.35$ is illuminated vertically with a wide beam of coherent light with a wavelength of λ as shown. (The thickness of the wedge-shaped film is exaggerated for clarity.) A sample ray is shown in the middle of the figure. The ray refracts into the thin film nearly vertically. When it reaches the bottom surface of film, part of it will continue to refract out of the film (ray r_1) and part of it will reflect twice within the film before it refract out of the bottom surface of the film (ray r_2). r_1 and r_2 interfere with each other and create a thin-film interference pattern on the screen below.



i. If the thickness of the film t_1 on the left end is given to be $415nm$, what is the wavelength λ in the visible spectrum that will give rise to a bright fringe at that location? Take the spectrum of visible light to be within $[380nm, 740nm]$.

ii. Using the same wavelength λ of light from part i, if one observes exactly 6 bright fringes and 5 dark fringes on the screen along the length of the thin film, what must be the thickness t_2 on the right end of the wedge-shaped thin film?

Solution:

i) Ray r_2 will go through two internal reflections but since the ray inside the thin film is reflecting off from air (bottom and top surfaces) with a lower index of refraction than the index of refraction of the material in the film, the electric field will not suffer a sign change from the two reflections. So, the only contribution to the phase difference between r_2 and r_1 will be from the extra path different $2t_1$ that r_2 will accumulate more as compare with r_1. For construction interference (bright fringe), we will then have the following condition,

$$2t_1 = m\frac{\lambda}{n}, \quad m = 1, 2, \cdots$$

Solving for λ, we have,

$$\lambda = \frac{2nt_1}{m}, \quad m = 1, 2, \cdots$$

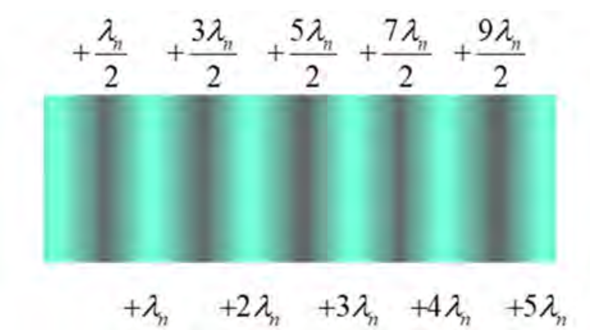
Now, we need to check which solution for the different *m* will give a light with wavelength in the visible range.

Putting $m = 1, 2, \cdots$ into the above equation , we have

$$\lambda = \frac{2(1.35)(415nm)}{1} = 1120.5nm \text{ (out of range)}$$
$$\lambda = \frac{2(1.35)(415nm)}{2} = 560.25nm \text{ (visible)}$$
$$\lambda = \frac{2(1.35)(415nm)}{3} = 373.5nm \text{ (out of range)}$$

All larger choice of *m* will give even shorter wavelengths. So the only visible light that will give a bright fringe on the left edge is for $m = 2$ and $\lambda = 560.nm$

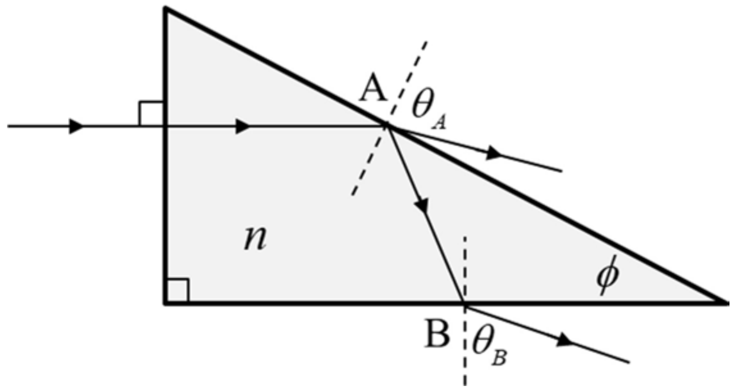
ii) The next dark fringe occurs when r_2 accumulates another half of a wavelength $\frac{\lambda_n}{2}$ and the next bright fringe happens when r_2 accumulates another one full wavelength λ_n. If within the full length of the screen across the thin film, we observed 5 dark fringes and 6 dark fringes with the 6th bright fringes exactly at the right edge, we should observe the following pattern on the screen,



So, at the right edge of the thin film, we know that ray r_2 will exactly accumulate FIVE additional full wavelength λ_n. Since, from the left edge, *m* started out at $m = 2$, five *additional* wavelengths will take us to $m = 7$, so we have

$$2t_2 = 7\frac{\lambda}{n}, \quad m = 7$$
$$t_2 = \frac{7}{2}\frac{560.25nm}{1.35} = 1452.5nm = 1450nm$$

4. (25 pts) A light ray enters a right triangular glass prism normally from the left edge as shown. The prism has an index of refraction of $n = 1.67$. Assume air is outside of the glass prism.



