1. (25pts)
Answer the following questions. Justify your answers. (Use the space provided below and the next page)

a). Two inertial observers are in relative motion. Which of the following quantities will they agree or disagree on? i) their relative speed; ii) the speed of light in vacuum; iii) the total energy of a moving particle; iv) the combined quantity -- \((E^2 - c^2 P^2)\) where \(E\) is the total energy and \(P\) is the linear momentum of a moving particle; v) the duration between two ticks of a clock.

b). Superman is running at high speed on a straight highway toward a pedestrian bridge which is oriented perpendicular to the highway. Which of the following length measurements will be contracted according to Superman? i) the length of the bridge; ii) the height of the bridge above the highway; iii) the distance to the bridge along the highway.

c). In a photoelectric effect experiment, no photoelectrons are observed to be ejected by an incident blue light. Will the experimenter observe ejected photoelectrons if the blue light is replaced by a red light within the same experimental setup?

d). In a Compton scattering experiment, incident X-rays are scattered from a stationary electron. Which of the following are the possible changes for the wavelength of the scattered X-rays: i) increase; ii) stay the same; iii) decrease?

Answers:
a) i) agree; ii) agree; iii) not agree; iv) agree; v) not agree
b) i) not contracted; ii) not contracted; iii) Lorenz contracted. Length is Lorenz contracted only in the direction of the relative motion.
c) Since red light will have lower frequency than blue light, red light will have lower energy. If blue light is not enough to eject photoelectrons, a lower energy red light will not be able to eject photoelectrons also.
d) By the conservation of energy, the scattered X-rays cannot have a higher energy than the incident X-rays since the scattered electron can have its kinetic energy larger than or equal to zero. Thus, the scattered X-rays can have the same (\(\phi=0\)) or a longer wavelength but it cannot have a shorter wavelength since the energy of a photon is inversely proportional to its wavelength.
2. (25 pts)
A distance galaxy is about \(1.00 \times 10^5\) light-years in diameter. The starship Enterprise enters the galaxy with a speed \(v = 0.990c\). i) How long does the Enterprise take to cross the galaxy from our viewpoint (stationary with respect to the galaxy)? ii) How long does the crew on Enterprise think the journey takes? iii) How wide is the galaxy (along the direction of the motion) according to the crew of the Enterprise?

i) From our viewpoint, the \(10^5\) light-years is the distance the Enterprise has to travel and its speed is \(0.99c\) so that the time that it takes to cross the galaxy is:

\[
\Delta t = \frac{L_p}{v} = \frac{10^5 \text{ light-years}}{0.99c} = 1.01 \times 10^5 \text{ years}
\]

ii) The time interval measured by the earth observer is not proper. Since the Enterprise is actually experiencing the two events (entering and leaving the galaxy) which defined this time interval, its measured time interval is the proper time. Then, from the time dilation formula, the Enterprise should measure a short time interval as compared with the stationary observer.

\[
\Delta t_p = \Delta t \cdot \frac{1}{\gamma} \quad \text{where} \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1/\sqrt{1 - 0.99^2} = 7.089
\]

\[
= \frac{1.01 \times 10^5 \text{ years}}{7.089}
\]

\[
= 1.42 \times 10^4 \text{ years}
\]

iii) In the Enterprise’s moving frame, the diameter of the galaxy along the direction of motion is Lorenz contracted:

\[
L = \frac{L_p}{\gamma} \quad \text{where} \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1/\sqrt{1 - 0.99^2} = 7.089
\]

\[
= \frac{10^5 \text{ light-years}}{7.089}
\]

\[
= 1.41 \times 10^4 \text{ light-years}
\]

So, in its viewpoint, the time that it takes to cross the contracted galaxy is:

\[
\Delta t_p = \frac{L}{v} = \frac{1.41 \times 10^4 \text{ light-years}}{0.99c} = 1.42 \times 10^4 \text{ years}
\]

This agrees with time dilation argument in part ii.
3. (25 pts)
An unstable particle with a rest mass of $3.50 \times 10^{-27}$ kg is initially at rest and is observed to decay into two smaller fragments with masses $m_1$ and $m_2$. The velocities of these two fragments are measured in the laboratory frame to be $0.987c$ (in the $+\hat{x}$ direction) and $-0.868c$ (in the $-\hat{x}$ direction). a) Using the conservation of relativistic energy and relativistic linear momentum, find the values of $m_1$ and $m_2$. b) Is the rest mass a conserved quantity in this process?

