1. (25pts) Answer the following questions. Justify your answers. (Use the space provided below and the next page)

a). An object (an arrow) is placed as shown in front of each of the following optical instruments. Describe the image produced in each case according to the following three sets of identifiers: real/virtual, upright/inverted, and larger/smaller?

b). A student is performing a Young’s double slit experiment and an interference pattern is observed on a wall at the other side of the room. He/she finds that the bright and dark lines (fringes) within the pattern are spaced too close together. If the student wants to increase the spacing between the fringes, what are the following modifications that will be effective: i) change the light source to one with a shorter wavelength, ii) decrease the separation between the two slits, iii) move the apparatus farther away from the wall?

c). On the ground of a gas station, a drop of an unknown oily substance is floating on top of a layer of water. At the thinnest part of this substance with a thickness much less than the wavelength of visible lights, one observes a dark spot. Is the index of refraction for this unknown substance larger or smaller then the index of refraction of water?

d). A real object is placed in front of a convex mirror. If one moves the object closer to the mirror, will the image move closer to or farther away from the mirror? With a real object, can the image ever be located at a distance larger than the focal length of the mirror?

Answers:

a) Converging lens: virtual, upright, bigger; Convex mirror: virtual, upright, smaller; Diverging lens: virtual, upright, smaller

b) To broaden the fringes, the experimenter can decrease the separation of the two slits and/or move the apparatus farther away from the wall.

c) Since the thin film is much thinner than a wavelength of the light, the dominate phase shift is from reflection only. In this case, since one observes a destructive interference (dark), the wave which reflected from the surface of the unknown substance and the wave which reflected from the interface between the unknown substance and the water must be exactly 180° out of phase. This requires that $n_{\text{water}} < n_{\text{unknown}}$ and $n_{\text{unknown}} > 1$ so that the wave reflecting off the unknown substance suffers a 180° phase shift while the wave reflecting off the water-unknown-substance interface does not.
d) The image distance for a convex mirror is given by \( \frac{1}{d_i} = -\left(\frac{1}{f} + \frac{1}{d_o}\right) \). For a real object (\(d_o is +\)), when \(d_o\) decreases, \(1/d_o\) gets bigger so that the magnitude of the right hand side of the equation will increase also. Thus, the magnitude of \(d_i\) will decrease, i.e., the virtual image (note the overall negative sign on the right hand side) will move closer to the mirror. Lastly, since \(d_o\) is positive for a real object, \(d_i\) will attain its maximal value when \(d_o = \infty\). In this case, the image is located exactly at the focal point of the mirror.
2. (25 pts)
A thin diverging lens with focal length \( f_1 = 10.0\, cm \) and a thin converging lens with focal length \( f_2 = 10.0\, cm \) are separated by 20.0 cm. An object is placed at a distance of 10.0 cm to the left of the first lens. i) Find the position of the final image; ii) draw the rays diagram for this situation; iii) what is the magnification of the final image; iv) is the final image virtual or real?

The black rays are from the original object and the blue rays are for the image formed by the converging lens using the intermediate image (blue arrow). The final image is indicated by the red arrow.

For the diverging lens (left), we have \( \frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} \) and for the converging lens (right), we have \( \frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} \).

By convention, \( f_1 \) is negative and \( f_2 \) is positive. From the first lens equation, we have

\[
\frac{1}{s_1'} = \frac{1}{-10} - \frac{1}{10} = -\frac{2}{10} \quad \Rightarrow \quad s_1' = -5.0\, cm.
\]

As indicated by the rays diagram, this image is a virtual image for lens #1 and it is real object for the converging lens and

\[
s_2 = 20\, cm + |s_1'| = +25\, cm.
\]

Putting this into the converging lens equation, we have...
\[ \frac{1}{s_2'} = \frac{1}{10} - \frac{1}{25} = \frac{3}{50} \quad \Rightarrow \quad s_2' = +16.7 \text{ cm}. \] The final image is real as indicated and is located 16.7 cm on the outgoing side of the converging lens.

