1. (25 pts) Answer the following questions. Justify your answers. (Use additional white paper if you need.)

a) (6 pts) A thin glass lens made with two different radii of curvature, R_1 and R_2 was used in an experiment. The following observations were reported:

i) the focal length of the lens changes with the color of light

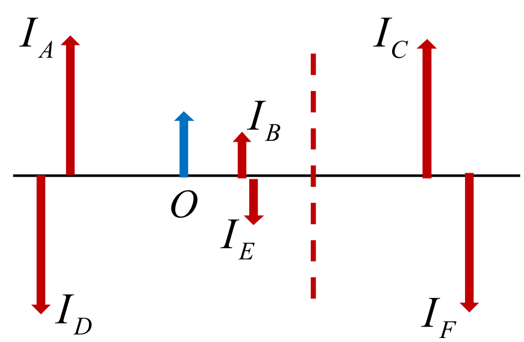
ii) the focal length of the lens changes from converging $(f > 0)$ to diverging $(f < 0)$ when the lens was submerged in an unknown liquid

iii) iii) the focal length changed when the orientation of the lens was flipped with respect to the incoming direction of the light.

Which of the above claim(s) is(are) physically possible?

b) (3 pts) In a single diffraction experiment, a diffraction pattern is viewed on a flat wall a few meters away. If the light source is changed from a green laser to a red laser, will the total number of visible fringes on the wall become more or less?

c) (12 pts) A real object (O) is placed in front of a thin lens. The location of the thin lens is indicated by the red dotted line. The optical axis of the lens is indicated by the horizontal line. The six red arrows labeled by $I_A, I_B, I_C, I_D, I_E, I_F$ indicate the possible location and orientation of the images of the object. (Note: The size of the red arrows and the distances from the lens are not drawn to scale.)



i. Which image(s) $(I_A, I_B, I_C, I_D, I_E, I_F)$ is (are) physically *possible* for a converging lens?

ii. Which image(s) $(I_A, I_B, I_C, I_D, I_E, I_F)$ is (are) physically *possible* for a diverging lens?

d) (4 pts)

i. With a real object, can a convex mirror produce an inverted image?

ii. With a real object, can a diverging lens produce a real image?

Solutions:

a)

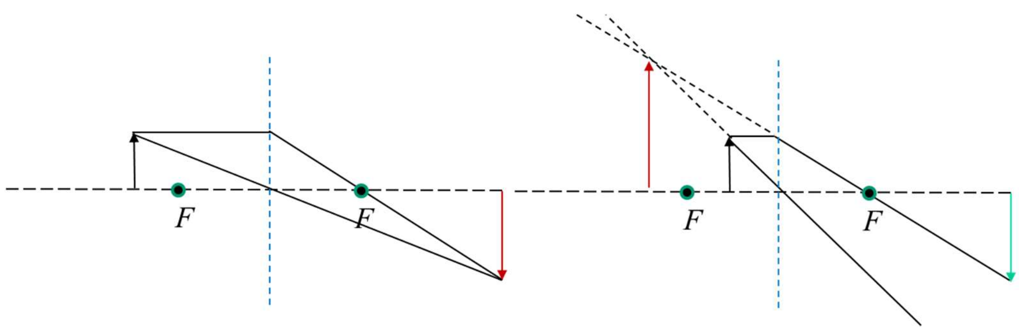
i. Possible: n is a function of the frequency (color) of the light so that f will depend on the color of light used as well.

ii. Possible: f depends on the factor $(n_{lens} - n_{out})$ For $n_{out} = 1$ (air), this factor is positive. However, if n_{out} is larger than n_{1ens}, this factor will become negative and *f* will change sign.

iii. Not possible: the lens' maker equation is symmetric with respect to switching R_1 and R_2. The left and right focal points have the same focal length.

