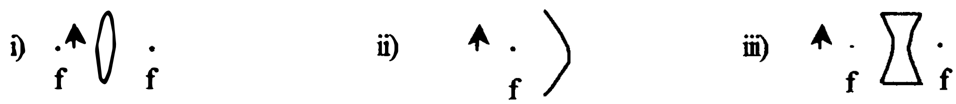
1. (25pts) Answer the following questions. Justify your answers. (Use the space provided below and the next page)

a). Which of the following objects (arrows) will produce real images? Which images will be inverted?

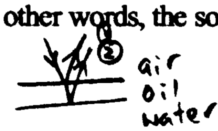


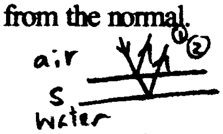
b). A spy plane sends a sonar signal toward the surface of the ocean. If the speed of propagation for sound waves is faster in water than in air, will the sonar signal tends to bend toward or away from the normal direction as it refracts from the air into the ocean?

c). On the ground of a gas station, one observes a bright spot at the thinnest region of an oil film on water. Near another corner, there is an unknown substance on top of a layer of water. At the thinnest part of this substance, one observes a dark spot. Is the index of refraction for this unknown substance larger or smaller then the index of refraction of oil?

d). A small aperture with a size of few millimeters is used in a diffraction experiment and you have a choice on the wavelength of the light source: x-rays ($\lambda \sim 10^{-10}m$), visible lights ($\lambda \sim 10^{-7}m$), infrared lights ($\lambda \sim 10^{-4}m$), and radio waves ($\lambda \sim 1\,m$), which one is most likely to produce measurable diffraction fringes?

a) i)virtual and upright; ii) real and inverted; iii) virtual and upright
b) By Snell's law, the refracted angle should be larger than the incident angle. In other words, the sonar signal will bend away from the normal.
c)





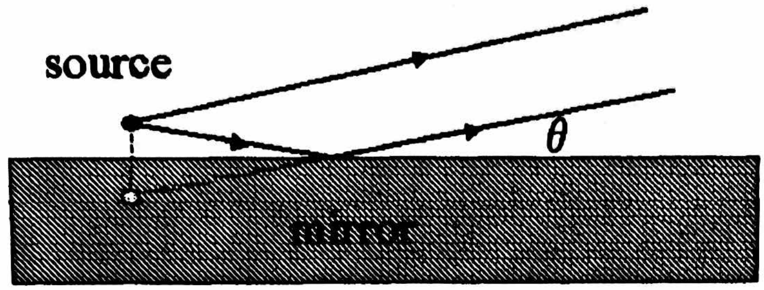
bright spot at the thinnest implies that rays 1 & 2 both suffer 180° phase shift so that the combined phase shift is a full wave length.
$\Rightarrow n_{air} < n_{oil} < n_{water}$

dark spot at thinnest
$\Rightarrow$ only 1 suffers a phase shift.
$\Rightarrow n_{air} < n_s$ and $n_s > n_{water}$

So we have $\boxed{n_s > n_{oil}}$

d) For the fringes to be observable, we should have $\lambda \lesssim a$ and the infrared light will best satisfy this criterion.

2.~~5~~. (25 pts)



A source of light with $\lambda = 550nm$ is placed 5.00mm above the surface of a mirror. At a horizontal distance of 100m away from the source, an interference pattern is observed. This pattern is formed when rays leaving the source directly interfere with rays reflected off the mirror surface as shown above. a) What is the vertical distance to the first bright fringe above the mirror (in mm)? b) What is the vertical distance to the first dark fringe above the mirror (in mm)?

The virtual image of source together with the source itself can be viewed as two coherent sources which will produce interference pattern. These two sources are separated by $(2 \times 5.00) = 10.0$ mm so that their path diff is

$\delta = d \sin\theta$ where $d = 10.0$ mm. $L = 100m$

This is different from regular double slits interference since the 2nd reflected ray will suffer a $\frac{\lambda}{2}$ phase shift due to reflection. So,

a). for constructive interference, $d \sin\theta = (m + \frac{1}{2})\lambda$ $m = 0, 1, 2, \ldots$

(1st bright fringe) : $d \frac{Y_b}{L} = \frac{\lambda}{2} \Rightarrow Y_b = \frac{(550 \times 10^{-9} m)(100m)}{2(10 \times 10^{-3} m)}$