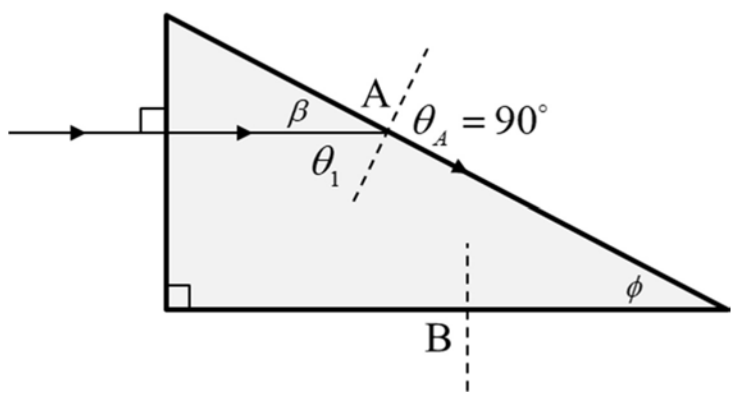
i. What is the critical value ϕ_c for the angle ϕ so that the refracting ray exiting the right face of the prism at point A will be barely skimming along the right face of the prism, i.e. $\theta_A = 90^\circ$?

ii. To completely eliminate the refracted ray from the right face (Total Internal Reflection at point A), should one choose a right triangular prism with ϕ larger or smaller the critical angle ϕ_c calculated in part i ?

iii. Now consider a *reflected* ray from point A. It will reach the base of the prism at point B. What is the critical value ϕ_c for the angle ϕ so that the refracting ray exiting the base of the prism at point B will be barely skimming along the bottom of the prism, i.e. $\theta_B = 90^\circ$? [Note: the condition for part iii is different from part i.]

Solution:

i. At point A, total internal reflection requires the angle of refraction $\theta_A = 90^\circ$. Then, Snell's Law gives,



$$n \sin\theta_1 = \sin 90^\circ = 1$$

We now need to link ϕ to θ. Since the incident ray is parallel to the base of the triangular prism, β is equal to ϕ and

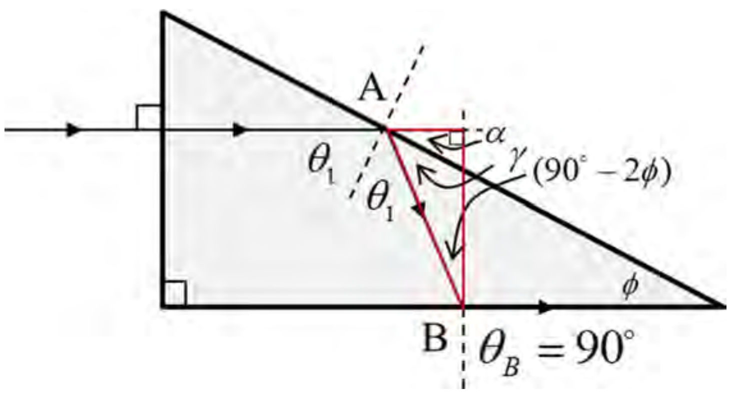
$$\theta_1 = 90^\circ - \beta = 90^\circ - \phi$$

So, we have

$$n \sin\left(90^\circ - \phi\right) = n\cos\phi = 1$$
$$\phi = \cos^{-1}\left(\frac{1}{n}\right) = \cos^{-1}\left(\frac{1}{1.67}\right) = 53.2^\circ$$

ii. For total internal reflection, we need the incident angle θ_1 at point A to be larger than the critical condition. Since $\theta_1 = 90^\circ - \phi$, for larger value of θ_1, we need smaller values of ϕ. So, for total internal reflection, we need $\phi < \phi_c = 53.2^\circ$.

iii. From the law of reflection, the angle of internal reflection at point A is also θ_1, the relevant picture describing the angle of incident at point B is:

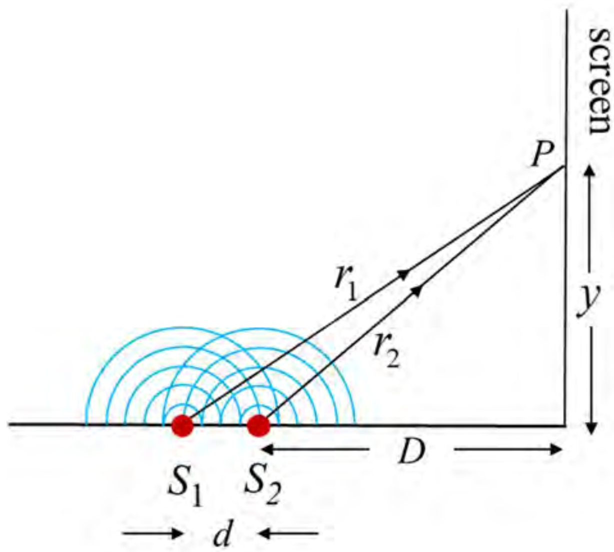


Using the red right triangular shown above, the two interior angles (α, γ) to the left can each be shown to be ϕ. First, since the top edge of the red triangle is parallel to the base of the prism, $\alpha = \phi$. Then, since $\theta_1 + \gamma = 90^\circ$ and $\theta_1 = 90^\circ - \phi$, so $\gamma = \phi$. Now, adding up all the interior angles of the red triangle and equating the sum to 180°, we can deduce that the angle of incident at point B is $90^\circ - 2\phi$.

Then, apply Snell's law with the critical condition for the refracting ray to be just skimming along the base, i.e., $\theta_B = 90^\circ$, we have

$$n \sin\left(90^\circ - 2\phi\right) = \sin 90^\circ = 1$$
$$n \cos 2\phi = 1$$
$$\phi = \frac{1}{2}\cos^{-1}\left(\frac{1}{n}\right) = 26.6^0$$

5. (25 pts) Two coherent point sources S_1 and S_2 emit light with the same wavelength λ and at the same amplitude in all directions as shown. The two light sources are separated by a distance of $d = 5.00\lambda$ on the horizontal axis. A vertical viewing screen is placed at a distance of $D = 20.0\lambda$ to the right of S_2. The figure also shows two sample rays of light from S_1 and S_2 reaching the vertical screen at point P at a height y above the horizontal axis. [Note: Since the screen is close to the two sources of light, you cannot treat r_1 and r_2 as nearly parallel.]



i. What is the path difference $\delta = r_1 - r_2$ between the two sources at $y = 0$ on the screen? Will there be a bright fringe (constructive interference maximum) or dark fringe (destructive interference minimum) at $y = 0$?

ii. As y increases, will the path difference δ increase or decrease?

iii. Write down an expression for the path difference δ in terms of the height y above the horizontal axis on the screen.

iv. To observe the **first** dark fringe (destructive interference minimum) above the horizontal axis on the screen, what is the required path difference δ (in units of λ)?

v. Calculate the path difference δ (in units of λ) for light from the two sources reaching the screen at a height of $y = 30.0\lambda$. Is the resulting intensity at this location on the screen close to a maximum (constructive interference) or a minimum (destructive interference)?

Solution:

The general principle for two waves interference is the following:

-For maximum intensity at locations with constructive interference, we need
$\delta = r_1 - r_2 = m\lambda, \quad m = 1,2,3,4,5$ and
-For minimum intensity at locations with destructive interference, we need

$$\delta = r_1 - r_2 = \left(m + \frac{1}{2}\right)\lambda, \quad m = 1,2,3,4 .$$

i. $\delta = d = 5\lambda$. Since δ is an integer multiple of a wavelength, it will be a bright fringe.

ii. By considering the geometry of the problem, as y increases going up the screen, the path difference δ between r_1 and r_2 will decrease.

iii. First, we need to express r_1 and r_2 in terms of y, d, and D,

$r_1 = \sqrt{y^2 + (d+D)^2}$ and $r_2 = \sqrt{y^2 + D^2}$

Then, the path difference between the two light rays is given by,

$\delta(y) = r_1 - r_2 = \sqrt{y^2 + (d+D)^2} - \sqrt{y^2 + D^2}$

iv. To observe the first dark fringe above the horizontal axis, the path difference must be $\left(m + \frac{1}{2}\right)\lambda$. From i, we know that $y = 0$ is a bright fringe (maximum) with $\delta = 5\lambda$. As y increases, δ decreases and the intensity will decrease as well. The first (next) minimum will occur when δ decreased by $\frac{1}{2}\lambda$ so, we will have $\delta = 4.50\lambda$ at the first minimum above the axis.

v. Substituting $y = 30.0\lambda$ into the equation from part iii, we have

$\delta(30\lambda) = \left[\sqrt{30^2 + (5+20)^2} - \sqrt{30^2 + 20^2}\right]\lambda = 2.996\lambda$

So, the path difference is almost at 3λ and the intensity at this location will be close to its maximum (constructive interference), $\delta = m\lambda$ with $m = 3$.