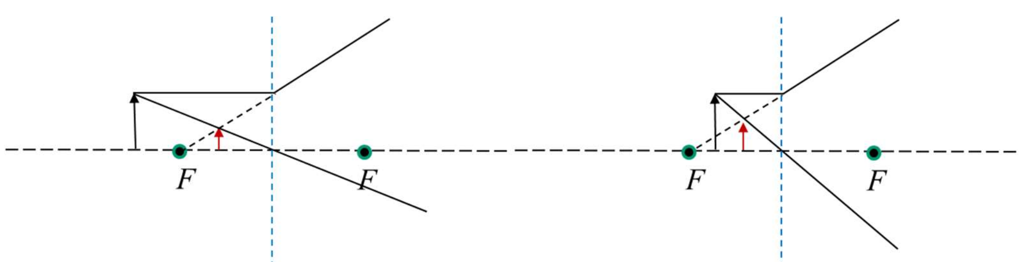
b) Since the locations of dark fringes in a single slit diffraction is given by $\sin\theta = m\lambda/a$, the pattern will get closer if one uses a shorter wavelength. Therefore, within the visible limit $\theta \in [-\pi, \pi]$, the light that will produce tighter spacing will produce the larger number of fringes. Since red light has a longer wavelength, it will produce less number of visible fringes on the wall than the green laser light.

c) For a real object, here are the two possible cases for a converging lens (O outside of F and O inside of F):



So, for question i: only I_A, I_F are physically possible.

For a diverging lens, here are the two possible cases for a converging lens (O outside of F and O inside of F):



So, for question ii: only I_B is physically possible.

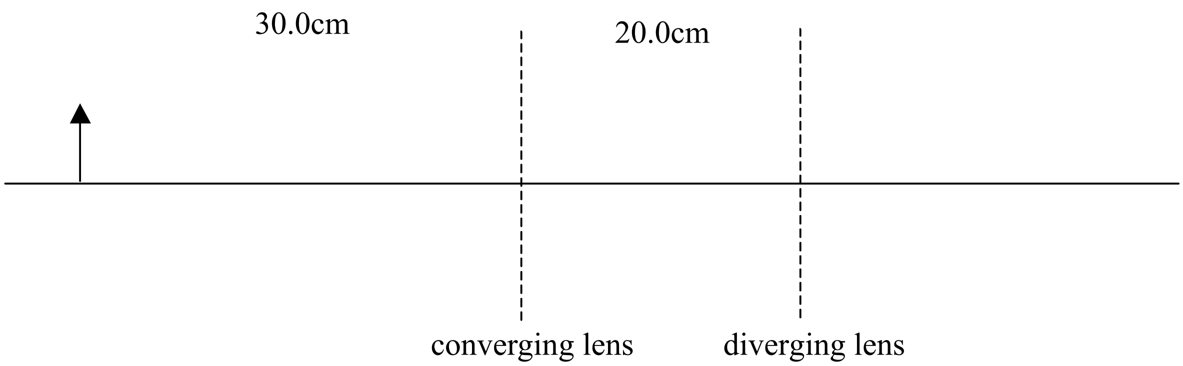
d)

i. No. With a real object, a convex mirror will always produce a upright image.

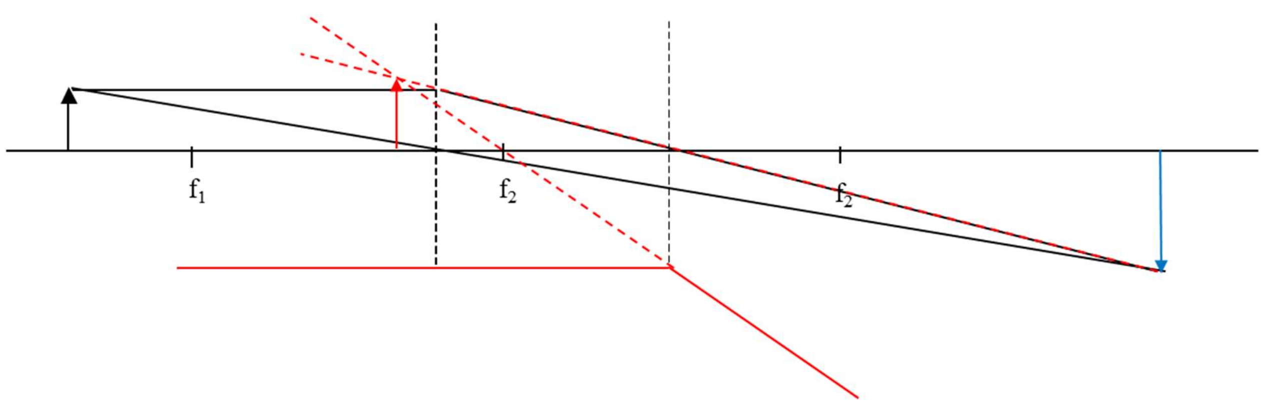
ii. No. With a real object, the images from a diverging lens will always be vitural.

