1. (25pts)
Answer the following questions. Justify your answers. (Use the space provided below and the next page)

a). Two inertial observers are in relative motion. Which of the following quantities will they agree or disagree on? i) their relative speed; ii) the speed of light in vacuum; iii) the total energy of a moving particle; iv) the combined quantity -- (E^2-c^2p^2) where E is the total energy and P is the linear momentum of a moving particle; v) the duration between two ticks of a clock.

b). Superman is running at high speed on a straight highway toward a pedestrian bridge which is oriented *perpendicular* to the highway. Which of the following length measurements will be contracted according to Superman? i) the length of the bridge; ii) the height of the bridge above the highway; iii) the distance to the bridge along the highway.

c). In a photoelectric effect experiment, no photoelectrons are observed to be ejected by an incident blue light. Will the experimenter observe ejected photoelectrons if the blue light is replaced by a red light within the same experimental setup?

d). In a Compton scattering experiment, incident X-rays are scattered from a stationary electron. Which of the following are the possible changes for the wavelength of the scattered X-rays: i)increase; ii) stay the same; iii) decrease?

Answers:
a) i) agree; ii) agree; iii) not agree; iv) agree; v) not agree
b) i) not contracted; ii) not contracted; iii) Lorenz contracted. Length is Lorenz contracted only in the direction of the relative motion.
c) Since red light will have lower frequency than blue light, red light will have lower energy. If blue light is not enough to eject photoelectrons, a lower energy red light will not be able to eject photoelectrons also.
d) By the conservation of energy, the scattered X-rays cannot have a higher energy than the incident X-rays since the scattered electron can have its kinetic energy larger than or equal to zero. Thus, the scattered X-rays can have the same (ϕ=0) or a longer wavelength but it cannot have a shorter wavelength since the energy of a photon is inversely proportional to its wavelength.

2. (25 pts)

A distance galaxy is about 1.00×10^5 light-years in diameter. The starship Enterprise enters the galaxy with a speed v=0.990c. i) How long does the Enterprise take to cross the galaxy from our viewpoint (stationary with respect to the galaxy)? ii) How long does the crew on Enterprise think the journey takes? iii) How wide is the galaxy (along the direction of the motion) according to the crew of the Enterprise?

i) From our viewpoint, the 10^5 light-years is the distance the Enterprise has to travel and its speed is 0.99c so that the time that it takes to cross the galaxy is:

$$\Delta t = \frac{L_p}{v} = \frac{10^5 \, light-years}{0.99c} = 1.01\times10^5 \, years$$

ii) The time interval measured by the earth observer is not proper. Since the Enterprise is actually experiencing the two events (entering and leaving the galaxy) which defined this time interval, its measured time interval is the proper time. Then, from the time dilation formula, the Enterprise should measure a short time interval as compared with the stationary observer.

$$\Delta t_p = \frac{\Delta t}{g} \quad where \quad g = (1-\frac{v^2}{c^2})^{-1/2} = 1/\sqrt{1-0.99^2} = 7.089$$

$$= \frac{1.01\times10^5 \, years}{7.089}$$

$$= 1.42\times10^4 \, years$$

iii) In the Enterprise's moving frame, the diameter of the galaxy along the direction of motion is Lorenz contracted:

$$L = \frac{L_p}{g} \quad where \quad g = (1-\frac{v^2}{c^2})^{-1/2} = 1/\sqrt{1-0.99^2} = 7.089$$

$$= \frac{10^5 \, light-years}{7.089}$$

$$= 1.41\times10^4 \, light-years$$

So, in its viewpoint, the time that it takes to cross the contracted galaxy is:

$$\Delta t_p = \frac{L}{v} = \frac{1.41\times10^4 \, light-years}{0.99c} = 1.42\times10^4 \, years$$

This agrees with time dilation argument in part ii.

3. (25 pts)
An unstable particle with a rest mass of $3.50\times10^{-27}\,kg$ is initially at rest and is observed to decay into two smaller fragments with masses m_1 and m_2. The velocities of these two fragments are measured in the laboratory frame to be 0.987c (in the $+\hat{x}$ direction) and –0.868c (in the $-\hat{x}$ direction). a) Using the conservation of relativistic energy and relativistic linear momentum, find the values of m_1 and m_2. b) Is the rest mass a conserved quantity in this process?

