Key Terms

de Broglie wavelength, *1350*
quantum mechanics, *1350*
wave function, *1350*
boundary condition, *1351*
electron diffraction, *1353*
Heisenberg uncertainty principle, *1357*
electron microscope, *1360*
probability distribution function, *1362*
stationary state, *1363*
Schrödinger equation, *1364*
wave packet, *1367*

Answer to Chapter Opening Question ?

The smallest detail visible in an image is comparable to the wavelength used to make the image. Electrons can easily be given a large momentum p and hence a short wavelength $\lambda = h/p$, and so can be used to resolve extremely fine details. (See Section 39.4.)

Answers to Test Your Understanding Questions

39.1 Answer: (i) From Example 39.1, the speed of a particle is $v = h/\lambda m$ and the kinetic energy is $K = \frac{1}{2}mv^2 = (m/2)(h/\lambda m)^2 = h^2/2\lambda^2 m$. This shows that for a given wavelength, the kinetic energy is inversely proportional to the mass. Hence the proton, with a smaller mass, has more kinetic energy than the neutron.

39.2 Answer: no The energy of a photon is $E = hf$, and the frequency of a photon is $f = c/\lambda$. Hence $E = hc/\lambda$ and $\lambda = hc/E = (4.136 \times 10^{-15}\ \text{eV} \cdot \text{s}) \times (2.998 \times 10^8\ \text{m/s})/(54\ \text{eV}) = 2.3 \times 10^{-8}$ m. This is more than 100 times greater than the wavelength of an electron of the same energy. While both photons and electrons have wavelike properties, they have different relationships between their energy and momentum and hence between their frequency and wavelength.

39.3 Answer: (i) and (iii) (tie), (ii) and (iv) (tie) According to the Heisenberg uncertainty principle, the smaller the uncertainty Δx in the x-coordinate, the greater the uncertainty Δp_x in the x-momentum. The relationship between Δx and Δp_x does not depend on the mass of the particle, and so is the same for a proton as for an electron.

39.5 Answer: no Equation (39.25) represents a superposition of wave functions with different values of wave number k and hence different values of energy $E = \hbar^2 k^2/2m$. The state that this combined wave function represents is not a state of definite energy, and therefore not a stationary state. Another way to see this is to note that the time-dependent wave function $\Psi(x, t)$ would include a factor $e^{-iEt/\hbar}$ inside the integral in Eq. (39.25), with a different value of E for each value of k. This wave function therefore has a very complicated time dependence, and the probability distribution function $|\Psi(x, t)|^2$ does depend on time.

PROBLEMS

For instructor-assigned homework, go to **www.masteringphysics.com**

Discussion Questions

Q39.1. In attempting to reconcile the wave and particle models of light, some people have suggested that the photon rides up and down on the crests and troughs of the electromagnetic wave. What things are *wrong* with this description?

Q39.2. If a proton and an electron have the same speed, which has the longer de Broglie wavelength? Explain.

Q39.3. If a proton and an electron have the same kinetic energy, which has the longer de Broglie wavelength? Explain.

Q39.4. Does a photon have a de Broglie wavelength? If so, how is it related to the wavelength of the associated electromagnetic wave? Explain.

Q39.5. When an electron beam goes through a very small hole, it produces a diffraction pattern on a screen, just like that of light. Does this mean that an electron spreads out as it goes through the hole? What does this pattern mean?

Q39.6. You have been asked to design a magnet system to steer a beam of 54-eV electrons like those described in Example 39.2 (Section 39.2). The goal is to be able to direct the electron beam to a specific target location with an accuracy of ± 1.0 mm. In your design, do you need to take the wave nature of electrons into account? Explain.

Q39.7. The upper half of the electron diffraction pattern shown in Fig. 39.6 is the mirror image of the lower half of the pattern. Would it be correct to say that the upper half of the pattern is caused by electrons that pass through the upper half of the slit? Explain.

Q39.8. A particular electron in the experimental setup shown in Fig. 39.6 lands on the photographic film a distance x above the center of the pattern. Given the value of x, is it possible to calculate the precise trajectory that the electron followed? Explain.