$\boxed{Y_b = 2.75 \text{ mm}}$

b). for destructive interference, $d \sin\theta = m\lambda$ $m = 1, 2, 3, \ldots$

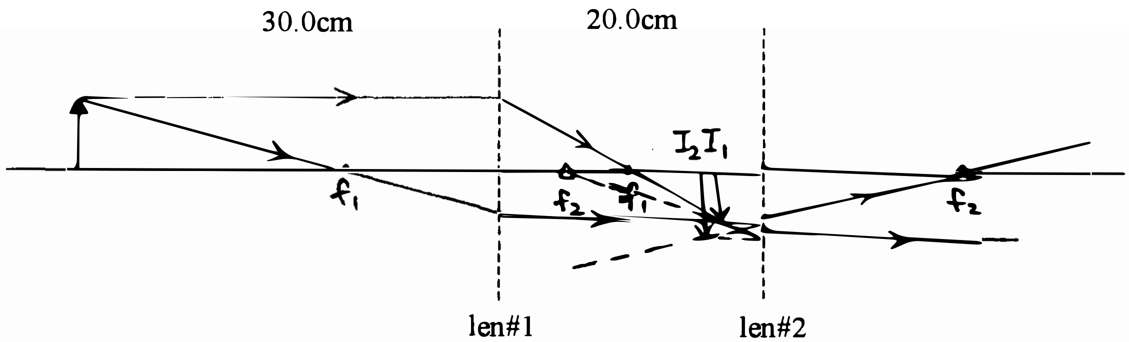
(1st dark fringe above mirror) : $\frac{d Y_d}{L} = \lambda \Rightarrow Y_d = \frac{(550 \times 10^{-9} m)(100m)}{(10 \times 10^{-3} m)}$

$\boxed{Y_d = 5.50 \text{ mm}}$

8/8

3. (25 pts)
Two thin converging lenses of focal lengths $f_1 = 10.0cm$ and $f_2 = 15.0cm$ are separated by 20.0cm. An object is placed at a distance of 30.0cm to the left of the first lens. i) Find the position of the final image; ii) draw the rays diagram for this situation; iii) what is the magnification of the final image; iv) is the final image virtual or real?

ii)



i). len #1 : $\frac{1}{P_1} + \frac{1}{q_1} = \frac{1}{f_1} \Rightarrow \frac{1}{30.0cm} + \frac{1}{q_1} = \frac{1}{10.0cm}$

$$\frac{1}{q_1} = \frac{1}{10.0cm} - \frac{1}{30.0cm}$$

$q_1 = 15.0$ cm I_1 is to the right of lens#1.

len #2 : $\frac{1}{P_2} + \frac{1}{q_2} = \frac{1}{f_2}$ use I_1 as object for lens #2.

$P_2 = 5.00$ cm to the left of lens# 2

$$\frac{1}{5.00cm} + \frac{1}{q_2} = \frac{1}{15.0cm}$$

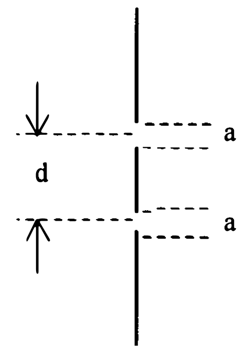
$\boxed{q_2 = -7.50cm}$ I_2 is 7.50 cm to the left of lens #2.

iii) $M = M_1 M_2 = \left(-\frac{q_1}{P_1}\right)\left(-\frac{q_2}{P_2}\right) = \left(-\frac{15.0}{30.0}\right)\left(-\frac{-7.50}{5.00}\right)$

$\boxed{M = -0.75}$ "–" → inverted.

iv) $\boxed{\text{virtual}}$

4. (25 pts)
In a double slit diffraction experiment, two slits with equal width $a = 0.030\,mm$ are separated by a distance of $d = 0.15\,mm$. Take the wavelength of the light to be 550nm. The observed fringes will be a superposition of both interference and diffraction effects. i) How many complete bright interference fringes appear within the central diffraction envelope? ii) Which bright interference fringes are missing from this pattern? iii) What is the relative intensity of the fourth interference fringe as compared to the central maximum?