Conservation of E:

$$3.5\times10^{-27}\,kg\ c^2 = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2$$

where $\gamma_1 = \dfrac{1}{\sqrt{1-\dfrac{v_1^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.987^2}} = 6.222$

$$\gamma_2 = \frac{1}{\sqrt{1-\dfrac{v_2^2}{c^2}}} = \frac{1}{\sqrt{1-0.868^2}} = 2.014$$

So, we have

$$3.50\times10^{-27}\,kg = 6.222m_1 + 2.014m_2 \qquad (1)$$

Conservation of linear momentum:

$$\gamma_1 m_1 v_1 = \gamma_2 m_2 v_2$$
$$6.222m_1(0.987c) = 2.014m_2(0.868c)$$
$$m_1 = 0.285m_2$$

Substitute this relation into (1), we have,

$$m_2 = 9.24\times10^{-28}\,kg$$
$$m_1 = 2.63\times10^{-28}\,kg$$

Note that
$m_1 + m_2 = 1.19\times10^{-27}\,kg < 3.50\times10^{-27}\,kg$ so that mass is not conserved in this process.

4. (25 pts)
In a photoelectric effect experiment, when a green light from a mercury lamp ($l = 546.1 nm$) is shined on a metal surface, a stopping potential of 0.387V is able to stop the photocurrent completely. i) What is the work function for this metal? ii) With the same setup as before, a yellow light from a sodium lamp ($\lambda = 587.5 nm$) is used in place of the green light. What will the new stopping potential in this case be? [express energy in units of eV]

i)

$$KE_{\max} = e\Delta V_s = hf - \phi$$

$$0.387 eV = \frac{hc}{\lambda} - \phi$$

$$\phi = \frac{hc}{\lambda} - 0.387 eV$$

$$\phi = \frac{6.63\times10^{-34}\,J\cdot s\left(3.00\times10^{8}\,m/s\right)}{546.1\times10^{-9}\,m}\left(\frac{1eV}{1.602\times10^{-19}\,J}\right) - 0.387 eV$$

$$\phi = 1.89 eV$$

ii)

$$e\Delta V_s^{'} = hf^{'} - \phi = \frac{hc}{\lambda^{'}} - \phi$$

$$e\Delta V_s^{'} = \frac{6.63\times10^{-34}\,J\cdot s\left(3.00\times10^{8}\,m/s\right)}{587.5\times10^{-9}\,m}\left(\frac{1eV}{1.602\times10^{-19}\,J}\right) - 1.885 eV$$

$$\Delta V_s^{'} = 0.227 V$$

5. (25 pts)

A photon with a wavelength of $l = 0.150nm$ collides with a stationary electron in a Compton scattering experiment. After the collision, the photon recoils backward and the scattered electron move in the forward direction. See the diagram below. i) What is the wavelength λ’ of the recoiled photon due to Compton shift? ii) What is the energy of the recoiled photon? iii) What is the kinetic energy of the scattered electron? [express energy in units of eV]

<u>before</u> <u>after</u>

i)

$$l^{'} = l_0 + \frac{h}{m_e c}(1 - \cos 180^{\circ})$$

$$l^{'} = l_0 + l_C(1+1)$$

$$l^{'} = 0.150nm + 0.00243nm \times 2$$

$$l^{'} = 0.15486nm = 0.155nm$$

ii)

$$E^{'} = hf^{'} = \frac{hc}{l^{'}} = \frac{6.626\times10^{-34}\,J\cdot s\left(3.00\times10^{8}\,m/s\right)}{0.15486\times10^{-9}\,m}\left(\frac{1eV}{1.602\times10^{-19}\,J}\right)$$

$$E^{'} = 8.01keV$$

iii)

Conservation of energy requires that

$$hf + m_e c^2 = hf^{'} + KE_e + m_e c^2$$

$$KE_e = hf - hf^{'}$$

$$KE_e = hc\left(\frac{1}{l_0} - \frac{1}{l^{'}}\right)$$

$$KE_e = 6.626\times10^{-34}\,J\cdot s\left(3.00\times10^{8}\,m/s\right)\left(\frac{1}{0.150nm} - \frac{1}{0.155nm}\right)\left(\frac{1eV}{1.602\times10^{-19}\,J}\right)$$

$$KE_e = 260eV$$