1. **(25 pts)** Answer the following questions. (Use the space provided below and the next page if needed.)

   a) **(3 pts)** You have two ideal gases with the same number of moles in your laboratory. You know that one of them is monatomic and the other is diatomic but you don’t know which one is which. You are asked to distinguish them by heating the gases up and to observe the temperature changes. For the same amount of heat absorbed by the two gases, you observed that gas #1 results in a higher temperature rise than gas #2. Which one is the diatomic ideal gas (gas #1 or gas #2)?

   b) **(4 pts)** In a reversible process, an ideal gas is allowed to compress isothermally. Then, by another reversible process, the gas is adiabatically expanded back to its original volume. Does the entropy of the gas increase, stay the same, or decrease after this two-stage process?

   c) **(6 pts)**
      i. Two thermal systems (not necessary an ideal gas) are with the same amount of internal energy, do they necessarily have the same temperature?
      ii. Can two thermal systems with different internal energy be at a thermal equilibrium?

   d) **(12 pts)** An ideal gas initially at an initial pressure, volume, and temperature is allowed to double its volume \(V \rightarrow 2V\) through the following processes:
      o Reversible isothermal expansion
      o Reversible isobaric expansion
      o Reversible adiabatic expansion
      o Adiabatic free expansion

      in four different experiments. When the experimenter compares the final temperature from the four experiments,
      i. Which process(es) will result in the largest temperature reduction?
      ii. Which process(es) will result in the smallest temperature reduction?
      iii. Which process(es) will result in a temperature increase?

   **SOLUTION:**

   a) According to Kinetic Theory of Gases, a diatomic ideal gas will have a larger heat capacity so that for the same amount of heat absorbed, it will result in a lower temperature raise \(dQ=nCdT\). Thus, **gas #2 is the diatomic gas**.

   b) In an isothermal compression, \(Q=W<0\) and entropy \(S_{\text{isotherm}} = Q/T\) will decrease. Now, since the adiabatic compression is reversible, the entropy change is 0. As a result, the two-stage process will result in a net **decrease** in entropy.

   c)
i. No. Two systems with the same internal energy do not necessarily have the same average translational kinetic energy and since temperature is a measure of the average translational kinetic energy of the molecules within the system only, it needs not be the same for the two systems.

ii. Yes. Two thermal systems are in thermal equilibrium if they have the same temperature. Two thermal systems can have different internal energy but with the same temperature.

d) The three reversible processes can be visualized in the PV diagram shown to the right. The adiabatic free expansion is a non-reversible process involving no heat exchange (adiabatic) and no work done (free expansion) so that it will involve no internal energy change so that an adiabatic free expansion will result in no temperature change similar to an isothermal process. From the PV diagram, we can see that,

i. The **reversible adiabatic expansion** will result in the largest temperature reduction.

ii. Both the **reversible isothermal expansion** and the **adiabatic free expansion** will result in zero temperature changes.

iii. The **reversible isobaric expansion** will result in a temperature increase. While the gas will do work on its environment while it expands but the heat absorbed by the gas needed to keep it at the same pressure will be larger resulting in an increase of its internal energy and its final temperature.
2. \textbf{(25 pts)} A 350.-g block of hot iron (Fe) at 400.\degree C is placed into 15.0 g of water at 20.0\degree C?  a) Will all water be converted into steam?  b) What will the final equilibrium temperature be?  

[\begin{align*}
c_{Fe} &= 0.470 \text{ J/g} \cdot \text{K} ; \\
L_v &= 22.6 \times 10^2 \text{ J/g} ; \\
c_{\text{water}} &= 4.186 \text{ J/g} \cdot \text{K} ; \\
c_{\text{steam}} &= 2.027 \text{ J/g} \cdot \text{K} 
\end{align*}] 

\textbf{SOLUTION:} 

[Note: There is a phase change point for water at 100\degree C (boiling point)] 

\begin{enumerate}
    \item a) Heat needed to convert all water into steam at 100\degree C:
        \begin{align*}
            \frac{\text{heat absorbed,}}{\text{water warming}} 
            \begin{cases}
                20 \degree C \rightarrow 100 \degree C \\
                \Delta T = 80 \degree K
            \end{cases}
        \end{align*}
        \begin{align*}
            + \left( \frac{\text{latent heat absorbed,}}{\text{water to steam}} @ 100 \degree C \right) 
            &= m_{\text{water}} c_{\text{water}} \cdot 80 \degree K + m_{\text{water}} L_v \\
            &= 15.0 \text{g} \left( (4.186 \text{ J/g} \cdot \text{K}) \times 80 \degree K + 22.6 \times 10^2 \text{ J/g} \right) \\
            &= 3.892 \times 10^4 \text{ J}
        \end{align*}