2. (25 pts) An object is placed at a distance of 30.0cm to the left of two thin lens separated by 20.0cm. The lens on the left is a converging lens with a focal length 20.0cm and the second lens on the right is a diverging lens with a focal length 15.0cm. (see diagram below).

i. Calculate the position of the final image.
ii. Use the rays tracing method to find the position of the final image.
iii. What is the lateral magnification of the final image? Is the final image real or virtual?



SOLUTION:



The black rays are from the original object and the red rays are from the image formed by the lens. The final image is indicated by the red arrow.

For the lens, we have $\frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}}$ and for the mirror, we have $\frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}}$.
By convention, f_1 and f_2 are positive. From the first lens equation, we have

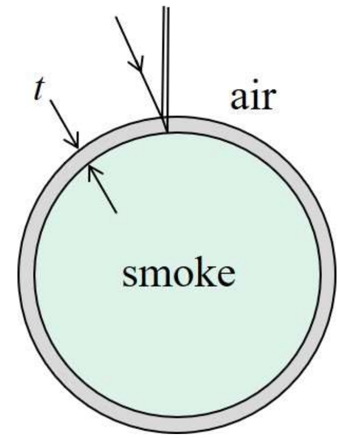
$\frac{1}{d_{i1}} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60} \quad \rightarrow \quad d_{i1} = 60cm$. As indicated by the rays diagram, this image will be a virtual object for the diverging lens and $d_{o2} = 20 - d_{i1} = -40cm$. Putting this into the mirror equation, we have

$\frac{1}{d_{i2}} = -\frac{1}{15} + \frac{1}{40} = -\frac{5}{120} \quad \rightarrow \quad d_{i2} = -24cm$. The final image is **virtual** as indicated and is located 24cm in front of the converging lens.

$$M = m_1 m_2 = \left(-\frac{60cm}{30cm}\right)\left(-\frac{-24cm}{-40cm}\right) = 1.2$$

3. (25 pts)

A circus performer blew a smoke-filled soap bubble with a thickness $t = 425.nm$ as shown to the right. Assume air to be outside of the bubble and consider nearly normal incidence light rays. Which wavelength of light within the visible range $(380.nm - 750.nm)$ will be missing due to total destructive interference of light reflected by the thin soap film (calculate all possible wavelengths within the given range)? Take the index of refraction of the soap film to be 1.47 and the index of refraction for smoke is 1.52.



SOLUTION:

Both waves reflecting off the outer and inner interface of the soap bubble will suffer a π phase change since the reflection is from a substance with a larger index of refraction (i.e., $n_{air} < n_{soap} < n_{smoke}$).

Let t be the thickness of the soap film, the condition for total destructive interference in this case is,

$$2t = (m + \frac{1}{2})\lambda_n \quad or \quad 2n_{soap}t = (m + \frac{1}{2})\lambda, \quad m = 0,1,2,\cdots$$

Note that the wavelength of light traveling through the thin soap film will be shorter by

$$\lambda_n = \frac{\lambda}{n_{soap}}$$

This then gives $\lambda = \dfrac{2n_{soap}t}{\left(m + \frac{1}{2}\right)}, \quad m = 0,1,2,\cdots$

