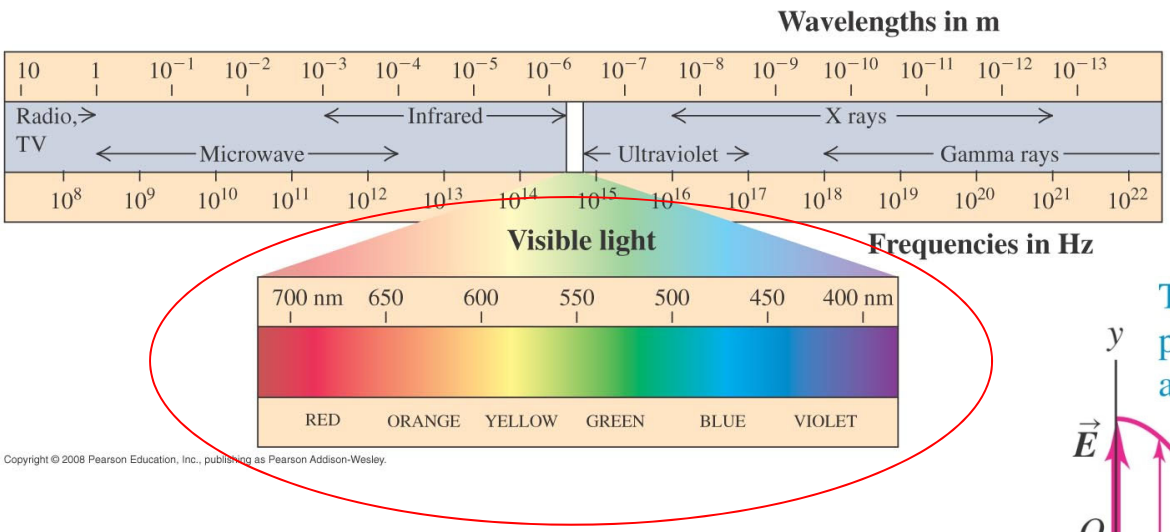
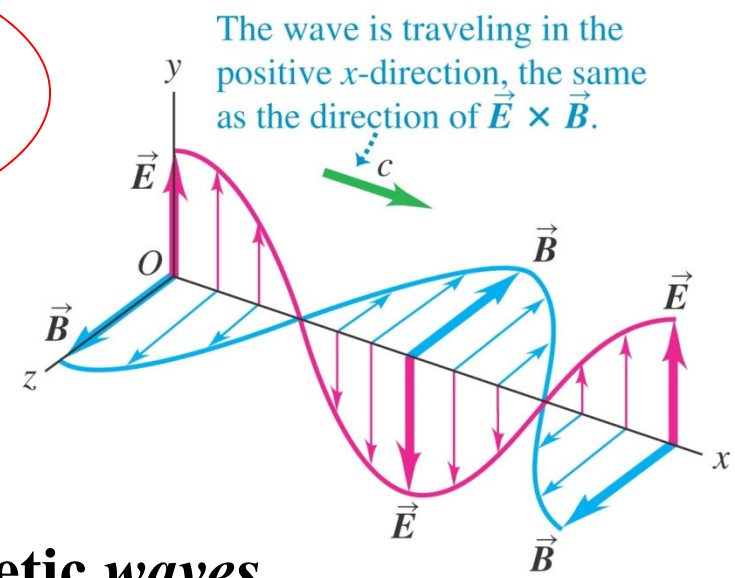
The Nature of Light





Light is a propagating **electromagnetic *waves***

Index of Refraction *n*:

- In materials, light interacts with atoms/molecules and travels *slower* than it can in vacuum, e.g.,

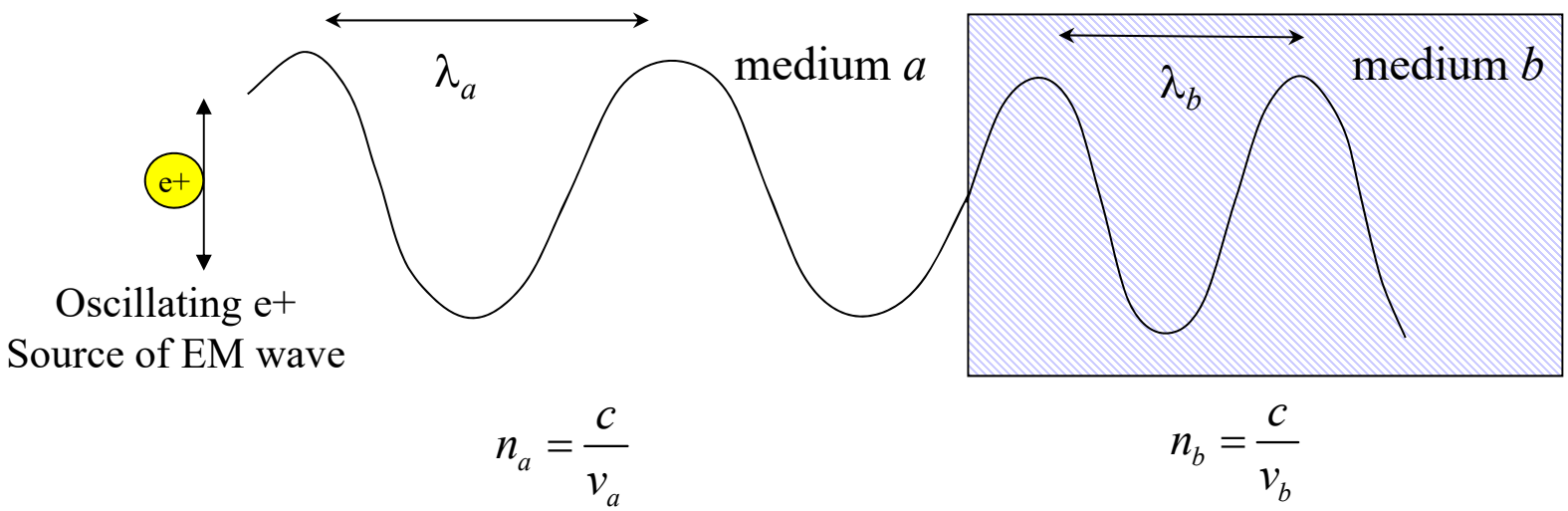
$$v_{water} \cong \frac{3}{4}c$$

- The optical property of transparent materials is called the **Index of Refraction**:

$$n \equiv \frac{c}{v_{material}}$$ (Table 33.1)

- Since $v_{\text{material}} < c$ always, $n > 1$!

Index of Refraction and Wave Aspects of Light



The wavelength of a light changes in different medium accordingly,

$$n_a \lambda_a = n_b \lambda_b$$

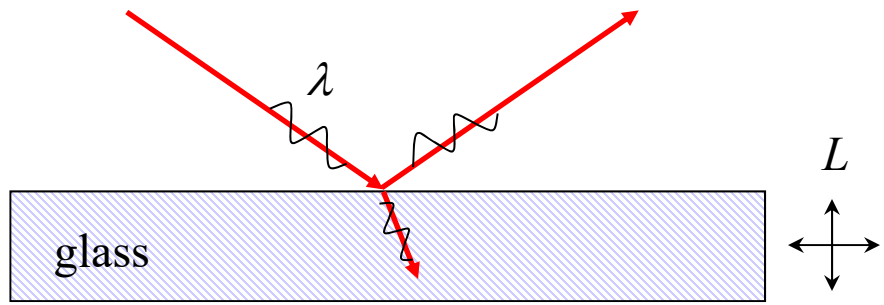
With one medium being a *vacuum*, we have $\lambda_n = \lambda / n$

The Study of Light: Optics

- Condition for Rays Optics:

$$L >> \lambda$$

Relevant system size >> wavelength



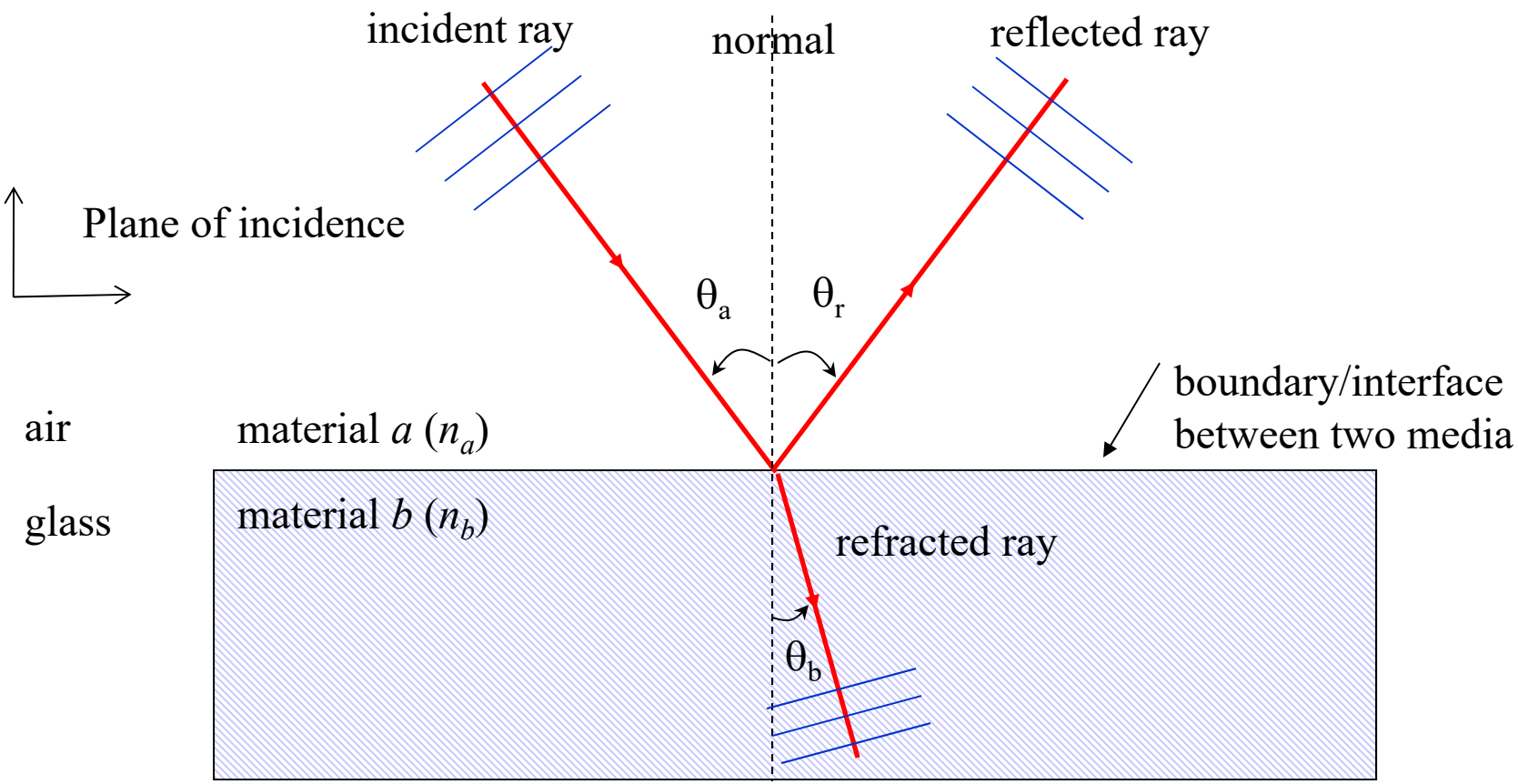
In this approximation, wave characteristic of light is not important and rays model of light gives accurate predictions.

