

Key Terms

particle in a box, 1376
 normalization, 1379
 potential well, 1380

square-well potential, 1381
 bound state, 1381
 potential barrier, 1384

tunneling, 1385
 harmonic oscillator, 1387

Answer to Chapter Opening Question ?

When an electron in one of these particles—called *quantum dots*—makes a transition from an excited level to a lower level, it emits a photon whose energy is equal to the difference in energy between the levels. The smaller the quantum dot, the larger the energy spacing between levels and hence the shorter (bluer) the wavelength of the emitted photons. See Example 40.4 (Section 40.2) for more details.

Answers to Test Your Understanding Questions

40.1 Answer: (v) Our derivation of the stationary-state wave functions for a particle in a box shows that they are superpositions of waves propagating in opposite directions, just like a standing wave on a string. One wave has momentum in the positive x -direction, while the other wave has an equal magnitude of momentum in the negative x -direction. The total x -component of momentum is zero.


40.2 Answer: (i) The energy levels are arranged as shown in Fig. 40.8b if $U_0 = 6E_\infty$, where $E_\infty = \pi^2\hbar^2/2mL^2$ is the ground-

level energy of an infinite well. If the well width L is reduced to $\frac{1}{2}$ of its initial value, E_∞ increases by a factor of 4 and so U_0 must also increase by a factor of 4. The energies E_1 , E_2 , and E_3 shown in Fig. 40.8b are all specific fractions of U_0 , so they will also increase by a factor of 4.

40.3 Answer: yes Figure 40.13 shows a possible wave function $\psi(x)$ for tunneling; since $\psi(x)$ is not zero within the barrier ($0 \leq x \leq L$), there is some probability that the particle can be found there.

40.4 Answer: (ii) If the second photon has a longer wavelength and hence lower energy than the first photon, the difference in energy between the first and second excited levels must be less than the difference between the ground level and the first excited level. This is the case for the hydrogen atom, for which the energy difference between levels decreases as the energy increases (see Fig. 38.9). By contrast, the energy difference between successive levels increases for a particle in a box (see Fig. 40.4b) and is constant for a harmonic oscillator (see Fig. 40.18).

PROBLEMS

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Discussion Questions

Q40.1. For the particle in a box, we chose $k = n\pi/L$ with $n = 1, 2, 3, \dots$ to fit the boundary condition that $\psi = 0$ at $x = L$. However, $n = 0, -1, -2, -3, \dots$ also satisfy that boundary condition. Why didn't we also choose those values of n ?

Q40.2. If ψ is normalized, what is the physical significance of the area under a graph of $|\psi|^2$ versus x between x_1 and x_2 ? What is the total area under the graph of $|\psi|^2$ when all x are included? Explain.

Q40.3. For a particle in a box, what would the probability distribution function $|\psi|^2$ look like if the particle behaved like a classical (Newtonian) particle? Do the actual probability distributions approach this classical form when n is very large? Explain.

Q40.4. In Chapter 15 we represented a standing wave as a superposition of two waves traveling in opposite directions. Can the wave functions for a particle in a box also be thought of as a combination of two traveling waves? Why or why not? What physical interpretation does this representation have? Explain.

Q40.5. A particle in a box is in the ground level. What is the probability of finding the particle in the right half of the box? (Refer to Fig. 40.5, but don't evaluate an integral.) Is the answer the same if the particle is in an excited level? Explain.

Q40.6. The wave functions for a particle in a box (see Fig. 40.5a) are zero at certain points. Does this mean that the particle can't move past one of these points? Explain.

Q40.7. For a particle confined to an infinite square well, is it correct to say that each state of definite energy is also a state of definite wavelength? Is it also a state of definite momentum? Explain. (*Hint:* Remember that momentum is a vector.)

Q40.8. For a particle in a finite potential well, is it correct to say that each bound state of definite energy is also a state of definite wavelength? Is it a state of definite momentum? Explain.

