1. (25pts) Answer the following questions. Justify your answers. (Use the space provided below and the next page)

a). (9 pts)
   i. With a real object, can a diverging lens produce an inverted image?
   ii. With a real object, can a concave mirror produce a virtual image?
   iii. With a real object, can the image of a convex mirror be larger than the object?

b). (4 pts) In a single slit diffraction experiment, the diffraction pattern is viewed on a flat wall a few meters away. If the light source is changed from a green laser to a red laser, will the total number of visible fringes on the wall become more or less?

c). (12 pts) A student made a thin glass lens with two different radii of curvature, \( R_1 \) and \( R_2 \). The student reported the following observations:
   i. the focal length of the lens depended on the color of the light being used;
   ii. the lens changed from being a converging lens to a diverging lens when it was submerged in an unknown oily liquid;
   iii. the focal length changed when the orientation of the lens was flipped with respect to the incoming direction of the light.

Which of the above claim(s) is(are) physically possible?

Solutions:

a)
   i. No. For a real object, a diverging lens will always produce an inverted image. From the Thin Lens Equation, \( s' = \frac{sf}{s-f} \), then \( m \equiv -\frac{s'}{s} = -\frac{f}{s-f} \). For a diverging lens, \( f \) is negative, i.e., \( f = -|f| \). This gives, \( m = \frac{|f|}{s+|f|} \) so \( m \) will always be positive if \( s \) is positive for a real object.
   
   ii. Yes. If a real object is placed inside the focal point of a concave mirror, the image is inverted. Again, \( s' = \frac{sf}{s-f} \), for a concave mirror, \( f \) is positive, i.e., \( f = +|f| \). So, \( s' = \frac{s|f|}{s-|f|} \) and \( s' \) will be negative if the object is inside the focal point, i.e., \( s < |f| \).
   
   iii. No. For a real object, the image of a convex mirror will always be smaller than the object. Similar to i, we have \( m \equiv -\frac{s'}{s} = -\frac{f}{s-f} \). For a convex mirror, we
again have its focal length being negative, so we have $f = -|f|$ and 
\[ m = \frac{|f|}{s + |f|} \]

So, if $s$ is positive, we have $s + |f| \geq |f|$ and $m \leq 1$.

b) Less. Since the locations of dark fringes in a single slit diffraction is given by 
\[ \sin \theta = \frac{m\lambda}{a} \], the pattern will get closer if one uses a shorter wavelength. Therefore, within the visible limit $\theta \in [-\pi, \pi]$, the light that will produce tighter spacing will produce the larger number of fringes. Since red light has a longer wavelength, it will produce less number of visible fringes on the wall than the green laser light.

c) i. Possible: $n$ is a function of the frequency (color) of the light so that $f$ will depend on the color of light used as well.

ii. Possible: $f$ depends on the factor $(n_{\text{lens}} - n_{\text{out}})$. For $n_{\text{out}} = 1$ for air, this factor is positive. If $n_{\text{out}}$ is larger than $n_{\text{lens}}$, this factor will become negative and $f$ will change sign.

iii. Not possible: the lens’ maker equation is symmetric with respect to switching $R_1$ and $R_2$. The left and right focal points have the same focal length.
2. (25 pts) A thin converging lens with a focal length \( f_1 = 10.0 \text{cm} \) and a thin diverging lens with a focal length \( f_2 = -10.0 \text{cm} \) are separated by 15.0 cm. An object is placed at a distance of 20.0 cm to the left of the converging lens.

i) Find the position of the final image.
ii) Draw the rays diagram for this situation.
iii) What is the magnification of the final image?
iv) Is the final image virtual or real?

\[ \begin{array}{c}
20.0\text{cm} \\
\text{converging lens} \\
\text{diverging lens}
\end{array} \]

Solution:

The black rays are from the original object and the light blue rays are from the intermediate image formed by the converging lens (blue arrow). The final image is indicated by the red arrow.
For the converging lens (left lens), we have \( \frac{1}{s'_1} = \frac{1}{f_1} - \frac{1}{s_1} \) and for the diverging lens (right lens), we have \( \frac{1}{s'_2} = \frac{1}{f_2} - \frac{1}{s_2} \).

By convention, \( f_1 \) is positive and \( f_2 \) is negative. From the lens equation for lens #1 (converging), we have

\[
\frac{1}{s'_1} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \quad \Rightarrow \quad s'_1 = +20.0 \text{cm}.
\]

As indicated by the rays diagram, this intermediate image (blue arrow) is on the outgoing side of the diverging lens so that it will be a virtual object for the diverging lens (right lens) and \( s_2 = -(s'_1 - 15) = -5.00 \text{cm} \).