(Visible light: $\lambda \sim 500$ nm $<< L$ → Rays Optics works well with typical optical instruments: mirror, lens, cameras, telescopes,…)

- Physical (Wave) Optics (Ch.35-36):

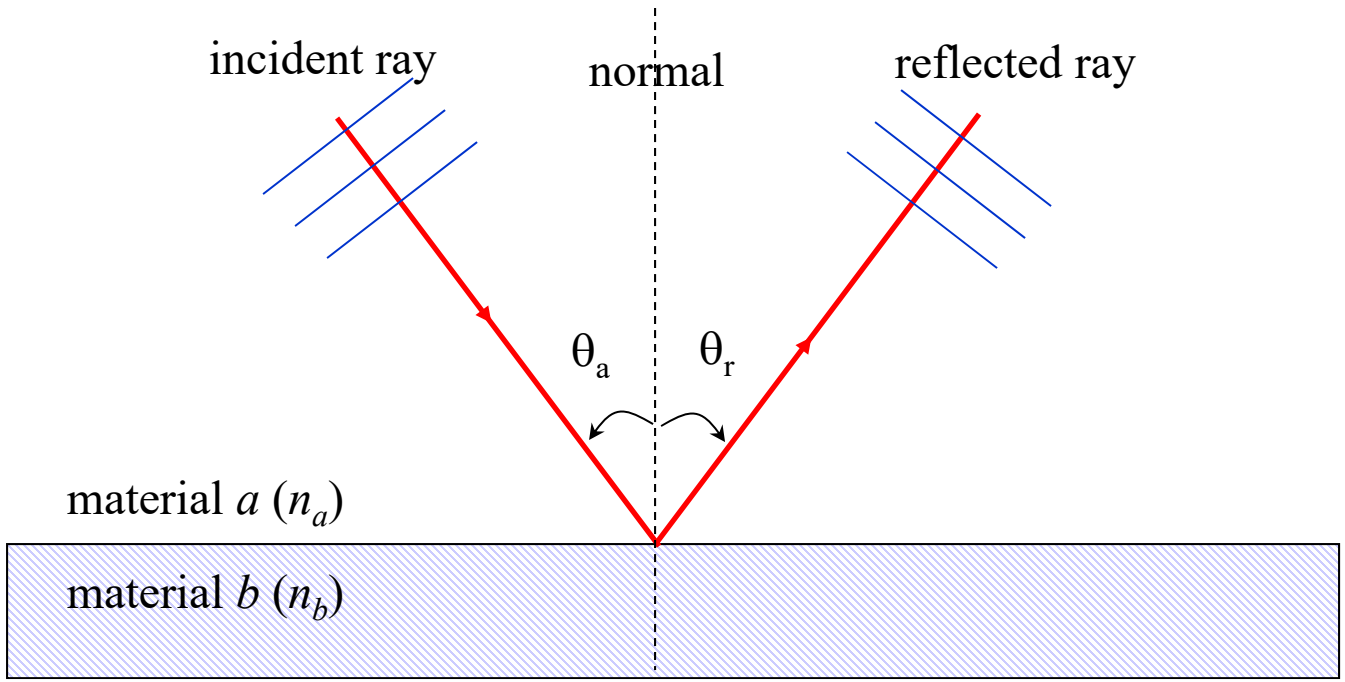
  The study of light when wave properties of light are important (diffraction and interference). $L \approx \lambda$

Reflection and Refraction



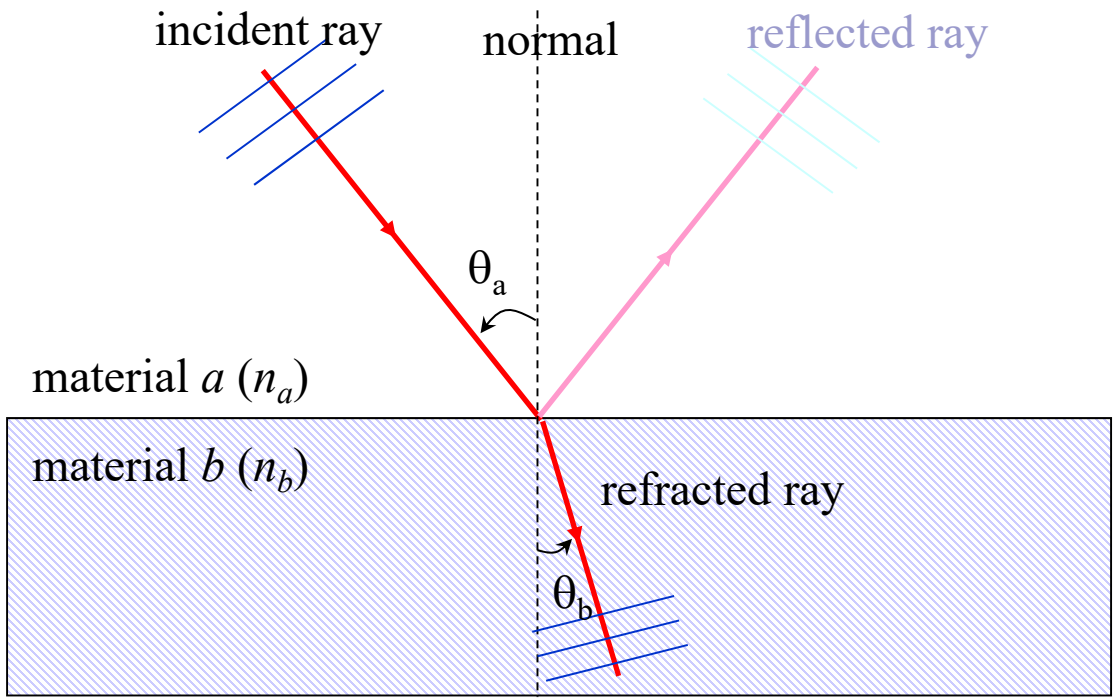
When light hits a *boundary*, typically a part of it will be *reflected* & a part of it will be *refracted*.

Law of Reflection



θ_r (angle of reflection) = θ_a (angle of incidence)

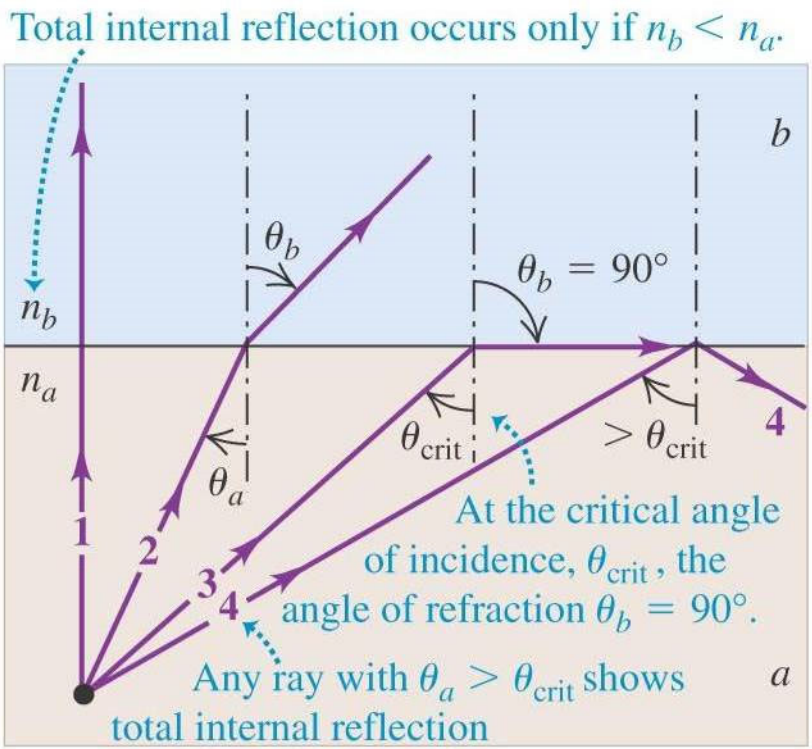
Law of Refraction (Snell's Law)



$$n_a \sin\theta_a = n_b \sin\theta_b$$

Total Internal Reflection

- Light moves from a medium with a *larger n* to one with a *smaller n.*
- As the angle of incidence becomes more and more acute, the light ceases to be transmitted, only reflected.



At the interface,

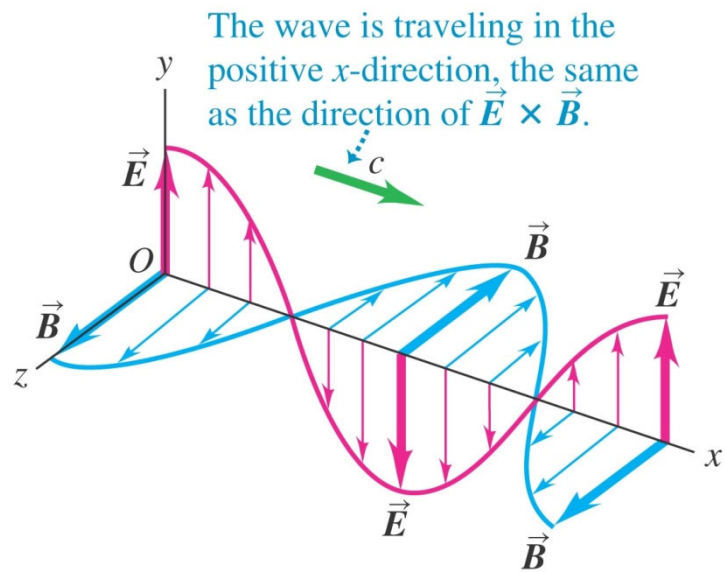
$$\sin\theta_{crit} = \frac{n_b}{n_a}$$

A Linearly Polarized EM Wave

For an electromagnetic wave, the direction of the *electric* field vector $\vec{\mathbf{E}}(x,t)$ gives the **polarization** of the wave.

An transverse electromagnetic wave with polarization in the *y*-direction:

$$\begin{cases} \vec{\mathbf{E}}(x,t) = E_{\max}\cos(kx-\omega t)\,\hat{\mathbf{j}} \\ \vec{\mathbf{B}}(x,t) = B_{\max}\cos(kx-\omega t)\,\hat{\mathbf{k}} \end{cases}$$



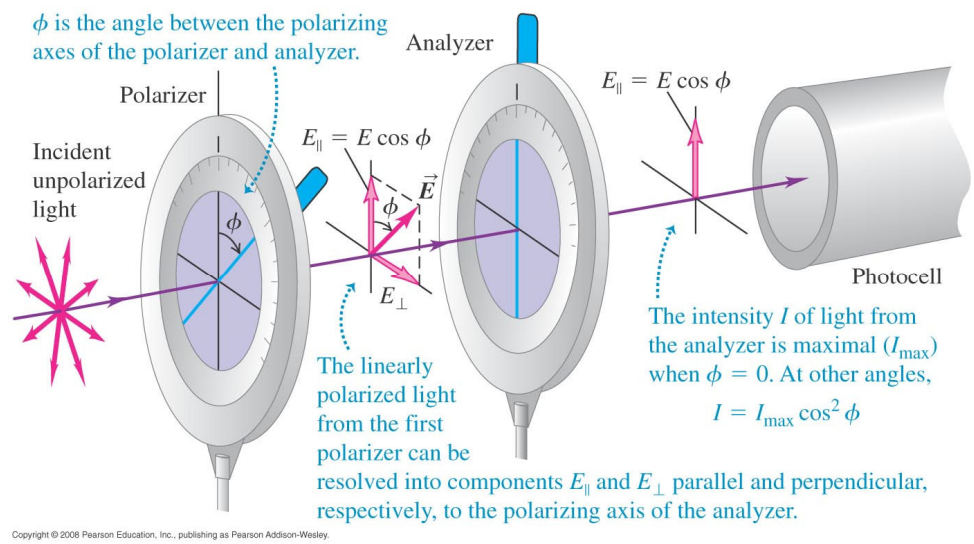
A polarized wave in a well defined direction is called a *linearly polarized* wave.