Conservation of E:

$$3.50 \times 10^{-27} \text{kg} \ c^2 = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2$$

where \( \gamma_1 = \frac{1}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.987^2}} = 6.222 \)

\( \gamma_2 = \frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.868^2}} = 2.014 \)

So, we have

$$3.50 \times 10^{-27} \text{kg} = 6.222m_1 + 2.014m_2 \quad (1)$$

Conservation of linear momentum:

$$\gamma_1 m_1 v_1 = \gamma_2 m_2 v_2$$

$$6.222m_1 (0.987c) = 2.014m_2 (0.868c)$$

$$m_1 = 0.285m_2$$

Substitute this relation into (1), we have,

$$m_2 = 9.24 \times 10^{-28} \text{kg}$$
$$m_1 = 2.63 \times 10^{-28} \text{kg}$$

Note that

$$m_1 + m_2 = 1.19 \times 10^{-27} \text{kg} < 3.50 \times 10^{-27} \text{kg}$$

so that mass is not conserved in this process.
4. (25 pts)
In a photoelectric effect experiment, when a green light from a mercury lamp \((\lambda = 546.1\; nm)\) is shined on a metal surface, a stopping potential of 0.387V is able to stop the photocurrent completely. 

i) What is the work function for this metal? 

\[
KE_{max} = e\Delta V_s = hf - \phi
\]
\[
0.387\text{eV} = \frac{hc}{\lambda} - \phi
\]
\[
\phi = \frac{hc}{\lambda} - 0.387\text{eV}
\]
\[
\phi = \frac{6.63 \times 10^{-34} \cdot s \cdot (3.00 \times 10^8 \text{ m/s}) \cdot 1\text{eV}}{546.1 \times 10^{-9} \text{ m} \cdot 1.602 \times 10^{-19} \text{ J}} - 0.387\text{eV}
\]
\[
\phi = 1.89\text{eV}
\]

ii) 

\[
e\Delta V_s = hf - \phi = \frac{hc}{\lambda} - \phi
\]
\[
e\Delta V_s = \frac{6.63 \times 10^{-34} \cdot s \cdot (3.00 \times 10^8 \text{ m/s}) \cdot 1\text{eV}}{587.5 \times 10^{-9} \text{ m} \cdot 1.602 \times 10^{-19} \text{ J}} - 1.885\text{eV}
\]
\[
\Delta V_s = 0.227\text{V}
\]
5. (25 pts)

A photon with a wavelength of $\lambda_0 = 0.150 \text{nm}$ collides with a stationary electron in a Compton scattering experiment. After the collision, the photon recoils backward and the scattered electron move in the forward direction. See the diagram below. i) What is the wavelength $\lambda'$ of the recoiled photon due to Compton shift? ii) What is the energy of the recoiled photon? iii) What is the kinetic energy of the scattered electron? [express energy in units of eV]

\[
\lambda' = \lambda_0 + \frac{h}{m_c} (1 - \cos 180^\circ)
\]

\[
\lambda' = \lambda_0 + \lambda_c (1+1)
\]

\[
\lambda' = 0.150 \text{nm} + 0.00243 \text{nm} \times 2
\]

\[
\lambda' = 0.15486 \text{nm} = 0.155 \text{nm}
\]

\[
E' = hf' = \frac{hc}{\lambda'} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s} \left(3.00 \times 10^8 \text{ m/s}\right)}{0.15486 \times 10^{-9} \text{ m}} \left(\frac{1 \text{eV}}{1.602 \times 10^{-19} \text{ J}}\right)
\]

\[
E' = 8.01 \text{keV}
\]

iii)

Conservation of energy requires that

\[
hf + m_e c^2 = hf' + KE_e + m_e c^2
\]

\[
KE_e = hf - hf'
\]

\[
KE_e = \frac{hc}{\lambda_0} - \frac{1}{\lambda}
\]

\[
KE_e = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \left(3.00 \times 10^8 \text{ m/s}\right) \left(\frac{1}{0.150 \text{nm}} - \frac{1}{0.155 \text{nm}}\right) \left(\frac{1 \text{eV}}{1.602 \times 10^{-19} \text{ J}}\right)
\]

\[
KE_e = 260 \text{eV}
\]