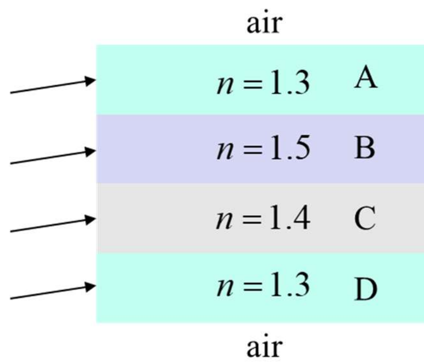
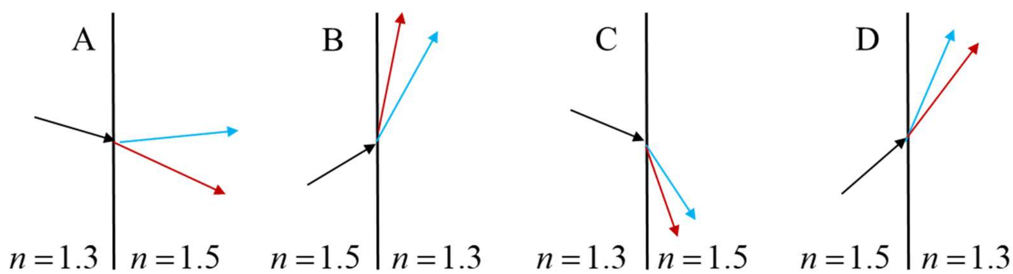
1. (25 pts) Answer the following questions. Justify your answers. (Use additional white paper if you need.)

a) (8 pts) Four light rays enter four long channels of optical materials from the left as show. The index of refraction for the four materials are as indicated. Air is on top of and below the four stacked optical channels. Through which channel (A, B, C, or D) could a light ray be most likely trapped totally within it after many internal reflections as it propagates toward the right?

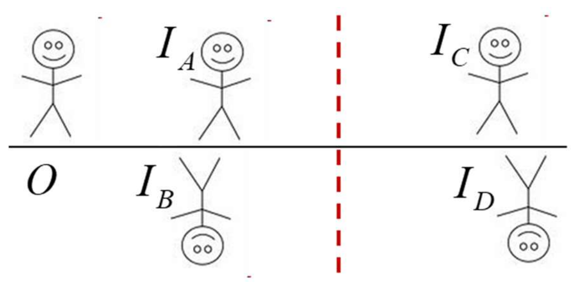



b) (8 pts)




The four panels above show four different situations when an incident ray (black arrow) composes of two colors (blue and red) refracts through an interface between two different materials. The approximate index of refraction for each material is indicated in the graph. Which panel(s) (A, B, C, and D) is (are) physically possible?

c) (9 pts) A stick figure ($O$) stands in front of a spherical mirror at a location indicated by the red dotted line. The optical axis of the spherical mirror is indicated by the horizontal line. The other four stick figures labeled by $I_A, I_B, I_C, I_D$ suggest the general locations where the images might be produced by the mirror. (Note: The stick figures are qualitative representation of the situations only. The size of the figures and the distances from the mirror are not drawn to scale.)




i. Which of the image(s) $(I_A, I_B, I_C, I_D)$ is (are) physically *impossible* for either a concave or convex mirror?

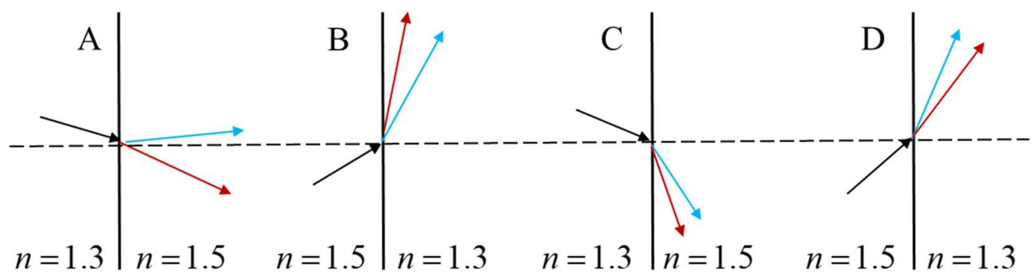
ii. Which of the image(s) $(I_A, I_B, I_C, I_D)$ could be produced by a concave mirror?

iii. Which of the image(s) $(I_A, I_B, I_C, I_D)$ could be virtual?