Polarization by a Polarizing Filter

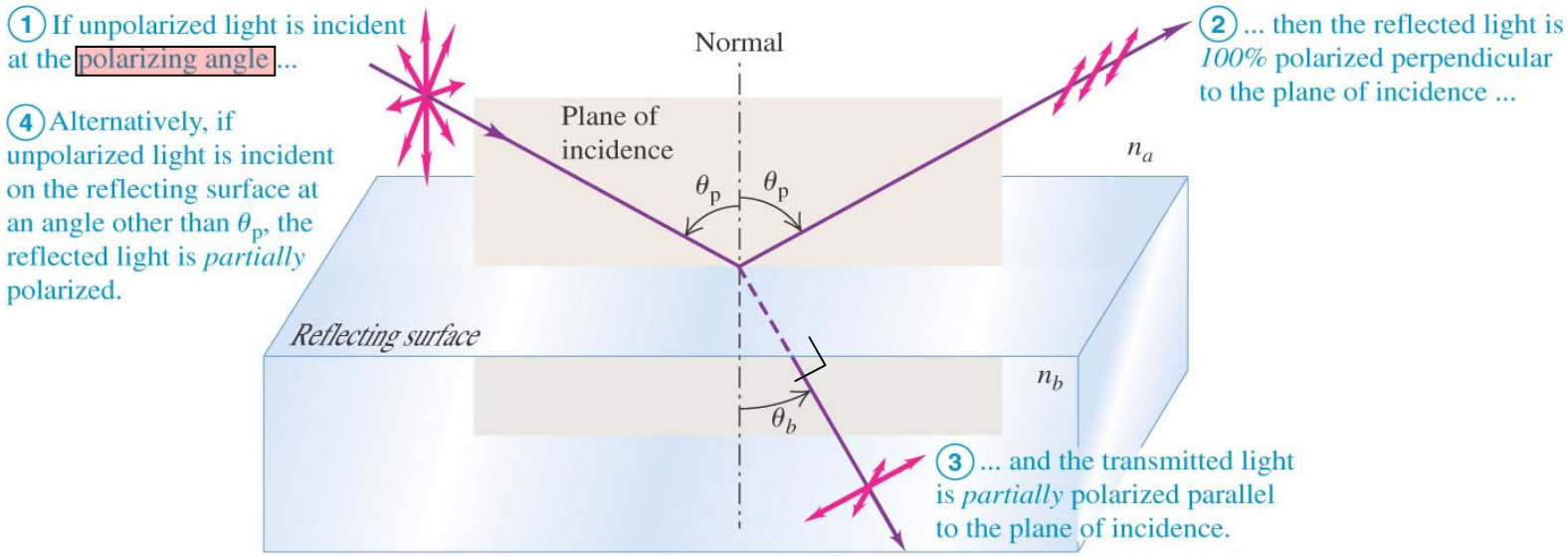
Since intensity (I) is proportional to E^2,

$$I_{trans} = I_{\max} \cos^2 \phi$$

(Malus's Law)
Transmitted intensity of
linearly polarized light
through a polarizer



Polarization by Reflection



At the special angle (**polarizing angle** or **Brewster's angle**) θ_p, the electric field component *parallel* to the "plane of incidence" will not be reflected !

(Brewster's Law) $\tan\theta_p = n_b / n_a$

Summary for Mirrors & Lens

The following are valid for both converging and diverging lens if we follow the proper sign conventions.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$ (object-image relation, lens & mirrors)

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$ (focal length, lens)

$$\frac{1}{f}=\frac{2}{R}$$ (focal length, mirrors)

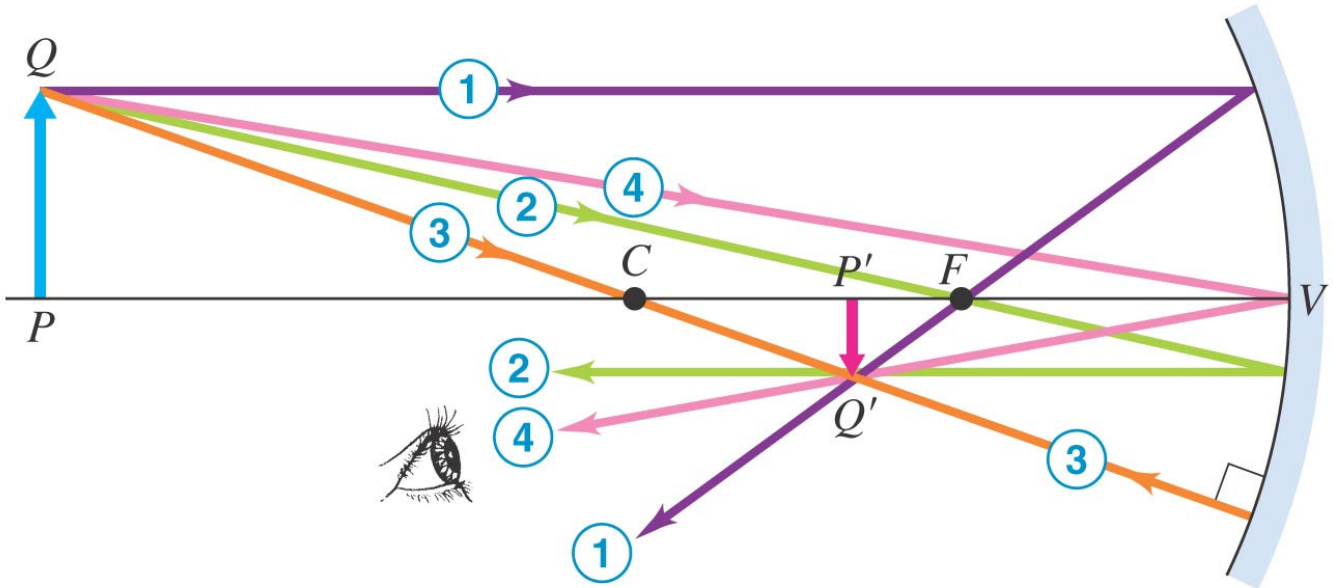
$$m=-\frac{s'}{s}$$ (lateral magnification, lens & mirrors)

Sign Rules for Mirrors & Lens

1. Object Distance:
   - *s* is + if the object is on the same side as the incoming light (for both reflecting and refracting surfaces) and *s* is – otherwise.
2. Image Distance:
   - *s'* is + if the image is on the same side as the outgoing light and is – otherwise.
3. Object/Image Height:
   - *y* (*y*') is + if the image (object) is erect or upright. It is – if it is inverted.
4. Radius of Curvature:
   - *R* is + when the center of curvature C is on the same side as the outgoing light and – otherwise.
5. Focus Length: (+ concave, - convex)
   (+ converging, - diverging)

Geometric Methods: Rays Tracing

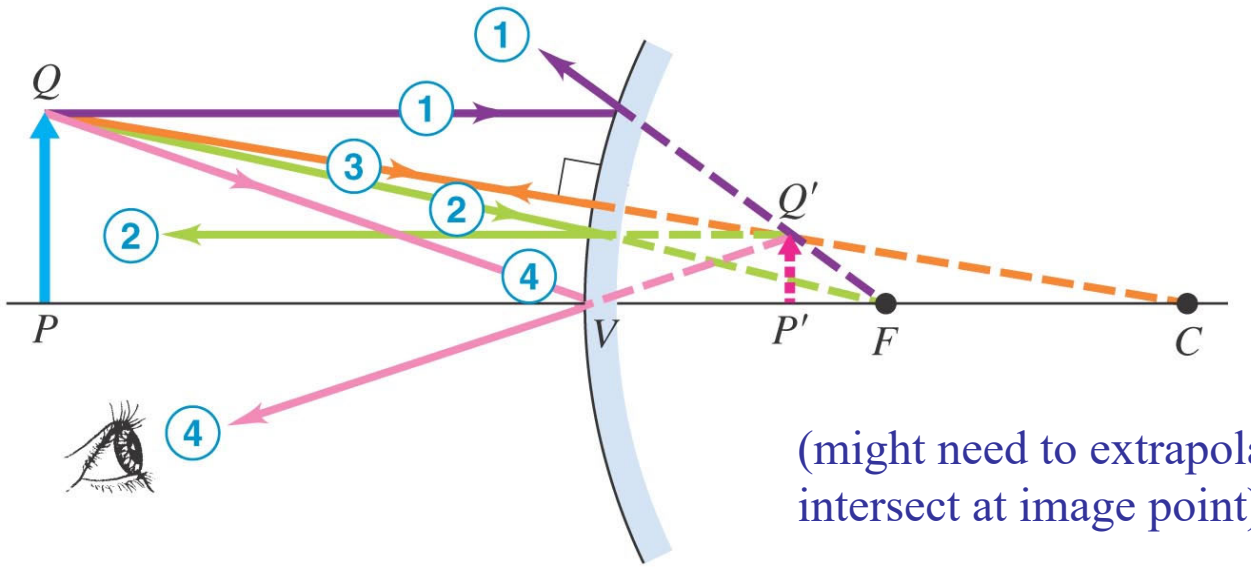
Principal rays for concave mirror



1. Ray parallel to axis reflects through focal point.
2. Ray through focal point reflects parallel to axis.
3. Ray through center of curvature intersects the surface normally and reflects along its original path.
4. Ray to vertex reflects symmetrically around optic axis.

Geometric Methods: Rays Tracing

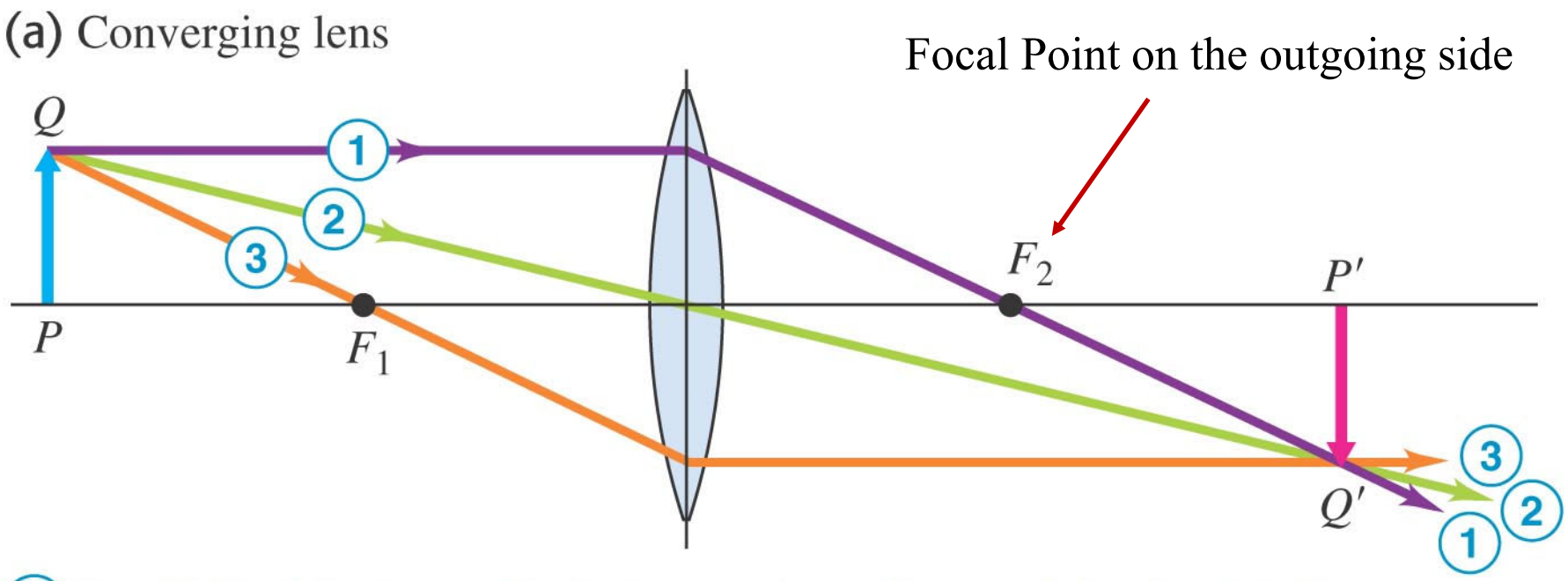
Principal rays for convex mirror



(might need to extrapolate lines to intersect at image point)