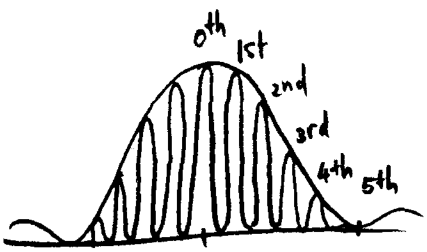
i) 1st min of diffraction envelope :

$$a\sin\theta = \lambda$$

locations of interference maxs :

$$d\sin\theta = m\lambda$$

Equating the two gives the largest m such that the mth order interference peak will still be in the 1st order min of diffraction envelope.



$$\frac{\lambda}{a} = \sin\theta = \frac{m\lambda}{d} \Rightarrow m = \frac{d}{a} = \frac{0.15\,mm}{0.030\,mm} = 5.$$

So, the 5th order interference peak falls exactly on top of the 1st min of diffraction envelope.

$\Rightarrow$ $\boxed{\text{9 complete bright interference fringes}}$

ii) $\boxed{5^{th}, 10^{th}, 15^{th}, \ldots \text{ will be missing.}}$

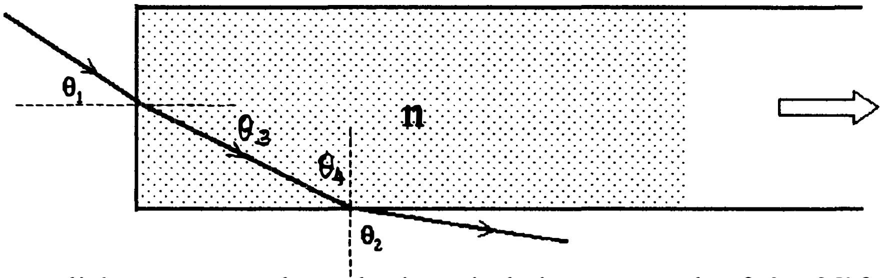
iii) $$I = I_{max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)\left(\frac{\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\frac{\pi a\sin\theta}{\lambda}}\right)^2$$

$$d\sin\theta = 4\lambda \Rightarrow \sin\theta = \frac{4\lambda}{d}$$

7/8

$$\Rightarrow \frac{I}{I_{max}} = \cos^2(1)\left(\frac{\sin\left(\frac{\pi a}{\lambda}\frac{4\lambda}{d}\right)}{4\pi a/d}\right)^2 = \left(\frac{\sin\frac{4\pi}{5}}{4\pi/5}\right)^2 = \boxed{0.055}$$

5. ~~2~~. (25 pts)



A light ray enters a long plastic optical pipe at an angle of $\theta_1 = 35^\circ$ from the left end. It exits from the bottom surface of the pipe at an angle of $\theta_2 = 80^\circ$. a) Determine the index of refraction n for this plastic pipe. b) If n=1.14, what is the maximum value for θ_1 such that the light ray will not exit from the bottom surface? [$\sin(90^\circ - \theta) = \cos\theta$]

a). Applying Snell's Law to both the left end and the bottom surface,

$$\begin{cases} \sin\theta_1 = n\sin\theta_3 & \text{(left) (1)} \\ n\sin\theta_4 = \sin\theta_2 & \text{(bottom) (2)} \end{cases}$$

Since $\theta_4 = 90^\circ - \theta_3$,

$$\sin\theta_4 = \sin(90^\circ - \theta_3) = \cos\theta_3$$

$$\Rightarrow \begin{cases} n\sin\theta_3 = \sin\theta_1 \\ n\cos\theta_3 = \sin\theta_2 \end{cases}$$

$\rightarrow$ Dividing these two equations, we have,

$$\tan\theta_3 = \frac{\sin\theta_1}{\sin\theta_2}$$

$$\theta_3 = \tan^{-1}\left[\frac{\sin 35^\circ}{\sin 80^\circ}\right] = 30.22^\circ$$

Substitute θ_3 back into the 1st eq,

$$n\sin 30.22^\circ = \sin 35^\circ$$

$$\boxed{n = \frac{\sin 35^\circ}{\sin 30.22^\circ} = 1.14}$$

b). We want the max θ_1 such that $\theta_2 \leq 90^\circ$.

So, from the 2nd eq., we have

$$n\cos\theta_3 = 1$$

$$\theta_3 = \cos^{-1}\left(\frac{1}{n}\right) = 28.66^\circ$$

back to eq 1, we have

$$\sin\theta_1 = n\sin\theta_3$$

$$\theta_1 = \sin^{-1}(n\sin 28.66^\circ)$$

$$\boxed{\theta_1 \leq 33.1^\circ}$$