# Solutions:

a) **Channel B**: To ensure that a light ray will be trapped totally within a particular optical channel after many internal reflections as it propagates toward the right, the optical channel must act like a fiber optic cable allowing total internal reflection to occur with it. For a particular channel, total internal reflection is possible only if the index of refraction of the material on top of and below it is lower than its own index of refraction. Among the four channels, only Channel B satisfies this condition with the index of refraction for both materials on the top and bottom less than *n*=1.5.

b)

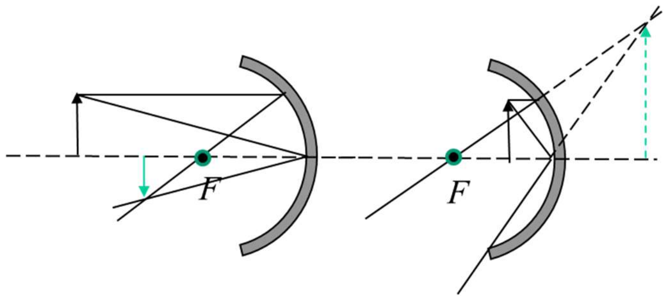



**Panel B is the only physically possible**: The incident ray refracting across the interface must satisfy Snell's law. The normal to the interface is added to aid the analysis.

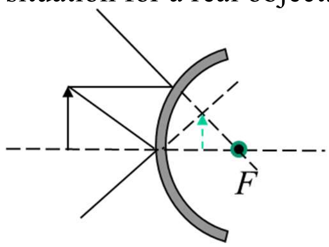
- Panel A is not possible: Different colors from the same incident light will disperse with different angle of refraction but they all must refract to the SAME SIDE with respect to the normal. Here, the blue and red ray diffract to different side of the normal.
- Panel C is not possible: According to Snell's law $n_L \sin\theta_L = n_R \sin\theta_R$, if $n_R > n_L$, the angle of refraction and the angle of incidence must have reverse relationship, i.e., $\theta_R < \theta_L$. Both refracted colored rays as compared with the incident ray have the opposite relationship in Panel C.
- Panel D is also not possible: According to dispersion, the index of refraction of a material varies with the color of light with blue light having a slightly larger index of refraction than a red light in the same material. Then, from Snell's Law, the red light with the smaller index of refraction will have a larger deflection as it refracts (bend away more from the normal) as compared with the blue light. The ordering of the refracted beams in Panel D is wrong.

Panel B is the situation that does not violate Snell's law, $\theta_{refraction} > \theta_{incident}$ with $n_{right} < n_{left}$ and has the right dispersion order for the red and blue light.

c) For a real object, here are the two possible cases for a concave mirror ( $O > f$ and $O < f$ ):




For a convex mirror, since $F$ is behind the mirror, this is the only possible situation for a real object.




These are all the possible images for spherical mirrors (concave or convex) with a real object. So,