1. Reflected parallel ray appears to come from focal point.
2. Ray toward focal point reflects parallel to axis.
3. As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
4. As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

Ray Tracing Methods for Lenses

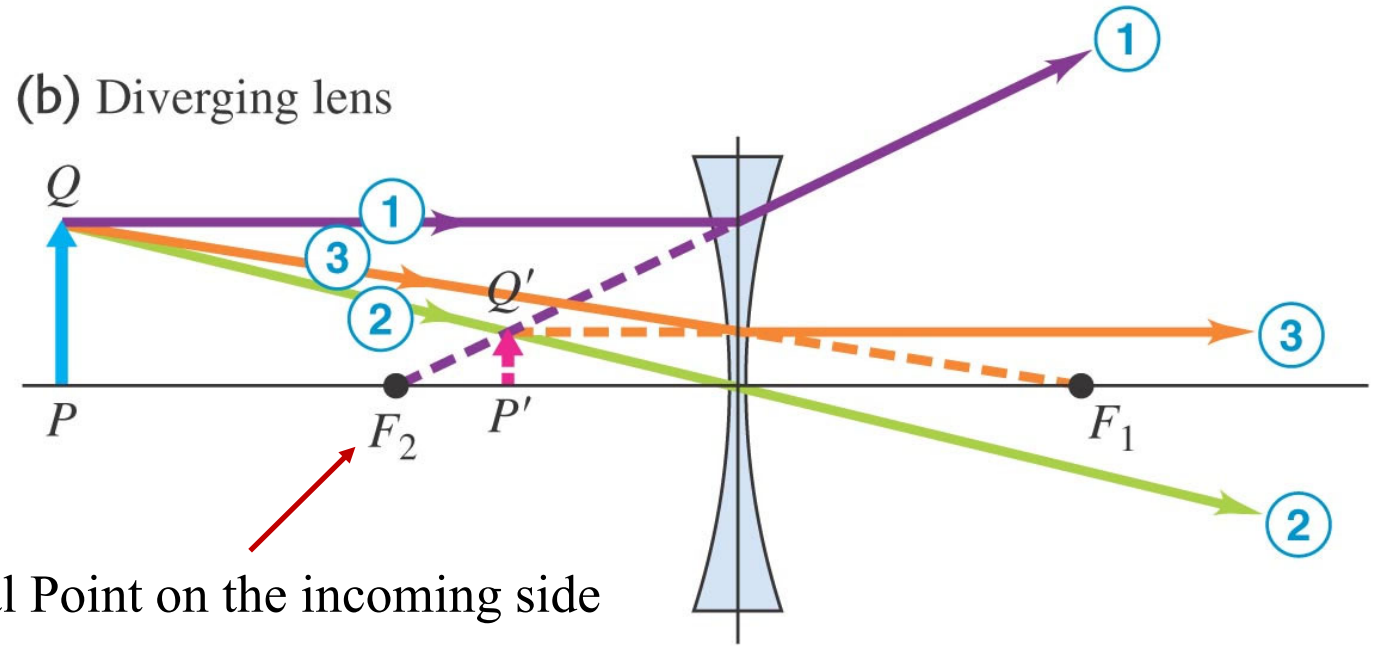


1 Parallel incident ray refracts to pass through second focal point F_2.

2 Ray through center of lens does not deviate appreciably.

3 Ray through the first focal point F_1 emerges parallel to the axis.

Rays Tracing Methods for Lenses



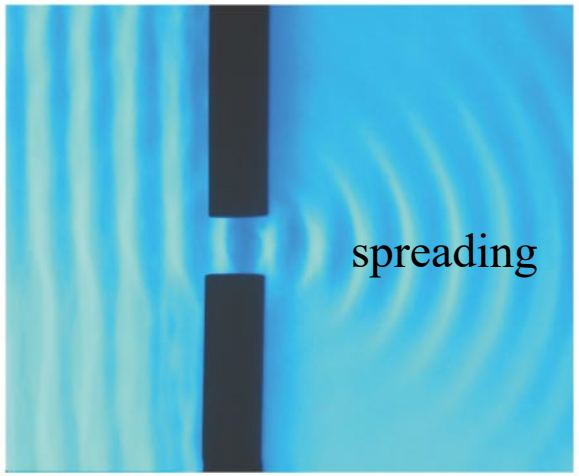
Focal Point on the incoming side

1. Parallel incident ray appears after refraction to have come from the second focal point F_2.
2. Ray through center of lens does not deviate appreciably.
3. Ray aimed at the first focal point F_1 emerges parallel to the axis.

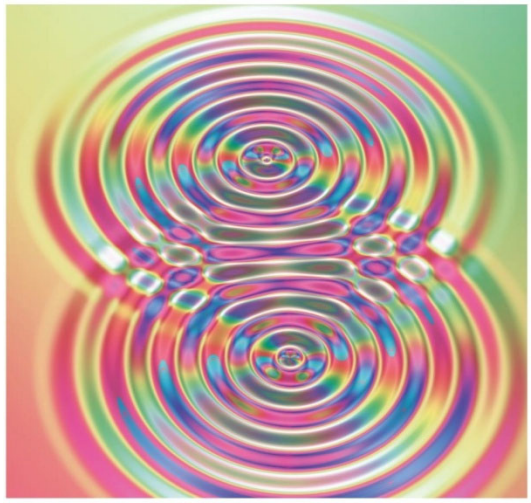
Wave Nature of Light

- Previous Chapters (Geometric Optics) $\lambda << L$
  - Rays Model is an approximation of EM waves with rays pointing in the direction of propagation
- Next Couple of Chapters (Wave/Physical Optics) $\lambda \sim L$
  - Like water waves, light *spreads* and *interferes* with each other.
  - Observed phenomena *cannot* be accounted for by rays:

Diffraction

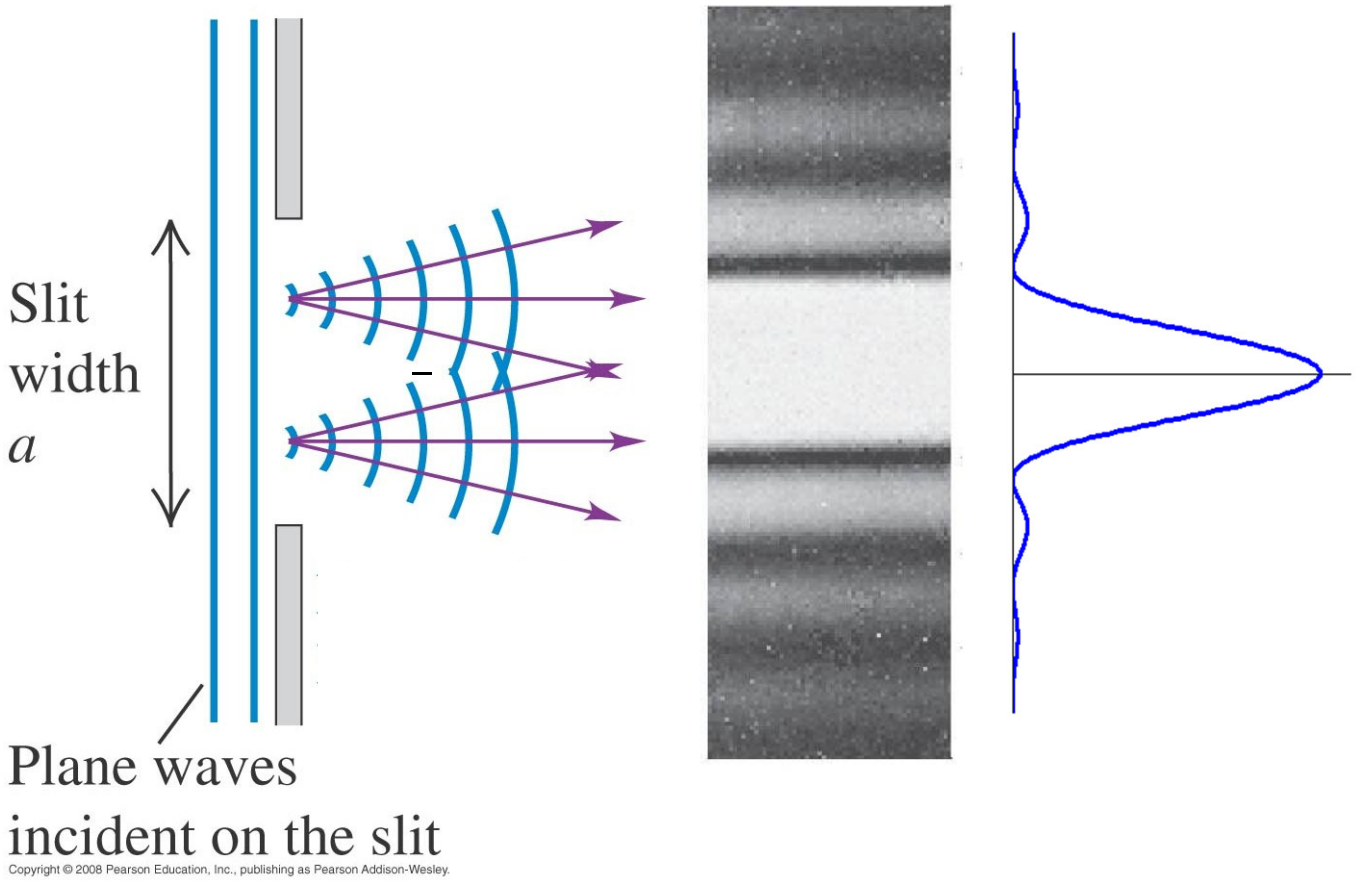


Interference



constructive/ destructive interference patterns

Single Slit Diffraction

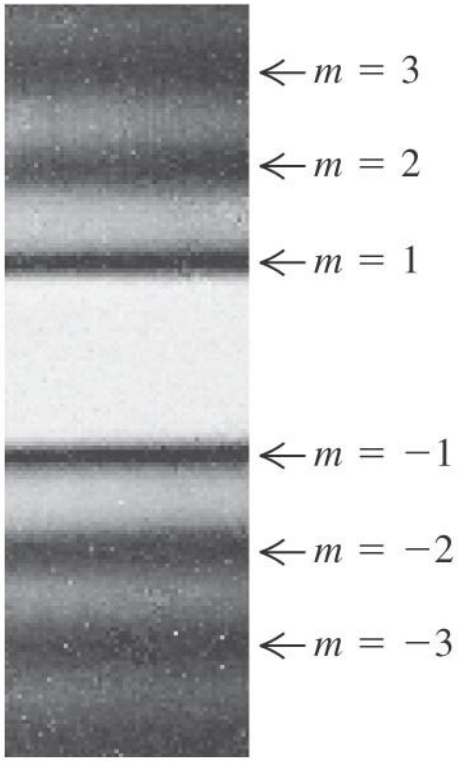


Single-Slit Diffraction: Dark Fringes

For higher order minimum with larger angular distance θ, we can use the same argument by subdividing the slit into more groups (6, 8, 10, etc.).