Q40.9. Equation 40.15 gives the time-dependent wave functions for a particle in a box. How can a wave function that has both a real part and an imaginary part describe the position (a real number) of a particle?

Q40.10. In Fig. 40.5b, the probability function is zero at the points $x = 0$ and $x = L$, the "walls" of the box. Does this mean that the particle never strikes the walls? Explain.

Q40.11. A particle is confined to a finite potential well in the region $0 < x < L$. How does the area under the graph of $|\psi|^2$ in the region $0 < x < L$ compare to the total area under the graph of $|\psi|^2$ when including all possible x ?

Q40.12. Compare the wave functions for the first three energy levels for a particle in a box of width L (see Fig. 40.5a) to the corresponding wave functions for a finite potential well of the same width (see Fig. 40.8a). How does the wavelength in the interval $0 \leq x \leq L$ for the $n = 1$ level of the particle in a box compare to the corresponding wavelength for the $n = 1$ level of the finite potential well? Use this to explain why E_1 is less than E_∞ in the situation depicted in Fig. 40.8b.

Q40.13. It is stated in Section 40.2 that a finite potential well always has at least one bound level, no matter how shallow the well. Does this mean that as $U_0 \rightarrow 0$, $E_1 \rightarrow 0$? Does this violate the Heisenberg uncertainty principle? Explain.

Q40.14. Figure 40.8a shows that the higher the energy of a bound state for a finite potential well, the more the wave function extends outside the well (into the intervals $x < 0$ and $x > L$). Explain why this happens.

Q40.15. In classical (Newtonian) mechanics, the total energy E of a particle can never be less than the potential energy U because the kinetic energy K cannot be negative. Yet in barrier tunneling (see

Section 40.3) a particle passes through regions where E is less than U . Is this a contradiction? Explain.

Q40.16. Figure 40.10 shows the scanning tunneling microscope image of 48 iron atoms placed on a copper surface, the pattern indicating the density of electrons on the copper surface. What can you infer about the potential-energy function inside the circle of iron atoms?

Q40.17. Qualitatively, how would you expect the probability for a particle to tunnel through a potential barrier to depend on the height of the barrier? Explain.

Q40.18. The wave function shown in Fig. 40.13 is nonzero for both $x < 0$ and $x > L$. Does this mean that the particle splits into two parts when it strikes the barrier, with one part tunneling through the barrier and the other part bouncing off the barrier? Explain.

Q40.19. The probability distributions for the harmonic oscillator wave functions (see Figs. 40.20 and 40.21) begin to resemble the classical (Newtonian) probability distribution when the quantum number n becomes large. Would the distributions become the same as in the classical case in the limit of very large n ? Explain.

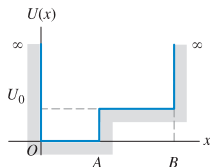
Q40.20. In Fig. 40.21, how does the probability of finding a particle in the center half of the region $-A < x < A$ compare to the probability of finding the particle in the outer half of the region? Is this consistent with the physical interpretation of the situation?

Q40.21. Compare the allowed energy levels for the hydrogen atom, the particle in a box, and the harmonic oscillator. What are the values of the quantum number n for the ground level and the second excited level of each system?

Q40.22. In the hydrogen atom the potential-energy function depends only on distance r from the nucleus, not on direction; that is, it is spherically symmetric. Would you expect all the corresponding wave functions for the electron in the hydrogen atom to be spherically symmetric? Explain.

Q40.23. Sketch the wave function for the potential-energy well shown in Fig. 40.24 when E_1 is less than U_0 and when E_3 is greater than U_0 .

Figure 40.24 Question Q40.23.



Exercises

Section 40.1 Particle in a Box

40.1. Ground-Level Billiards. (a) Find the lowest energy level for a particle in a box if the particle is a billiard ball ($m = 0.20\text{kg}$) and the box has a width of 1.5 m, the size of a billiard table. (Assume that the billiard ball slides without friction rather than rolls. That is, ignore the rotational kinetic energy.) (b) Since the energy in part (a) is all kinetic, to what speed does this correspond? How much time would it take at this speed for the ball to move from one side of the table to the other? (c) What is the difference in energy between the $n = 2$ and $n = 1$ levels? (d) Are quantum-mechanical effects important for the game of billiards?