Now, let see what wavelengths of light will be reflected destructively:

$$m = 0: \quad \lambda = \frac{2n_{soap}t}{(1/2)} = 4(1.47)(425nm) = 2499nm$$

$$m = 1: \quad \lambda = \frac{2n_{soap}t}{(3/2)} = \frac{4}{3}(1.47)(425nm) = 833nm$$

$$m = 2: \quad \lambda = \frac{2n_{soap}t}{(5/2)} = \frac{4}{5}(1.47)(425nm) = 500nm$$

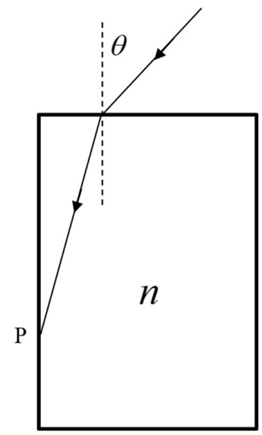
$$m = 3: \quad \lambda = \frac{2n_{soap}t}{(7/2)} = \frac{4}{7}(1.47)(425nm) = 357nm$$

The first two wavelengths are longer than the reddest visible red light and the last wavelength is just outside of the visible violet range so that the ONLY answer should be:

$\lambda = 500.\,nm \quad (green)$

4. (25 pts)

Three different color lights (red, green, and violet) enter a clear rectangular block of ice from the top at an angle of $\theta = 60.0^0$ as shown in the diagram. Using the index of refraction for the different color of light given in the table below, determine which color light will be able to exit the left face of the ice block at *P*. Calculate the exiting angle of refraction θ' at *P* for the light which will exit the ice block. [Assume that the ice block is surrounded by air and don't consider rays reflected multiple times within the block.]

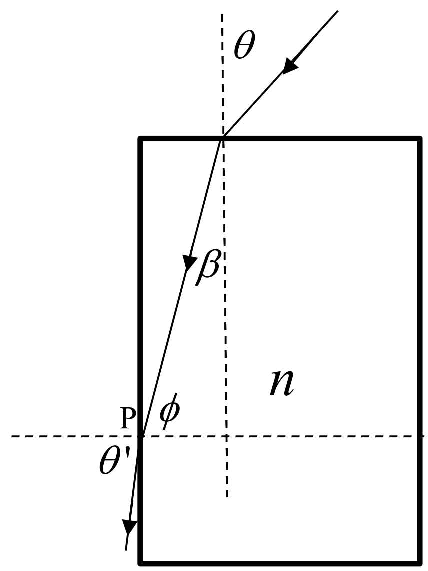


COLOR	λ	n
red	650nm	1.310
green	500nm	1.321
violet	450nm	1.335

SOLUTION:

First, we will write down Snell's law for the top and left faces of the ice block when light enters and exits it:

$$\sin\theta = n\sin\beta \quad (top)$$
$$n\sin\phi = \sin\theta' \quad (left)$$



From geometry, we know that

$$\phi = 90^\circ - \beta$$

Now, for a specific color light to exit the left face, it cannot suffer a total internal reflection meaning that $\sin\theta' \le 1$. So, we will calculate this expression using the two equations for the different color lights:

$$\sin\theta' = n\sin\left(90^\circ - \beta\right) = n\cos\beta$$
$$= n\cos\left[\arcsin\left(\frac{\sin\theta}{n}\right)\right]$$

red: $\sin\theta' = (1.310)\cos\left[\arcsin\left(\frac{\sin 60^\circ}{1.310}\right)\right] = 0.9829$

red light will exit with $\theta' = \arcsin(0.9829) = 79.4^\circ$

green: $\sin\theta' = (1.321)\cos\left[\arcsin\left(\dfrac{\sin 60^\circ}{1.321}\right)\right] = 0.9975$

green light will exit with $\theta' = \arcsin(0.9975) = 86.0^\circ$

violet: $\sin\theta' = (1.335)\cos\left[\arcsin\left(\dfrac{\sin 60^\circ}{1.335}\right)\right] = 1.0160$

violet light will be totally internally reflected!