This leads to the following general formula for the dark fringes:

$$a \sin\theta = m\lambda, \quad m = \pm 1, \pm 2, \cdots$$



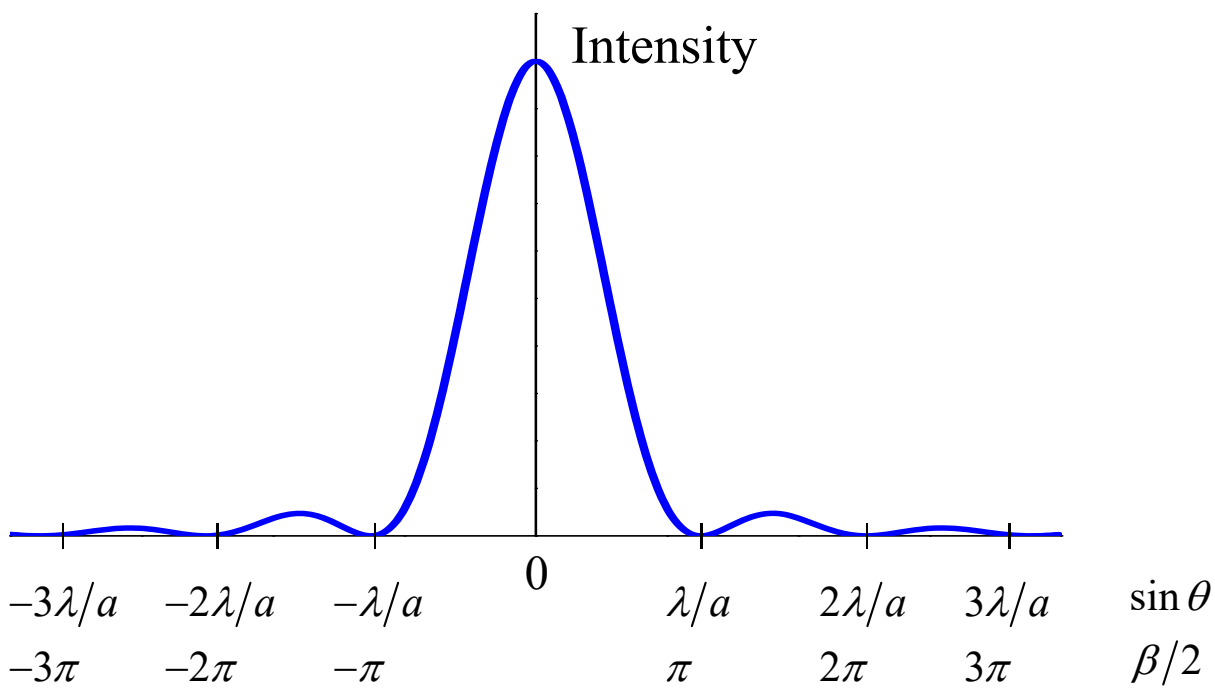
Note:

1. $m = 0$ is *not* the first minimum !
   In fact, it is the location for the central max.
2. Secondary maximum occurs *near* $3\lambda/2$, $5\lambda/2$, etc. but not exactly.

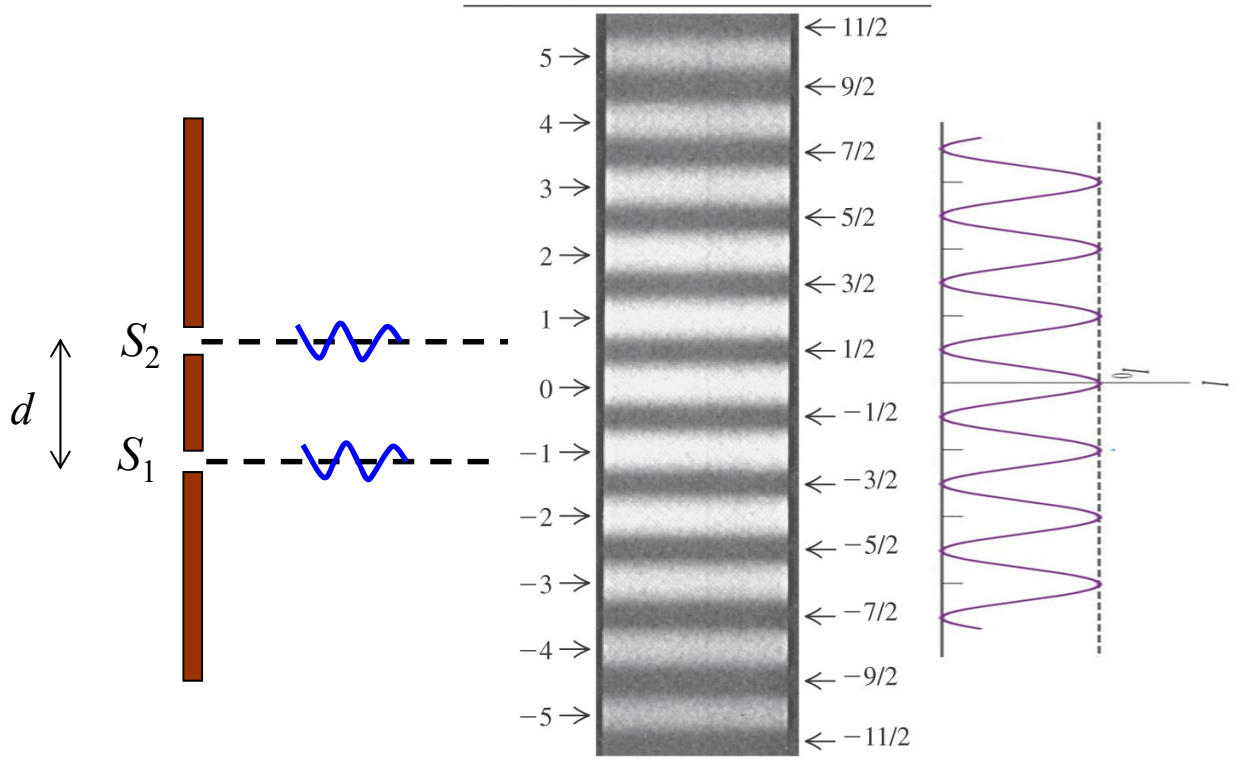
Intensity in Single-Slit Pattern

Then, lastly wiith $\beta = \frac{2\pi}{\lambda} a \sin\theta$,the intensity of the pattern as a function of θ is,

$$I = I_0 \left[\frac{\sin\left(\pi a \sin\theta/\lambda\right)}{\pi a \sin\theta/\lambda} \right]^2$$



Double-Slit Interference Pattern (w/o diffraction)



Constructive/Destructive Two-Slit Interference

Applying the conditions for constructive/ destructive interference, we have the following conditions:

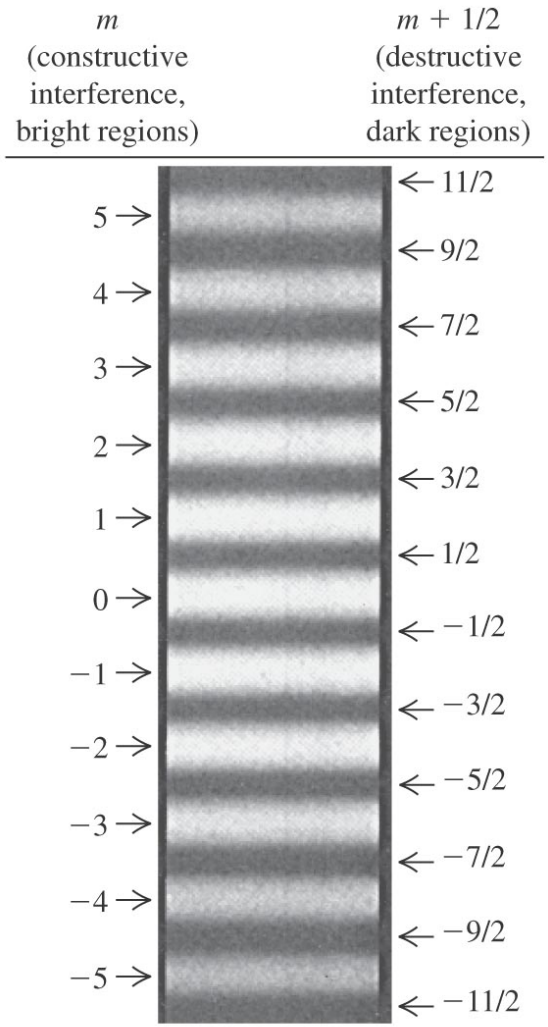
Constructive Interference: Two Slit Interference

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \cdots)$$

Destructive Interference: Two Slit Interference

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \cdots)$$

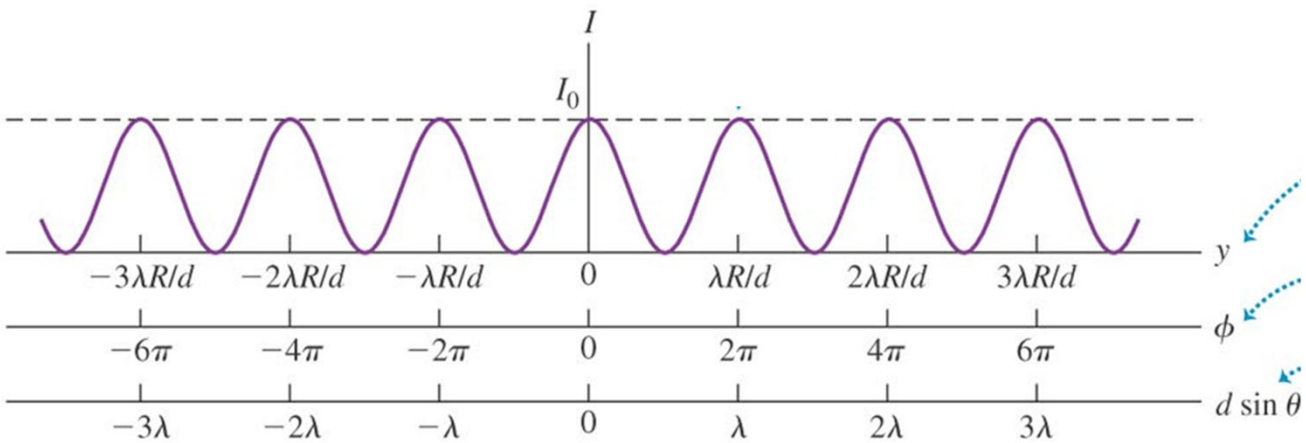
- The bright/dark bands in the pattern are called fringes
- m is the *order* of the fringes



Intensity in Two-Slit Interference

Putting this expression for the phase difference into our previous intensity equation for a two-slit interference pattern, we have,