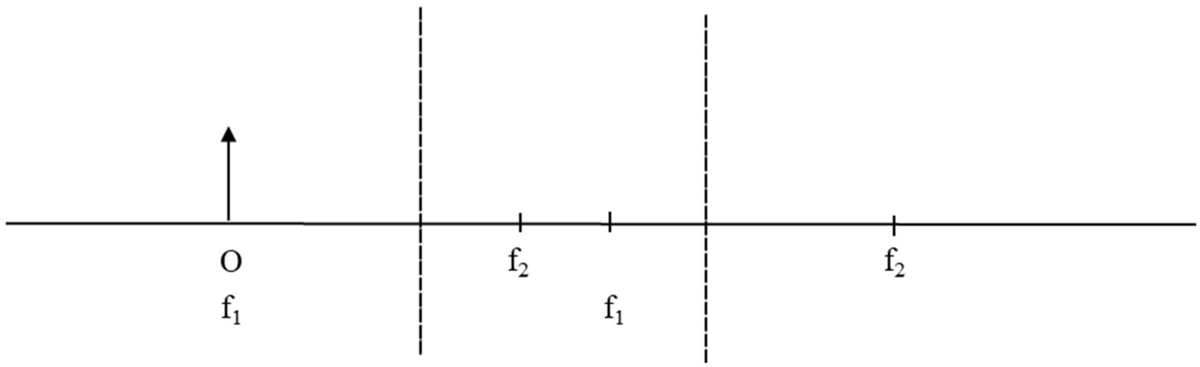
**i.** Images $\mathbf{I_A}$ and $\mathbf{I_D}$ are NOT POSSIBLE**.**

ii. Images $\mathbf{I_B}$ and $\mathbf{I_C}$ can be produced by a concave mirror.

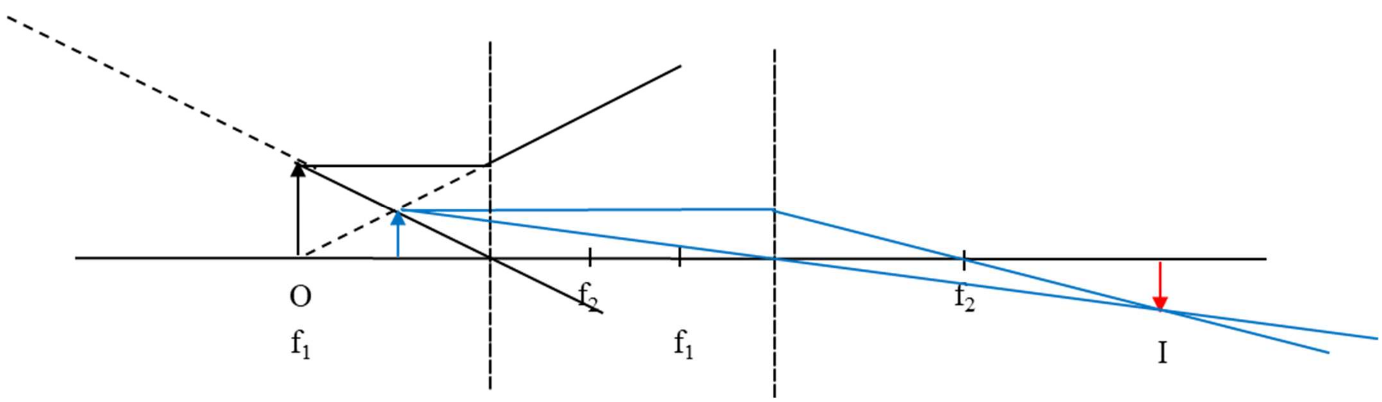
iii. Image $\mathbf{I_C}$ is virtual.

2. (25 pts) A thin diverging lens with a focal length $f_1 = -10.0cm$ on the left and a thin converging lens with a focal length $f_2 = 10.0cm$ on the right are separated by $15.0cm$. An object is placed at a distance $10.0cm$ to the left of the first lens.

i. Find the position of the final image.
ii. Draw the rays diagram for this situation.
iii. What is the magnification of the final image?
iv. Is the final image virtual or real?




**Solution:**




The black rays are from the original object and the blue rays are from the intermediate image (blue arrow). The final image is indicated by the red arrow.