To compare, heat released from the hot iron from 400\degree C to 100\degree C will be:

\begin{align*}
    \frac{\text{heat released,}}{\text{Fe cooling}} 
    \begin{cases}
        400 \degree C \rightarrow 100 \degree C
    \end{cases}
    \rightarrow m_{Fe} c_{Fe} \left( 373.15 \degree K - 673.15 \degree K \right) \\
    &= -4.935 \times 10^4 \text{ J}
\end{align*}

Comparing the two values, the amount of heat released by the iron is more than enough to turn all water to steam at 100\degree C.

\item b) Let the \( T > 100 \degree C \) be the final temperature.  By identifying all heat exchange processes on both sides of the boiling point,

\begin{align*}
    \sum Q &= 0 \\
    \left( \frac{\text{heat released,}}{\text{Fe cooling}} \right) + \left( \frac{\text{heat absorbed,}}{\text{water warming}} \right) + \left( \frac{\text{latent heat absorbed,}}{\text{water to steam}} @ 100 \degree C \right) + \left( \frac{\text{heat absorbed,}}{\text{steam warming}} \right) &= 0
\end{align*}

\begin{align*}
    m_{Fe} c_{Fe} \left( T - 673.15 \degree K \right) + m_{\text{water}} c_{\text{water}} \left( 373.15 \degree K - 293.15 \degree K \right) + m_{\text{water}} L_v + m_{\text{water}} c_{\text{steam}} \left( T - 373.15 \degree K \right) &= 0
\end{align*}

Rearranging terms, we then can solve for \( T \),

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\[
T = \frac{m_2 c_{Fe} (673.15 K) - m_a \left( L_v + c_v (80 K) - c_s (373.15 K) \right)}{m_2 c_{Fe} + m_s (c_s)}
\]

\[
= \frac{(350 g) (0.470 J / g \cdot C)(673.15 K) - 15g \left( 22.6 \times 10^2 J / g + (4.186 J / g \cdot K)(80 K) - (2.027 J / g \cdot K)(373.15 K) \right)}{350g (0.470 J / g \cdot K) + 15g (2.027 J / g \cdot K)}
\]

\[
= 426.65 K = 153.50^\circ C
\]

\[
= 427 K = 154^\circ C
\]
3. (25 pts) One mole of an ideal monatomic gas starting at state \(a\) \((P_a = 150\text{ kPa}; V_a=15.0\text{ L})\) is adiabatically expanded to a volume of \(V_b =30.0\text{ L}\) at state \(b\). Then, the gas is isothermally compressed back to its initial volume at state \(c\) with \(V_c =V_a\). To complete the cycle, the gas is returned to state \(a\) through an isochoric process.

i) Calculate the net work \(W\) done by the gas per cycle.

ii) What is the total amount of heat absorbed \(Q\) by the gas in one cycle?

iii) What is the efficiency \(e\) of this cycle?

\([1L = 10^{-3} m^3, 1 \text{ atm} = 1.013 \times 10^5 Pa]\)

SOLUTION:

i) First, we use the ideal gas law to find the numerical value of the initial temperature \(T_a\):

\[
T_a = \frac{P_a V_a}{nR} = \frac{(1.50 \times 10^5 \text{ Pa})(15 \times 10^{-3} \text{ m}^3)}{(1\text{ mol})(8.314 \text{ J/ mol} \cdot \text{ K})} = 270.63K
\]

Using the adiabatic process, we can also calculate \(T_b\):

\[
T_b = T_a \left(\frac{V_a}{V_b}\right)^{\gamma -1} = 270.63K \left(\frac{15}{30}\right)^{\frac{2}{3}} = 170.48K
\]

To calculate the net work done by the ideal gas in cycle, we need to calculate the work done from each branches of the cycle:

In an adiabatic process, \(Q_{ab} = 0\) so that from 1st law, we have

\[
W_{ab} = -\Delta U_{ab} = -nC_v(T_b - T_a) = -(1\text{ mole})\left(\frac{3R}{2}\right)(170.48K - 270.63K) = 1.24888kJ
\]

Now, for the isothermal compression, we can calculate \(W_{bc}\) using

\[
W_{bc} = nRT_b \ln \left(\frac{V_c}{V_b}\right) = (1\text{ mole})R(170.48K)\ln \left(\frac{15}{30}\right) = -982.47J
\]

Lastly, for the isochoric process, \(\Delta V = 0\) so \(W_{ca} = 0\). This then gives,

\[
W = W_{ab} + W_{bc} + W_{ca} = 1248.88J - 982.47J + 0J = 266.41J = 266J
\]

ii) By definition, the adiabatic process has \(Q_{ab} = 0\).
Then, for the isothermal process, from 1st law, we have
\[ Q_{bc} = W_{bc} = -982.47 \text{ J} \quad (\text{heat released}) \]

In the isochoric process, we have

\[ Q_{ca} = \Delta U_{ca} + W_{ca} = nC_v (T_a - T_c) = (1 \text{ mole}) \left( \frac{3R}{2} \right) (270.63K - 170.48K) \]
\[ = +1.24888 kJ \quad (\text{heat absorbed}) \]

So, all heat absorbed in one cycle is just \( Q_{ca} \cdot Q_h = Q_{ca} = 1250 \text{ J} \)

iii) The efficiency of this cycle is

\[ e = \frac{W}{Q_h} = \frac{266J}{1250J} = 21.3\% \]

If this is a Carnot cycle, the efficiency is

\[ e_{\text{carnot}} = 1 - \frac{T_c}{T_h} = 1 - \frac{170.48K}{270.63K} = 37.0\% \]

As it should be, the efficiency of this non-Carnot cycle is less than the theoretical maximum given by the Carnot cycle.
4. (25 pts) One mole of a monatomic ideal gas at a temperature $T_i = 350.0 \text{ K}$ is being compressed from $V_i = V$ to $V_f = V/3$. Calculate the final temperature and the entropy change for the three different reversible processes shown on the right:

a) an adiabatic compression;
b) an isothermal compression; and
c) an isobaric compression.

SOLUTION:

Since all three processes are reversible, we can use the expression: $dS = dQ/T$ for all of them.

a) For an adiabatic process, we have the relation: $T_f V_f^{\gamma - 1} = T_i V_i^{\gamma - 1}$. For a monatomic gas, we also know that $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$. Solving for $T_f$ gives,

$$T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma - 1} = 350 K \left( \frac{3}{2} \right)^{2/3} = 728 K$$

Since $dQ = 0$ for an adiabatic process, we will have no entropy change for this process, i.e.,

$$\Delta S = 0 J / K$$

b) For an isothermal process, we have by definition $T_f = T_i = 350 K$

To calculate the entropy change for an isothermal compression, we start by rearranging the 1st Law of Thermodynamics:

$$dU = dQ - dW = 0$$

$$dQ = dW = pdV = nRT \frac{dV}{V}$$

Then, for a reversible process $dS = dQ/T$ and we have,
$$\Delta S = \int d\frac{Q}{T} = \int \frac{nRT}{TV} dV = nR \ln(V_f/V_i)$$

$$\Delta S = (1.00mol)(8.314J/mol \cdot K) \ln \left(\frac{1}{3}\right) = -9.13J/K$$

c) For an isobaric process, $T$ and $V$ are directly proportional to each other through the Ideal Gas Law and we can rewrite

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = \frac{1}{3} \quad \rightarrow \quad T_f = \frac{T_i}{3} = \frac{350K}{3} = 117K$$

Using the definition of molar specific heat at constant pressure, we also have,

$$dQ = nC_v dT$$

Substituting this into the definition for entropy, we then have

$$\Delta S = \int d\frac{Q}{T} = \int \frac{nC_v dT}{T} = nC_v \ln(T_f/T_i)$$

For a monatomic gas, we can express

$$C_p = C_v + R = \frac{3R}{2} + R = \frac{5R}{2}$$

Putting everything together, we arrive at,

$$\Delta S = (1.00mol) \frac{5}{2} (8.314 J/mol \cdot K) \ln \left(\frac{1}{3}\right) = -22.8J/K$$