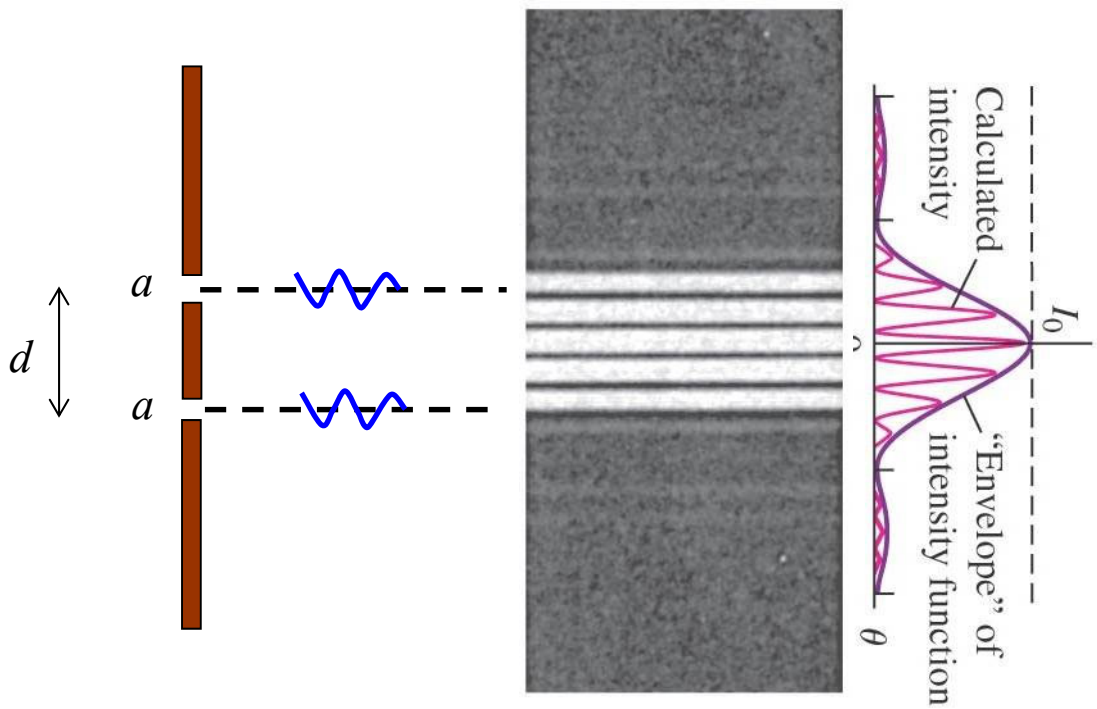
$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

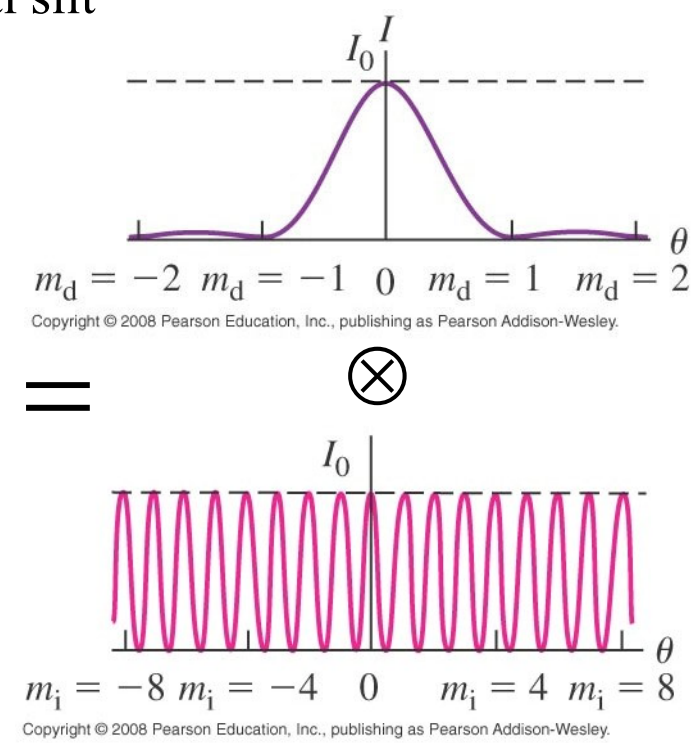


Full Double-Slit Diffraction Pattern

Combined effects of → interference from both slits

→ diffraction from individual slit

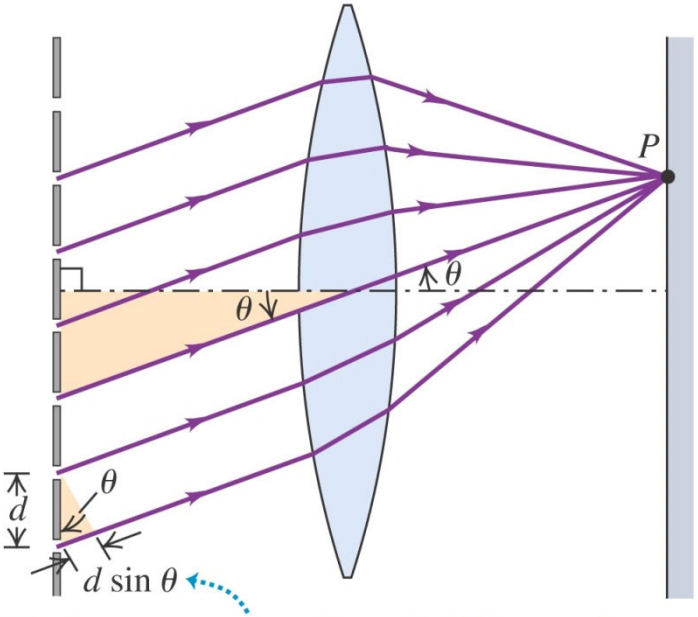


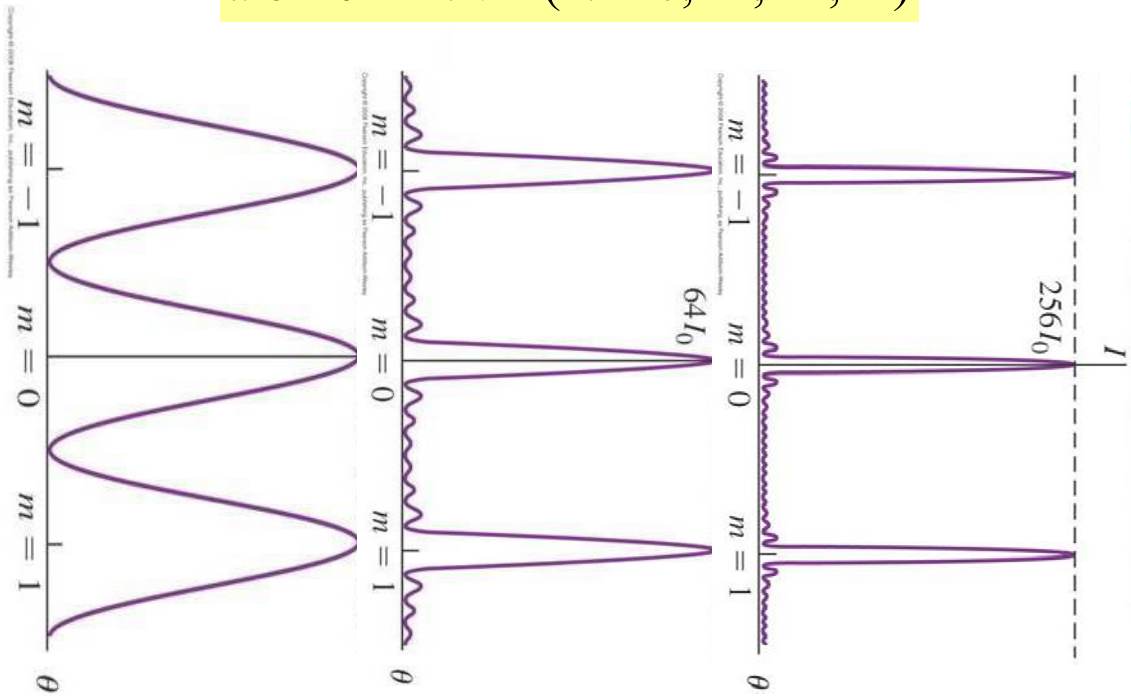


Diffraction Patterns from Multiple Slits

locations of Maxs stay the same

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \cdots)$$



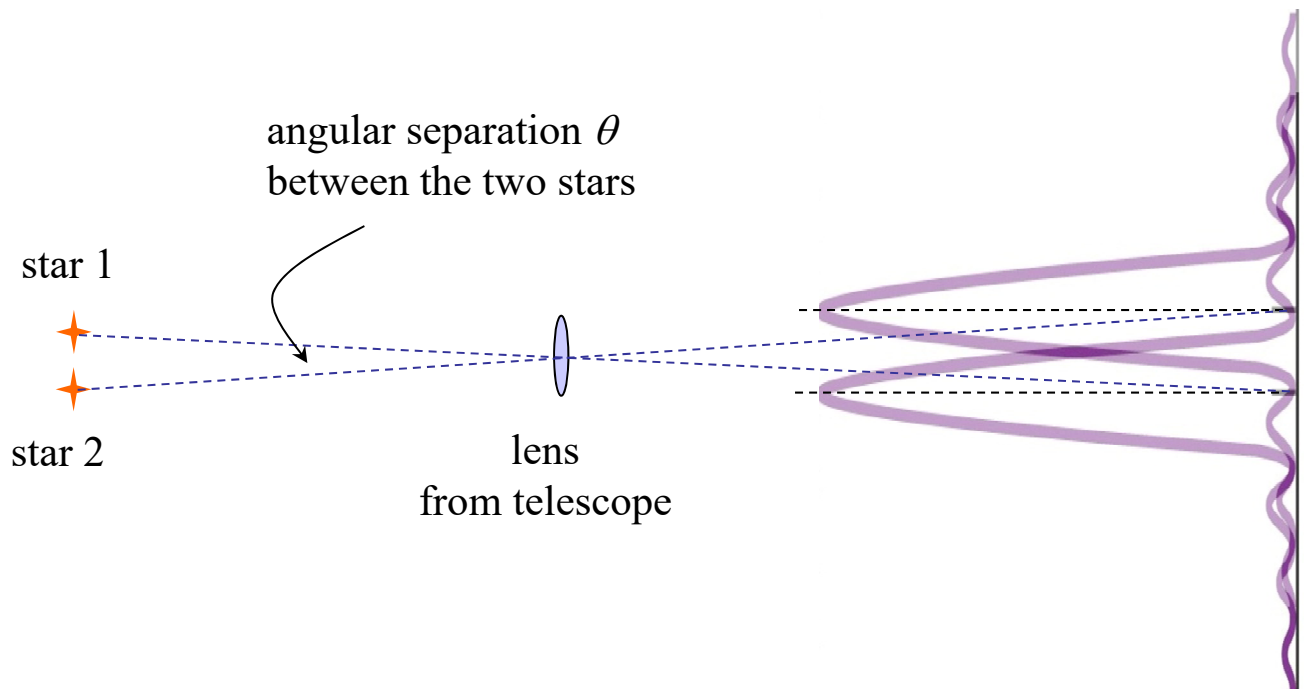


$N \rightarrow$ larger

peaks get sharper

Resolving Power for Circular Apertures

Consider two *non-coherent* point sources (so that they don't interfere), i.e. two distant stars,

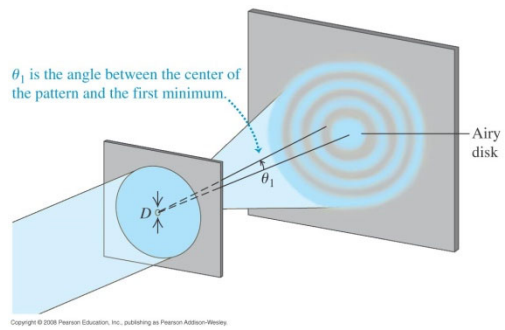


We will observe two diffraction patterns on top of each others.