For the lens #1, we have $\frac{1}{s'_1} = \frac{1}{f_1} - \frac{1}{s_1}$ and for lens #2, we have $\frac{1}{s'_2} = \frac{1}{f_2} - \frac{1}{s_2}$.
By convention, $f_1$ is negative (diverging) and $f_2$ is positive (converging). From the first lens equation, we have

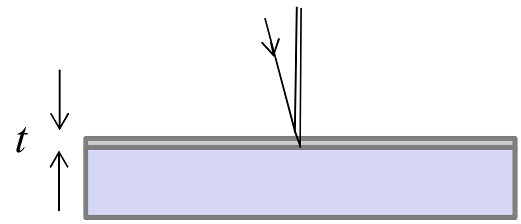
$$\frac{1}{s'_1} = -\frac{1}{10} - \frac{1}{10} = -\frac{2}{10} \quad \rightarrow \quad s'_1 = -5.00cm .$$

As indicated by the rays diagram, this image will be a virtual object for the diverging lens Then, from the diagram, we can see that $s_2 = 15cm - s'_1 = +20cm$. The intermediate image is a real object for len #2. Putting this into the equation for lens #2, we have

$$\frac{1}{s'_2} = \frac{1}{10} - \frac{1}{20} = \frac{2-1}{20} \quad \rightarrow \quad s'_2 = +20.0cm.$$

The final image is **real** as indicated and is located 20 cm behind len #2 (or 35cm behind lens #1).

$M = m_{div} m_{con} = -\frac{s'_1}{s_1}\left(-\frac{s'_2}{s_2}\right) = -\frac{-5}{10}\left(-\frac{20}{20}\right) = -0.5$. The final image is **real**, inverted, and smaller.




3. (25 pts) An optical coating with a thickness $t = 330$nm and an index of refraction $n_{coating} = 1.60$ has been applied to the surface of an optical device made with glass ($n_{glass} = 1.50$). Which wavelengths of light within the visible range (380nm – 750nm) are strongly reflected (constructively interfered) by this optical coating (calculate all possible values within the given range)? Assume nearly normal incidence for your considerations.

SOLUTION:

Only waves reflecting off the top surface of the coating will suffer a $\pi$ phase change since the reflection is from a substance with a larger index of refraction (i.e., $n_{air}<n_{coating}$ and $n_{coating}>n_{glass}$).

Let $t$ be the thickness of the soap film, the condition for constructive interference in this case is,

$$2t = (m+\frac{1}{2})\lambda_n \quad or \quad 2n_{coating}t = (m+\frac{1}{2})\lambda, \quad m = 0,1,2,\cdots$$

Note that the extra path difference occurs in the optical coating so that the relevant wavelength is $\lambda_n = \dfrac{\lambda}{n_{coating}}$

Now, let see what wavelengths of light will be reflected constructively:

$$m=0: \quad \lambda = \frac{2n_{coating}t}{(1/2)} = 4(1.6)(330nm) = 2110nm$$

$$m=1: \quad \lambda = \frac{2n_{coating}t}{(3/2)} = \frac{4}{3}(1.6)(330nm) = 704nm$$

$$m=2: \quad \lambda = \frac{2n_{coating}t}{(5/2)} = \frac{4}{5}(1.6)(330nm) = 422nm$$

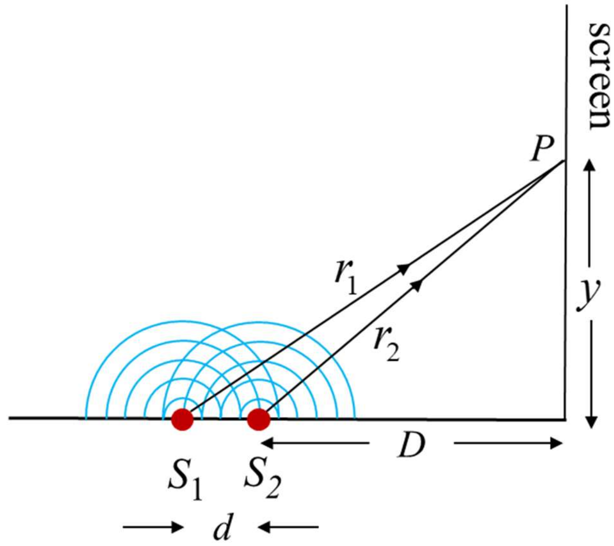
$$m=3: \quad \lambda = \frac{2n_{coating}t}{(7/2)} = \frac{4}{7}(1.6)(330nm) = 302nm$$