Putting this into the lens equation for lens #2 (diverging), we have

\[
\frac{1}{s'_2} = \frac{1}{10} - \left( \frac{1}{-5} \right) = +10.0 \text{cm} \quad \Rightarrow \quad s'_2 = +10.0 \text{cm}.
\]

The final image (red arrow) is real as indicated and is located 10cm on the outgoing side of the diverging lens.

\[
M = m_1m_2 = \left( -\frac{20 \text{cm}}{20 \text{cm}} \right) \left( -\frac{10 \text{cm}}{-5 \text{cm}} \right) = -2
\]

The final image is inverted and it is twice as high as the original object.
3. (25 pts)
An anti-reflective coating with a given thickness \( t \) and an index of refraction \( n_{\text{coating}} = 1.38 \) is to be applied to the surface of an optical device made with glass \( n_{\text{glass}} = 1.52 \).

As a design criterion, the intermediate wavelengths in the visible spectrum are to be suppressed leaving the light on short wavelength end (380 nm, violet) and the long wavelength end (760 nm, red) of the visible spectrum to be reflected by this optical coating. By considering constructive interference for the violet and red wavelengths, what is the minimum value for the thickness \( t \) needed for this anti-reflective coating? Assume nearly normal incidence for your consideration.

Solution:

Both waves reflecting off the top and bottom interfaces of the coating suffer a \( \pi \) phase change since \( n_{\text{air}} < n_{\text{coating}} < n_{\text{glass}} \).

Let \( t \) be the thickness of the optical coating, the condition for constructive interference in this case is,

\[
2t = m\lambda_n \quad \text{or} \quad 2n_{\text{coating}}t = n\lambda, \quad m = 0, 1, 2, \ldots
\]

Note that the extra path difference occurs in the optical coating so that the relevant wavelength is \( \lambda_n = \frac{\lambda}{n_{\text{coating}}} \)

Now, for the two colors (\( \lambda = 380 \text{nm} \) and \( \lambda' = 760 \text{nm} \)) of light reflected constructive, we can write down two conditions for two different still-unknown values of \( m \) and \( m' \):

\[
2n_{\text{coating}}t = m\lambda \quad (1a)
\]

\[
2n_{\text{coating}}t = m'\lambda' \quad (1b)
\]

Dividing these two equations and simplifying, we have

\[
\frac{\lambda}{\lambda'} = \frac{m'}{m} \Rightarrow m = \frac{m' \cdot 380 \text{nm}}{760 \text{nm}} = \frac{1}{2}
\]

So, the smallest integers that will satisfy the above relationship are:

\( m' = 1 \)

\( m = 2 \)

Putting this back into either Eq. 1a or 1b, we can then solve for \( t \):
\[2(1.38)t = 1(760\text{nm}) \quad \rightarrow \quad t = \frac{380\text{nm}}{1.38} = 275\text{nm}\]

Or,

\[2(1.38)t = 2(380\text{nm}) \quad \rightarrow \quad t = \frac{380\text{nm}}{1.38} = 275\text{nm}\]
4. (25 pts)
A light ray enters the left open end of an optical fiber at an angle $\theta$ as shown below. It is then refracted into the fiber at an angle $\beta$. For an entry angle $\theta$ sufficiently small, the refracted light ray, reaching the top edge of the glass fiber at $P$, can be totally internally reflected so that the light ray stays within the fiber. You are given the index of refraction for the glass fiber to be $n_{\text{glass}} = 1.50$ and the index of refraction for the cladding material encasing the fiber to be $n_{\text{cladding}} = 1.40$. Calculate the largest entry angle $\theta$ for light entering this fiber so that Total Internal Reflection occurs at $P$? [Hint: work backward for the critical angle for Total Internal Reflection at point $P$.]

Solutions:

Working backward from the top edge of the glass fiber at $P$, when the light traveling along the fiber hits the top wall at the critical incident angle $\gamma_c$, it will be total internally reflected, i.e.,

$$n_{\text{glass}} \sin \gamma_c = n_{\text{cladding}} \sin 90^\circ$$

$$\sin \gamma_c = \frac{n_{\text{cladding}}}{n_{\text{glass}}} = \frac{1.40}{1.50} = 0.9333$$

$$\gamma_c = 68.961^\circ$$

Now, at the open left end of the fiber where the light first enters the fiber, from Snell’s Law, we have
\[ n_{\text{air}} \sin \theta = n_{\text{glass}} \sin \beta \]

Looking at the geometry, one can easily see that

\[ \beta = 90^\circ - \gamma_c \]

So, we have

\[ n_{\text{air}} \sin \theta = n_{\text{glass}} \sin \beta = n_{\text{glass}} \sin \left( 90^\circ - \gamma_c \right) = n_{\text{glass}} \cos \gamma_c \]

This then gives,

\[ \sin \theta = \frac{n_{\text{glass}}}{n_{\text{air}}} \cos \gamma_c = 1.50 \cos \left( 68.96^\circ \right) = 0.5385 \]

\[ \theta = 32.6^\circ \]