Q39.9. Does the uncertainty principle have anything to do with marksmanship? That is, is the accuracy with which a bullet can be aimed at a target limited by the uncertainty principle? Explain.

Q39.10. Suppose a two-slit interference experiment is carried out using an electron beam. Would the same interference pattern result if one slit at a time is uncovered instead of both at once? If not, why not? Doesn't each electron go through one slit or the other? Or does every electron go through both slits? Discuss the latter possibility in light of the principle of complementarity.

Q39.11. Equation (39.13) states that the energy of a system can have uncertainty. Does this mean that the principle of conservation of energy is no longer valid? Explain.

Q39.12. Laser light results from transitions from long-lived metastable states. Why is it more monochromatic than ordinary light?

Q39.13. Could an electron-diffraction experiment be carried out using three or four slits? Using a grating with many slits? What sort of results would you expect with a grating? Would the uncertainty principle be violated? Explain.

Q39.14. As the lower half of Fig. 39.5 shows, the diffraction pattern made by electrons that pass through aluminum foil is a series of concentric rings. But if the aluminum foil is replaced by a single crystal of aluminum, only certain points on these rings appear in the pattern. Explain.

Q39.15. Why can an electron microscope have greater magnification than an ordinary microscope?

Q39.16. If quantum mechanics replaces the language of Newtonian mechanics, why don't we have to use wave functions to describe the motion of macroscopic bodies such as baseballs and cars?

Q39.17. A student remarks that the relationship of ray optics to the more general wave picture is analogous to the relationship of Newtonian mechanics, with well-defined particle trajectories, to quantum mechanics. Comment on this remark.

Q39.18. As Eq. (39.14) indicates, the time-dependent wave function for a stationary state is a complex number having a real part and an imaginary part. How can this function have any physical meaning, since part of it is *imaginary?*

Q39.19. When you check the air pressure in a tire, a little air always escapes; the process of making the measurement changes the quantity being measured. Think of other examples of measurements that change or disturb the quantity being measured.

Q39.20. Why must the wave function of a particle be normalized?

Q39.21. If a particle is in a stationary state, does that mean that the particle is not moving? If a particle moves in empty space with constant momentum $\vec{p}$ and hence constant energy $E = p^2/2m$, is it in a stationary state? Explain your answers.

Q39.22. Some lasers emit light in pulses that are only 10^{-12} s in duration. The length of such a pulse is $(3 \times 10^8 \text{ m/s})(10^{-12} \text{ s}) = 3 \times 10^{-4} \text{ m} = 0.3$ mm. Can pulsed laser light be as monochromatic as light from a laser that emits a steady, continuous beam? Explain.

Exercises

Section 39.1 De Broglie Waves

39.1. (a) An electron moves with a speed of 4.70×10^6 m/s. What is its de Broglie wavelength? (b) A proton moves with the same speed. Determine its de Broglie wavelength.

39.2. For crystal diffraction experiments (discussed in Section 39.2), wavelengths on the order of 0.20 nm are often appropriate. Find the energy in electron volts for a particle with this wavelength if the particle is (a) a photon; (b) an electron; (c) an alpha particle $(m = 6.64 \times 10^{-27} \text{ kg})$.

39.3. An electron has a de Broglie wavelength of 2.80×10^{-10} m. Determine (a) the magnitude of its momentum and (b) its kinetic energy (in joules and in electron volts).

39.4. Wavelength of an Alpha Particle. An alpha particle $(m = 6.64 \times 10^{-27} \text{ kg})$ emitted in the radioactive decay of uranium-238 has an energy of 4.20 MeV. What is its de Broglie wavelength?

39.5. In the Bohr model of the hydrogen atom, what is the de Broglie wavelength for the electron when it is in (a) the $n = 1$ level and (b) the $n = 4$ level? In each case, compare the de Broglie wavelength to the circumference $2\pi r_n$ of the orbit.

39.6. (a) A nonrelativistic free particle with mass m has kinetic energy K. Derive an expression for the de Broglie wavelength of the particle in terms of m and K. (b) What is the de Broglie wavelength of an 800-eV electron?