The first and the last wavelengths are outside of the visible range so that the ONLY answers should be:

$$\lambda = 704nm \quad (red)$$

$$\lambda = 422nm \quad (violet)$$

4. (25 pts) Two coherent point sources $S_1$ and $S_2$ emit light with the same wavelength $\lambda$ and at the same amplitude in all directions as shown. The two light sources are separated by a distance of $d = 5.00\lambda$ on the horizontal axis. A vertical viewing screen is placed at a distance of $D = 20.0\lambda$ to the right of $S_2$. The figure also shows two sample rays of light from $S_1$ and $S_2$ reaching the vertical screen at point $P$ at a height $y$ above the horizontal axis. [Note: Since the screen is close to the two sources of light, you cannot treat $r_1$ and $r_2$ as nearly parallel.]




i. What is the path difference $\delta = r_1 - r_2$ between the two sources at $y = 0$ on the screen? Will there be a bright fringe (constructive interference maximum) or dark fringe (destructive interference minimum) at $y = 0$?
ii. As $y$ increases, will the path difference $\delta$ increase or decrease?
iii. Write down an expression for the path difference $\delta$ in terms of the height $y$ above the horizontal axis on the screen.
iv. To observe the **first** dark fringe (destructive interference minimum) above the horizontal axis on the screen, what is the required path difference $\delta$ (in units of $\lambda$)?
v. Calculate the path difference $\delta$ (in units of $\lambda$) for light from the two sources reaching the screen at a height of $y = 30.0\lambda$. Is the resulting intensity at this location on the screen close to a maximum (constructive interference) or a minimum (destructive interference)?

**Solution**:

The general principle for two waves interference is the following:

-For maximum intensity at locations with constructive interference, we need
$\delta = r_1 - r_2 = m\lambda, \quad m = 1,2,3,4,5$ and
-For minimum intensity at locations with destructive interference, we need
$$\delta = r_1 - r_2 = \left(m + \frac{1}{2}\right)\lambda, \quad m = 1,2,3,4 .$$

i. $\delta = d = 5\lambda$. Since $\delta$ is an integer multiple of a wavelength, it will be a bright fringe.

ii. By considering the geometry of the problem, as $y$ increases going up the screen, the path difference $\delta$ between $r_1$ and $r_2$ will decrease.

iii. First, we need to express $r_1$ and $r_2$ in terms of $y, d$, and $D$,

$r_1 = \sqrt{y^2 + (d+D)^2}$ and $r_2 = \sqrt{y^2 + D^2}$

Then, the path difference between the two light rays is given by,

$\delta(y) = r_1 - r_2 = \sqrt{y^2 + (d+D)^2} - \sqrt{y^2 + D^2}$

iv. To observe the first dark fringe above the horizontal axis, the path difference must be $\left(m + \frac{1}{2}\right)\lambda$. From i, we know that $y = 0$ is a bright fringe (maximum) with $\delta = 5\lambda$. As $y$ increases, $\delta$ decreases and the intensity will decrease as well. The first (next) minimum will occur when $\delta$ decreased by $\frac{1}{2}\lambda$ so, we will have $\delta = 4.50\lambda$ at the first minimum above the axis.

v. Substituting $y = 30.0\lambda$ into the equation from part iii, we have

$\delta(30\lambda) = \left[\sqrt{30^2 + (5+20)^2} - \sqrt{30^2 + 20^2}\right]\lambda = 2.996\lambda$

So, the path difference is almost at $3\lambda$ and the intensity at this location will be close to its maximum (constructive interference), $\delta = m\lambda$ with $m = 3$.