40.2. A proton is in a box of width L . What must the width of the box be for the ground-level energy to be 5.0 MeV, a typical value

for the energy with which the particles in a nucleus are bound? Compare your result to the size of a nucleus—that is, on the order of 10^{-14} m.

40.3. Find the width L of a one-dimensional box that would correspond to the absolute value of the ground state of a hydrogen atom.

40.4. When a hydrogen atom undergoes a transition from the $n = 2$ to the $n = 1$ level, a photon with $\lambda = 122$ nm is emitted. (a) If the atom is modeled as an electron in a one-dimensional box, what is the width of the box in order for the $n = 2$ to $n = 1$ transition to correspond to emission of a photon of this energy? (b) For a box with the width calculated in part (a), what is the ground-state energy? How does this correspond to the ground-state energy of a hydrogen atom? (c) Do you think a one-dimensional box is a good model for a hydrogen atom? Explain. (*Hint:* Compare the spacing between adjacent energy levels as a function of n .)

40.5. A certain atom requires 3.0 eV of energy to excite an electron from the ground level to the first excited level. Model the atom as an electron in a box and find the width L of the box.

40.6. Recall that $|\psi|^2 dx$ is the probability of finding the particle that has normalized wave function $\psi(x)$ in the interval x to $x + dx$. Consider a particle in a box with rigid walls at $x = 0$ and $x = L$. Let the particle be in the ground level and use ψ_1 as given in Eq. (40.13). (a) For which values of x , if any, in the range from 0 to L is the probability of finding the particle zero? (b) For which values of x is the probability highest? (c) In parts (a) and (b) are your answers consistent with Fig. 40.8? Explain.

40.7. Repeat Exercise 40.6 for the particle in the first excited level.

40.8. (a) Show that $\psi = A \sin kx$ is a solution to Eq. (40.3) if $k = \sqrt{2mE}/\hbar$. (b) Explain why this is an acceptable wave function for a particle in a box with rigid walls at $x = 0$ and $x = L$ only if k is an integer multiple of π/L .

40.9. (a) Repeat Exercise 40.8 for $\psi = A \cos kx$. (b) Explain why this cannot be an acceptable wave function for a particle in a box with rigid walls at $x = 0$ and $x = L$ no matter what the value of k .

40.10. (a) Find the excitation energy from the ground level to the third excited level for an electron confined to a box that has a width of 0.125 nm. (b) The electron makes a transition from the $n = 1$ to $n = 4$ level by absorbing a photon. Calculate the wavelength of this photon.

40.11. An electron is in a box of width 3.0×10^{-10} m. What are the de Broglie wavelength and the magnitude of the momentum of the electron if it is in (a) the $n = 1$ level; (b) the $n = 2$ level; (c) the $n = 3$ level? In each case how does the wavelength compare to the width of the box?

40.12. Show that the time-dependent wave function given by Eq. (40.15) is a solution to the one-dimensional Schrödinger equation, Eq. (40.1).

Section 40.2 Potential Wells

40.13. (a) Show that $\psi = A \sin kx$, where k is a constant, is not a solution of Eq. (40.1) for $U = U_0$ and $E < U_0$. (b) Is this ψ a solution for $E > U_0$?

40.14. An electron is moving past the square well shown in Fig. 40.6. The electron has energy $E = 3U_0$. What is the ratio of the de Broglie wavelength of the electron in the region $x > L$ to the wavelength for $0 < x < L$?

40.15. An electron is bound in a square well of depth $U_0 = 6E_\infty$. What is the width of the well if its ground-state energy is 2.00 eV?

40.16. An electron is bound in a square well of width 1.50 nm and depth $U_0 = 6E_\infty$. If the electron is initially in the ground level and