Resolving Power for an Apertures

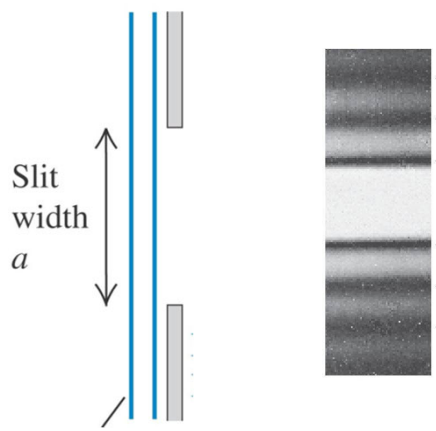
The **Limit of Resolution** for an aperture is defined as the smallest angular separation between two light sources that can be resolved according to the *Rayleigh's Criterion* and it is given by:

Circular Aperture



$$\sin\theta_{\min} = 1.22\frac{\lambda}{D}$$

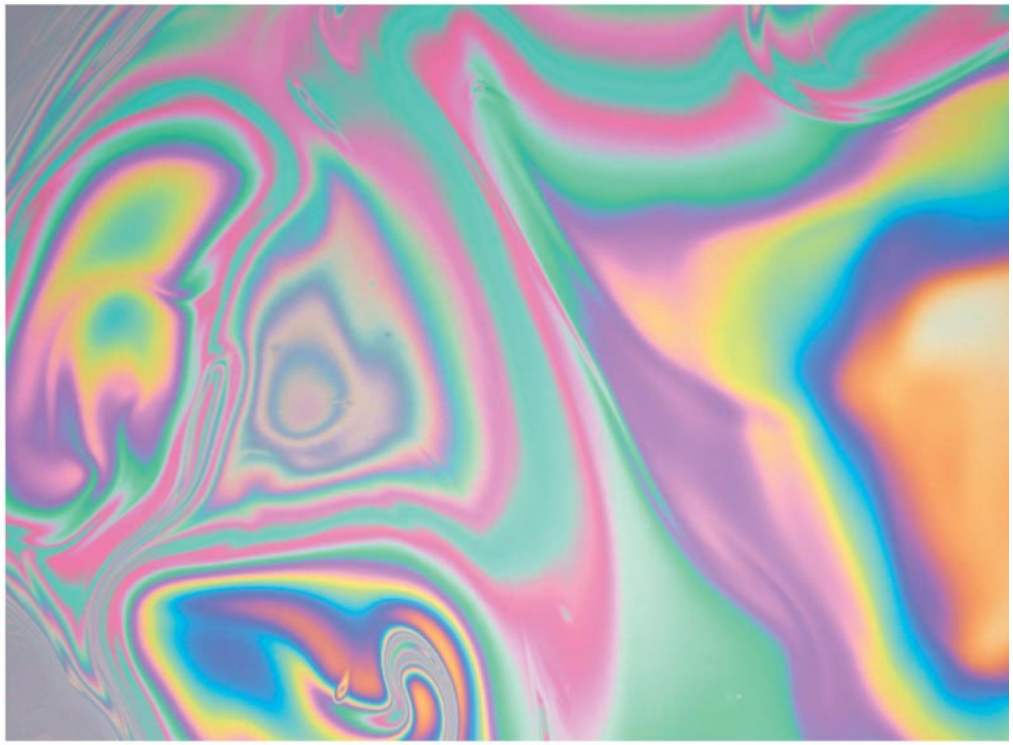
Single Aperture



$$\sin\theta_{\min} = \frac{\lambda}{a}$$

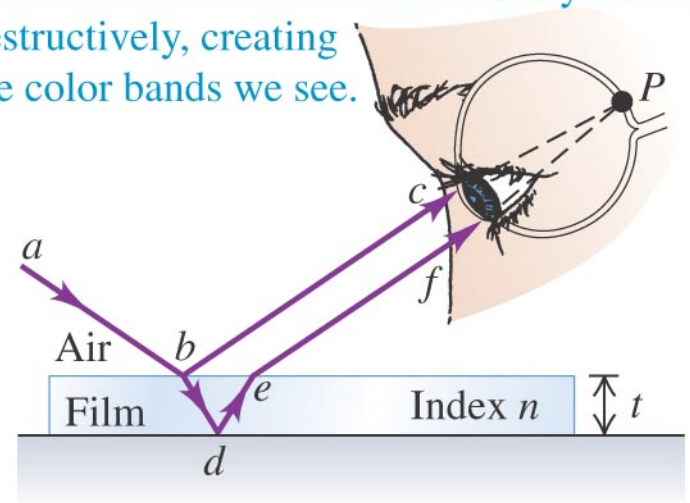
Interference in Thin Films

Color fringes observed from an oil slick on water or on a soap bubble are the white-light *interference* patterns produced by the *reflected* light off a *thin film* of oil or soap.



Light reflected from the upper and lower surfaces of the film comes together in the eye at *P* and undergoes interference.

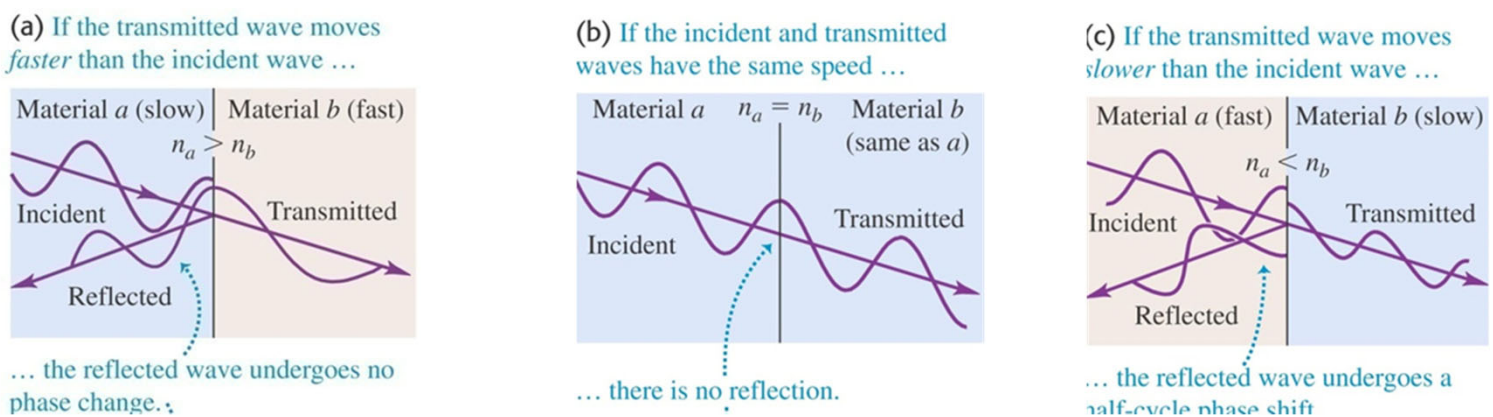
Some colors interfere constructively and others destructively, creating the color bands we see.



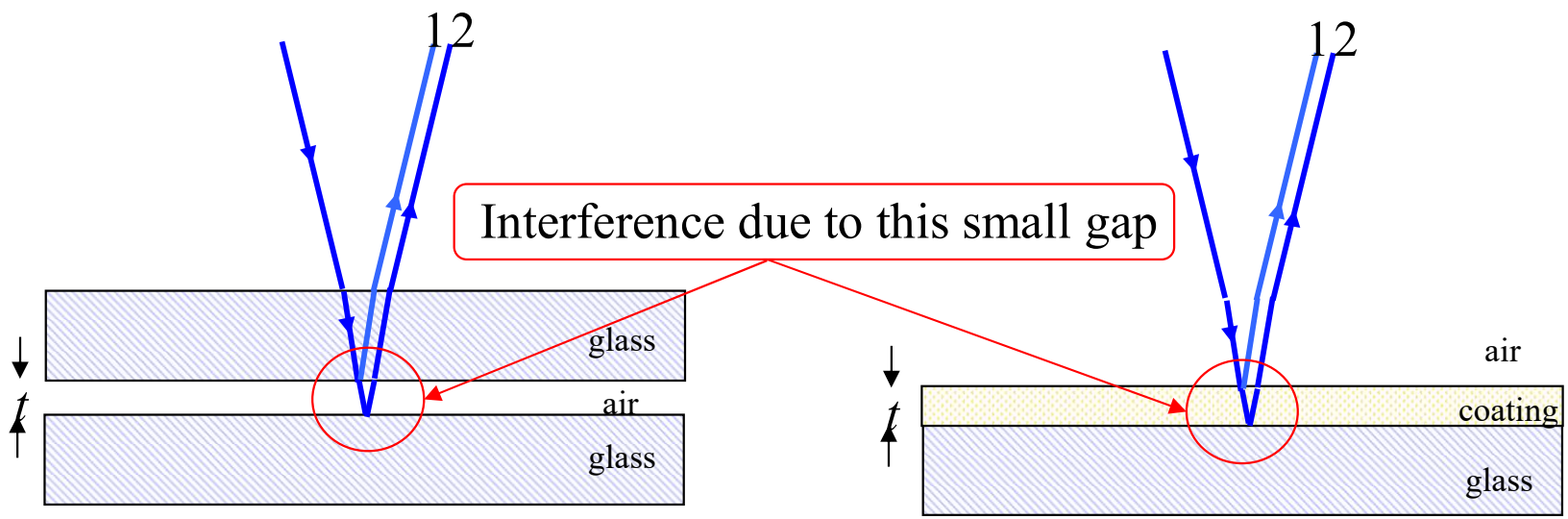
Phase Shifts During Reflection

From Maxwell's Equations, one can show that the reflected wave will suffer a 180° or $\lambda/2$ phase shift if it is reflected off from a medium with a *higher n*.

$$E_r = \frac{n_a - n_b}{n_a + n_b} E_i \qquad \text{(for normal incidence)}$$



Interference from a Thin Film



$n_{glass} > n_{air} < n_{glass}$ $\qquad$ $n_{air} < n_{coating} < n_{glass}$

Constructive: $2t = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \cdots$ $\qquad$ $2n_{coating}t = m\lambda, \quad m = 0, 1, 2, \cdots$

Destructive: $2t = m\lambda, \quad m = 0, 1, 2, \cdots$ $\qquad$ $2n_{coating}t = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \cdots$

CHAPTER 33 SUMMARY

Video Tutor Solutions

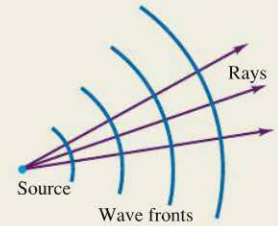
Light and its properties: Light is an electromagnetic wave. When emitted or absorbed, it also shows particle properties. It is emitted by accelerated electric charges.

A wave front is a surface of constant phase; wave fronts move with a speed equal to the propagation speed of the wave. A ray is a line along the direction of propagation, perpendicular to the wave fronts.

When light is transmitted from one material to another, the frequency of the light is unchanged, but the wavelength and wave speed can change. The index of refraction n of a material is the ratio of the speed of light in vacuum c to the speed v in the material. If λ_0 is the wavelength in vacuum, the same wave has a shorter wavelength λ in a medium with index of refraction n. (See Example 33.2.)

$$n = \frac{c}{v} \quad (33.1)$$

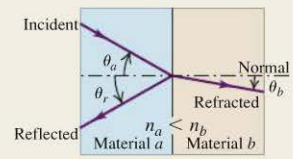
$$\lambda = \frac{\lambda_0}{n} \quad (33.5)$$



Reflection and refraction: At a smooth interface between two optical materials, the incident, reflected, and refracted rays and the normal to the interface all lie in a single plane called the plane of incidence. The law of reflection states that the angles of incidence and reflection are equal. The law of refraction relates the angles of incidence and refraction to the indexes of refraction of the materials. (See Examples 33.1 and 33.3.)