\[ M = \frac{m_{\text{con}} m_{\text{div}}}{s_1} = -\frac{s_1'}{s_1} \left( -\frac{s_2'}{s_2} \right) = -\frac{5}{10} \left( -\frac{50}{25} \right) = -\frac{1}{3}. \] The final image is inverted and smaller (about 1/3 of its original length).
3. (25 pts)
In a double slit interference experiment, a thin sheet of plastic (n=1.60) is covering the lower slit so that the wave entering the bottom slit has an additional phase of 90° compared to the top slit. The two slits are d=0.0600mm apart and the whole apparatus is illuminated by a light source with \( \lambda = 550\text{nm} \). An interference pattern is observed on a screen 10.0m away from the slits. a) What is the thickness of the plastic sheet? b) At what distance y with respect to the midline of the apparatus is the central maximum located on the screen? c) What is the vertical distance between the first order and the central maximum on the screen?

\[ a) \quad \frac{t}{\lambda_n} - \frac{t}{\lambda} = \frac{1}{4} \quad \Rightarrow \quad t = \frac{\lambda}{4(n-1)} = \frac{550\text{nm}}{4(1.60-1)} = 229\text{nm} \]

b) For constructive interference, we need the TOTAL phase difference for the two waves between the slits to be a multiple of a wavelength. In this case, since the initial phase difference between the waves from the two slits is already 90° or \( \lambda/4 \) out of phase, we need,

\[ d \sin \theta + \frac{\lambda}{4} = m\lambda \quad \Rightarrow \quad d \sin \theta = (m-\frac{1}{4})\lambda, \text{ where } m = 0, \pm 1, \pm 2, \cdots \]

\[ d \sin \theta_0 = -\frac{\lambda}{4} \]

So, the central maximum (m=0) is located at

\[ \theta_0 = \sin^{-1}\left(-\frac{550 \times 10^{-9} m}{4(0.06 \times 10^{-3} m)}\right) = -0.00229 \]

For small angles, we can approximate \( \tan \theta = \sin \theta = \theta \) so that the vertical location of the central maximum is at \( y_0 = 10.0m(-0.00229) = -0.0229m \) below the midline of slits.

c) The first order maxima are located at \( d \sin \theta_1 = 3\lambda/4 \) (or \( 5\lambda/4 \)). The vertical separation between either one and the central maximum is

\[ \Delta y = 10.0m \left(\frac{\lambda}{d}\right) = 10.0m \left(\frac{550 \times 10^{-9} m}{0.06 \times 10^{-3} m}\right) = 0.0917m \]
4. (25 pts)
A light ray enters a glass prism \((n=1.50)\) at an angle of \(\theta = 35^\circ\) with respect to the normal to the face \(ab\). Find the largest value for \(\phi\) such that the ray will be totally internally reflected at the face \(ac\) of the prism. Assume that the prism is surrounded by air.

At the face \(ab\), Snell’s law gives

\[
\sin \theta = n \sin \alpha \\
\alpha = \sin^{-1} \left( \frac{\sin \theta}{n} \right) = \sin^{-1} \left( \frac{\sin 35^\circ}{1.5} \right) = 22.48^\circ
\]

At the face \(ac\), for total internal reflection, we need

\[
n \sin \beta = 1 \\
\beta = \sin^{-1} \left( \frac{1}{n} \right) = \sin^{-1} \left( \frac{1}{1.5} \right) = 41.81^\circ
\]

Thus, \(\beta\) is the critical incident angle that a light ray can make on the inside face of \(ac\) so that it will be totally internally reflected if \(\beta > 41.81^\circ\).

Now, from geometry, if we consider the red 4-sided (red) polygon, we can relate all these angles together.

\[
90^\circ + (90^\circ + \alpha) + \beta + (90^\circ + \phi) = 360^\circ \\
\beta = 90^\circ - \alpha - \phi = 67.52^\circ - \phi
\]

We know that for total internal reflection at the interface \(ac\), \(\beta > 41.81^\circ\). Thus, \(\phi\) can’t be larger than \(67.52^\circ - 41.81^\circ = 25.71^\circ\).

\(\phi \leq 25.7^\circ\)