39.7. Why Don't *We* Diffract? (a) Calculate the de Broglie wavelength of a typical person walking through a doorway. Make reasonable approximations for the necessary quantities. (b) Will the person in part (a) exhibit wave-like behavior when walking through the "single slit" of a doorway? Why?

39.8. What is the de Broglie wavelength for an electron with speed (a) $v = 0.480c$ and (b) $v = 0.960c$? (*Hint:* Use the correct relativistic expression for linear momentum if necessary.)

39.9. (a) If a photon and an electron each have the same energy of 20.0 eV, find the wavelength of each. (b) If a photon and an electron each have the same wavelength of 250 nm, find the energy of each. (c) You want to study an organic molecule that is about 250 nm long using either a photon or an electron microscope. Approximately what wavelength should you use, and which probe, the electron or the photon, is likely to damage the molecule the least?

39.10. Hydrogen gas (H_2) is at 0°C. The mass of a hydrogen *atom* is 1.67×10^{-27} kg. (a) What is the average de Broglie wavelength of the hydrogen molecules? (b) How fast would an electron have to move to have the same de Broglie wavelength as the hydrogen? Do we need to consider relativity for this electron? (c) What would be the energy of a photon having the same wavelength as the H_2 molecules and the electrons? Compare it to the kinetic energy of the hydrogen molecule in part (a) and the electron in part (b).

39.11. Wavelength of a Bullet. Calculate the de Broglie wavelength of a 5.00-g bullet that is moving at 340 m/s. Will the bullet exhibit wavelike properties?

Section 39.2 Electron Diffraction

39.12. Through what potential difference must electrons be accelerated so they will have (a) the same wavelength as an x ray of wavelength 0.150 nm and (b) the same energy as the x ray in part (a)?

39.13. (a) Approximately how fast should an electron move so it has a wavelength that makes it useful to measure the distance between adjacent atoms in typical crystals (about 0.10 nm)? (b) What is the kinetic energy of the electron in part (a)? (c) What would be the energy of a photon of the same wavelength as the electron in part (b)? (d) Which would make a more effective probe of small-scale structures, electrons or photons? Why?

39.14. A beam of electrons is accelerated from rest through a potential difference of 0.100 kV and then passes through a thin slit. The diffracted beam shows its first diffraction minima at ±11.5° from the original direction of the beam when viewed far from the slit. (a) Do we need to use relativity formulas? How do you know? (b) How wide is the slit?

39.15. A beam of neutrons that all have the same energy scatters from the atoms that have a spacing of 0.0910 nm in the surface plane of a crystal. The $m = 1$ intensity maximum occurs when the angle θ in Fig. 39.3 is 28.6°. What is the kinetic energy (in electron volts) of each neutron in the beam?

39.16. A beam of 188-eV electrons is directed at normal incidence onto a crystal surface as shown in Fig. 39.4b. The $m = 2$ intensity maximum occurs at an angle $\theta = 60.6°$. (a) What is the spacing between adjacent atoms on the surface? (b) At what other angle or angles is there an intensity maximum? (c) For what electron energy (in electron volts) would the $m = 1$ intensity maximum occur at $\theta = 60.6°$? For this energy, is there an $m = 2$ intensity maximum? Explain.

39.17. A CD-ROM is used instead of a crystal in an electron-diffraction experiment like that shown in Fig. 39.3. The surface of the CD-ROM has tracks of tiny pits with a uniform spacing of 1.60 μm. (a) If the speed of the electrons is 1.26×10^4/s, at which values of θ will the $m = 1$ and $m = 2$ intensity maxima appear? (b) The scattered electrons in these maxima strike at normal incidence a piece of photographic film that is 50.0 cm from the CD-ROM. What is the spacing on the film between these maxima?

Section 39.3 Probability and Uncertainty

39.18. A pesky 1.5-mg mosquito is annoying you as you attempt to study physics in your room, which is 5.0 m wide and 2.5 m high. You decide to swat the bothersome insect as it flies toward you, but you