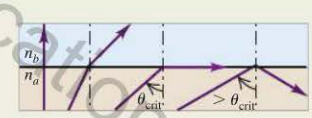
$$\theta_r = \theta_a \quad (33.2)$$
(law of reflection)

$$n_a \sin\theta_a = n_b \sin\theta_b \quad (33.4)$$
(law of refraction)



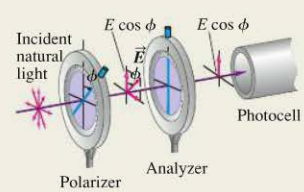
Total internal reflection: When a ray travels in a material of greater index of refraction n_a toward a material of smaller index n_b, total internal reflection occurs at the interface when the angle of incidence exceeds a critical angle θ_{crit}. (See Example 33.4.)

$$\sin\theta_{crit} = \frac{n_b}{n_a} \quad (33.6)$$



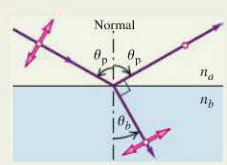
Polarization of light: The direction of polarization of a linearly polarized electromagnetic wave is the direction of the $\vec{E}$ field. A polarizing filter passes waves that are linearly polarized along its polarizing axis and blocks waves polarized perpendicularly to that axis. When polarized light of intensity I_{max} is incident on a polarizing filter used as an analyzer, the intensity I of the light transmitted through the analyzer depends on the angle ϕ between the polarization direction of the incident light and the polarizing axis of the analyzer. (See Example 33.5.)

$$I = I_{max}\cos^2\phi \quad (33.7)$$
(Malus's law)

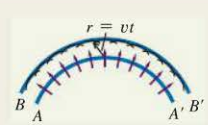


Polarization by reflection: When unpolarized light strikes an interface between two materials, Brewster's law states that the reflected light is completely polarized perpendicular to the plane of incidence (parallel to the interface) if the angle of incidence equals the polarizing angle θ_p. (See Example 33.6.)

$$\tan\theta_p = \frac{n_b}{n_a} \quad (33.8)$$
(Brewster's law)



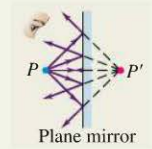
Huygens's principle: Huygens's principle states that if the position of a wave front at one instant is known, then the position of the front at a later time can be constructed by imagining the front as a source of secondary wavelets. Huygens's principle can be used to derive the laws of reflection and refraction.



CHAPTER 34 SUMMARY

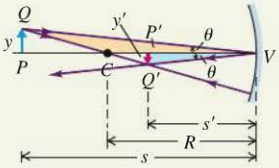
Video Tutor Solutions

Reflection or refraction at a plane surface: When rays diverge from an object point P and are reflected or refracted, the directions of the outgoing rays are the same as though they had diverged from a point P' called the image point. If they actually converge at P' and diverge again beyond it, P' is a real image of P; if they only appear to have diverged from P', it is a virtual image. Images can be either erect or inverted.

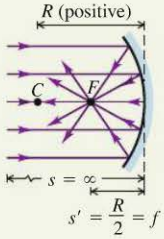


Lateral magnification: The lateral magnification m in any reflecting or refracting situation is defined as the ratio of image height y' to object height y. When m is positive, the image is erect; when m is negative, the image is inverted.

$$m = \frac{y'}{y} \qquad (34.2)$$



Focal point and focal length: The focal point of a mirror is the point where parallel rays converge after reflection from a concave mirror, or the point from which they appear to diverge after reflection from a convex mirror. Rays diverging from the focal point of a concave mirror are parallel after reflection; rays converging toward the focal point of a convex mirror are parallel after reflection. The distance from the focal point to the vertex is called the focal length, denoted as f. The focal points of a lens are defined similarly.



Relating object and image distances: The formulas for object distance s and image distance s' for plane and spherical mirrors and single refracting surfaces are summarized in the table. The equation for a plane surface can be obtained from the corresponding equation for a spherical surface by setting $R = \infty$. (See Examples 34.1–34.7.)

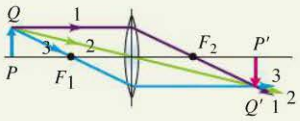
	Plane Mirror	Spherical Mirror	Plane Refracting Surface	Spherical Refracting Surface
Object and image distances	$\frac{1}{s} + \frac{1}{s'} = 0$	$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$	$\frac{n_a}{s} + \frac{n_b}{s'} = 0$	$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$
Lateral magnification	$m = -\frac{s'}{s} = 1$	$m = -\frac{s'}{s}$	$m = -\frac{n_a s'}{n_b s} = 1$	$m = -\frac{n_a s'}{n_b s}$

Object–image relationships derived in this chapter are valid only for rays close to and nearly parallel to the optic axis; these are called paraxial rays. Nonparaxial rays do not converge precisely to an image point. This effect is called spherical aberration.

Thin lenses: The object–image relationship, given by Eq. (34.16), is the same for a thin lens as for a spherical mirror. Equation (34.19), the lensmaker's equation, relates the focal length of a lens to its index of refraction and the radii of curvature of its surfaces. (See Examples 34.8–34.11.)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad (34.16)$$

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (34.19)$$



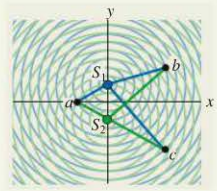
Sign rules: The following sign rules are used with all plane and spherical reflecting and refracting surfaces.

- $s > 0$ when the object is on the incoming side of the surface (a real object); $s < 0$ otherwise.
- $s' > 0$ when the image is on the outgoing side of the surface (a real image); $s' < 0$ otherwise.
- $R > 0$ when the center of curvature is on the outgoing side of the surface; $R < 0$ otherwise.
- $m > 0$ when the image is erect; $m < 0$ when inverted.

CHAPTER 35 SUMMARY

Video Tutor Solutions

Interference and coherent sources: Monochromatic light is light with a single frequency. Coherence is a definite, unchanging phase relationship between two waves. The overlap of waves from two coherent sources of monochromatic light forms an interference pattern. The principle of superposition states that the total wave disturbance at any point is the sum of the disturbances from the separate waves.

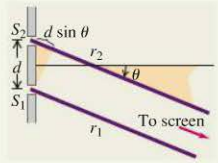


Two-source interference of light: When two sources are in phase, constructive interference occurs where the difference in path length from the two sources is zero or an integer number of wavelengths; destructive interference occurs where the path difference is a half-integer number of wavelengths. If two sources separated by a distance d are both very far from a point P, and the line from the sources to P makes an angle θ with the line perpendicular to the line of the sources, then the condition for constructive interference at P is Eq. (35.4). The condition for destructive interference is Eq. (35.5). When θ is very small, the position y_m of the mth bright fringe on a screen located a distance R from the sources is given by Eq. (35.6). (See Examples 35.1 and 35.2.)

$$d\sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$
(constructive interference) (35.4)

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$
$$(m = 0, \pm 1, \pm 2, \ldots)$$
(destructive interference) (35.5)

$$y_m = R\frac{m\lambda}{d}$$
(bright fringes) (35.6)

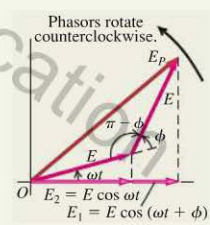


Intensity in interference patterns: When two sinusoidal waves with equal amplitude E and phase difference ϕ are superimposed, the resultant amplitude E_P and intensity I are given by Eqs. (35.7) and (35.10), respectively. If the two sources emit in phase, the phase difference ϕ at a point P (located a distance r_1 from source 1 and a distance r_2 from source 2) is directly proportional to the difference in path length $r_2 - r_1$. (See Example 35.3.)

$$E_P = 2E\left|\cos\frac{\phi}{2}\right| \quad (35.7)$$

$$I = I_0\cos^2\frac{\phi}{2} \quad (35.10)$$

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1) \quad (35.11)$$



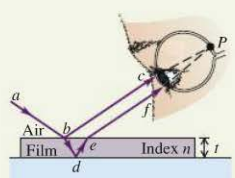
Interference in thin films: When light is reflected from both sides of a thin film of thickness t and no phase shift occurs at either surface, constructive interference between the reflected waves occurs when $2t$ is equal to an integral number of wavelengths. If a half-cycle phase shift occurs at one surface, this is the condition for destructive interference. A half-cycle phase shift occurs during reflection whenever the index of refraction in the second material is greater than that in the first. (See Examples 35.4–35.7.)

$$2t = m\lambda \quad (m = 0, 1, 2, \ldots)$$
(constructive reflection from thin film, no relative phase shift) (35.17a)

$$2t = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$$
(destructive reflection from thin film, no relative phase shift) (35.17b)

$$2t = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$$
(constructive reflection from thin film, half-cycle relative phase shift) (35.18a)

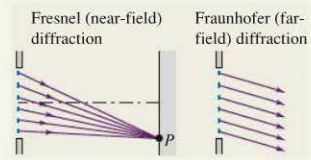
$$2t = m\lambda \quad (m = 0, 1, 2, \ldots)$$
(destructive reflection from thin film, half-cycle relative phase shift) (35.18b)



CHAPTER 36 SUMMARY

Video Tutor Solutions

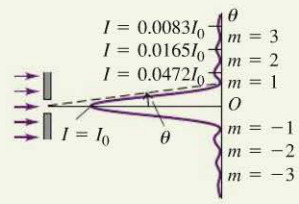
Fresnel and Fraunhofer diffraction: Diffraction occurs when light passes through an aperture or around an edge. When the source and the observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called Fraunhofer diffraction. When the source or the observer is relatively close to the obstructing surface, it is Fresnel diffraction.



Single-slit diffraction: Monochromatic light sent through a narrow slit of width a produces a diffraction pattern on a distant screen. Equation (36.2) gives the condition for destructive interference (a dark fringe) at a point P in the pattern at angle θ. Equation (36.7) gives the intensity in the pattern as a function of θ. (See Examples 36.1–36.3.)

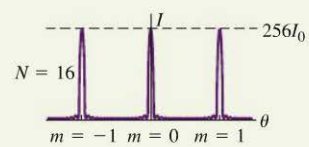
$$\sin\theta = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \ldots) \tag{36.2}$$

$$I = I_0\left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2 \tag{36.7}$$



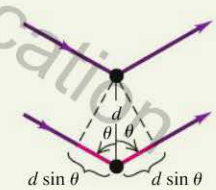
Diffraction gratings: A diffraction grating consists of a large number of thin parallel slits, spaced a distance d apart. The condition for maximum intensity in the interference pattern is the same as for the two-source pattern, but the maxima for the grating are very sharp and narrow. (See Example 36.4.)

$$d\sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \pm 3, \ldots) \tag{36.13}$$



X-ray diffraction: A crystal serves as a three-dimensional diffraction grating for x rays with wavelengths of the same order of magnitude as the spacing between atoms in the crystal. For a set of crystal planes spaced a distance d apart, constructive interference occurs when the angles of incidence and scattering (measured from the crystal planes) are equal and when the Bragg condition [Eq. (36.16)] is satisfied. (See Example 36.5.)

$$2d\sin\theta = m\lambda \qquad (m = 1, 2, 3, \ldots) \tag{36.16}$$



Circular apertures and resolving power: The diffraction pattern from a circular aperture of diameter D consists of a central bright spot, called the Airy disk, and a series of concentric dark and bright rings. Equation (36.17) gives the angular radius θ_1 of the first dark ring, equal to the angular size of the Airy disk. Diffraction sets the ultimate limit on resolution (image sharpness) of optical instruments. According to Rayleigh's criterion, two point objects are just barely resolved when their angular separation θ is given by Eq. (36.17). (See Example 36.6.)

$$\sin\theta_1 = 1.22\frac{\lambda}{D} \tag{36.17}$$