

1. (25 pts)

Answer the following questions.

- a) (6 pts) An ideal gas is being compressed isothermally and then expanded adiabatically back to its original volume. Both the compression and expansion are done reversibly. After the two-stage process,

- i) Will $T_f < T_i$, $T_f = T_i$, or $T_f > T_i$?
- ii) Will $\Delta U < 0$, $\Delta U = 0$, or $\Delta U > 0$?

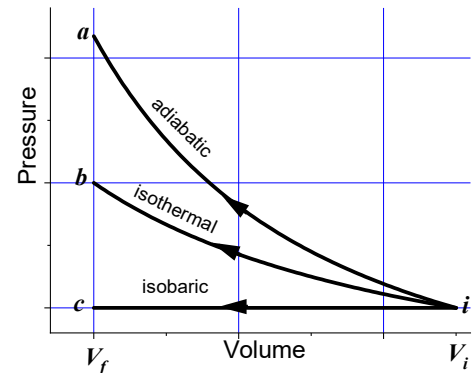
- b) (4 pts) An online advertisement claims that a heat engine operating between a high temperature reservoir at 120°C and a low temperature reservoir at 40°C can achieve a thermal efficiency of 30%. Is this claim theoretically possible?

- c) (6 pts) Systems A and B are in thermal equilibrium with each other.

- i) Do they necessarily have the same internal energy?
- ii) Will they have the same temperature?
- iii) If system A is also in thermal equilibrium with another system C, what can you say about the temperature for system C as compare with system B?

- d) (9 pts) An ideal gas is being compressed through three separate thermodynamic processes from V_i to V_f as shown to the right, please indicate whether its internal energy U will **increase** ($\Delta U > 0$), **decrease** ($\Delta U < 0$), or **stay the same** ($\Delta U = 0$):

- i) A reversible adiabatic compression **a**:
(**increase, decrease, stay the same**)
- ii) A reversible isothermal compression **b**:
(**increase, decrease, stay the same**)
- iii) A reversible isobaric compression **c**:
(**increase, decrease, stay the same**)



SOLUTION:

- a) Although the gas is compressed and expanded back to the same volume, it does not mean that the gas returns to the same **initial** state with the same P and T .
- i) Since the temperature decreases in a reversible adiabatic expansion while the temperature stays the same in an isothermal compression, the final temperature will be lower than the initial temperature after the two-stage process. So, $T_f < T_i$.

- ii) Since the internal energy of an ideal gas is a function of the temperature only and from part i, we know that $\Delta T < 0$. So, $\Delta U < 0$.
- b) The theoretical maximum efficiency of a heat engine is given by the efficiency of a Carnot Cycle, $e_c = 1 - \frac{T_C}{T_H}$. So for $T_H = 393.15K$ and $T_C = 313.15K$ (**temp must be in K**), the thermal efficiency for any heat engines cannot be larger than the Carnot Cycle's efficiency, $e_c = 1 - \frac{313.15K}{393.15K} = 20\%$. So, the claim of 30% thermal efficiency cannot be possible.
- c)
- No. The fact that systems A and B are in thermal equilibrium simply means they have the same temperature and this fact implies that microscopic elements within the system will have the same average translational KE per molecule but the amount of microscopic elements within each system might be different and their molecular structure might be different (e.g. monoatomic vs. diatomic) such that the total internal energy for the two systems are not necessary the same even if they are in thermal equilibrium.
 - Yes. If not, there will be heat exchanges between the two systems.
 - System C must have the same temperature as system B. 0th Law.
- d)
- For an adiabatic compression, temperature increases and since internal energy is a function of temperature only for an ideal gas, $\Delta U > 0$ (**increase**).
 - For any isothermal processes, temperature will stay the same so internal energy will also **stay the same**, $\Delta U = 0$.
 - For a compression, we have $V_f < V_i$. Then, from the Ideal Gas Law and with pressure being constant, we also know that $\frac{V_i}{T_i} = \frac{V_f}{T_f}$ or $\frac{T_f}{T_i} = \frac{V_f}{V_i}$. So, if we have $V_f < V_i$ for a compression, we will have $T_f < T_i$ for an isobaric compression. Since internal energy is a function of temperature only, we also have $\Delta U < 0$.

2. (25 pts)

0.0270 kg of ice at a temperature of $-5.00\text{ }^{\circ}\text{C}$ is placed in thermal contact with 0.320 kg of an unknown liquid at a temperature of $30.0\text{ }^{\circ}\text{C}$. After the system comes to thermal equilibrium, the combined liquid substance has an equilibrium temperature of $15.0\text{ }^{\circ}\text{C}$. Assuming that heat is only exchanged between the ice and the unknown liquid, and the unknown liquid didn't go through a phase change, calculate the specific heat c_{liquid} of the unknown liquid.

$$[L_f = 3.34 \times 10^5 \text{ J/kg}; c_{ice} = 2.100 \times 10^3 \text{ J/kg} \cdot \text{C}; c_{water} = 4.186 \times 10^3 \text{ J/kg} \cdot \text{C}]$$

SOLUTION:

Let the specific heat for the unknown liquid be c_{liq} . While the unknown liquid does not go through a phase change, the ice will at its freezing point at $0.0\text{ }^{\circ}\text{C}$. Writing out all the individual processes,

$$\sum Q = 0 \text{ gives}$$

$$\left(\begin{array}{l} \text{heat released,} \\ \text{liquid} \\ \text{cooling} \\ 30^{\circ}\text{C} \rightarrow 15^{\circ}\text{C} \end{array} \right) + \left(\begin{array}{l} \text{heat absorbed,} \\ \text{ice warming} \\ -5^{\circ}\text{C} \rightarrow 0^{\circ}\text{C} \end{array} \right) + \left(\begin{array}{l} \text{latent heat absorbed} \\ \text{by ice to melt into} \\ \text{water at } 0^{\circ}\text{C} \end{array} \right) + \left(\begin{array}{l} \text{heat absorbed,} \\ \text{water warming} \\ 0^{\circ}\text{C} \rightarrow 15^{\circ}\text{C} \end{array} \right) = 0$$

Writing them into equation form gives

$$m_{liq}c_{liq}(15^{\circ}\text{C} - 30^{\circ}\text{C}) + m_i c_i (0^{\circ}\text{C} - (-5^{\circ}\text{C})) + m_i L_f + m_i c_w (15^{\circ}\text{C} - 0^{\circ}\text{C}) = 0$$

Rearranging terms, we then can solve for c_{liq} ,

$$c_{liq} = \frac{m_i c_i (5^{\circ}\text{C}) + m_i L_f + m_i c_w (15^{\circ}\text{C})}{m_{liq} (15^{\circ}\text{C})}$$

$$c_{liq} = \frac{(0.0270\text{kg}) \left[(2100\text{J/kg} \cdot \text{C})(5.0^{\circ}\text{C}) + (3.34 \times 10^5 \text{ J/kg}) + (4186\text{J/kg} \cdot \text{C})(15.0^{\circ}\text{C}) \right]}{(0.320\text{kg})(15^{\circ}\text{C})}$$

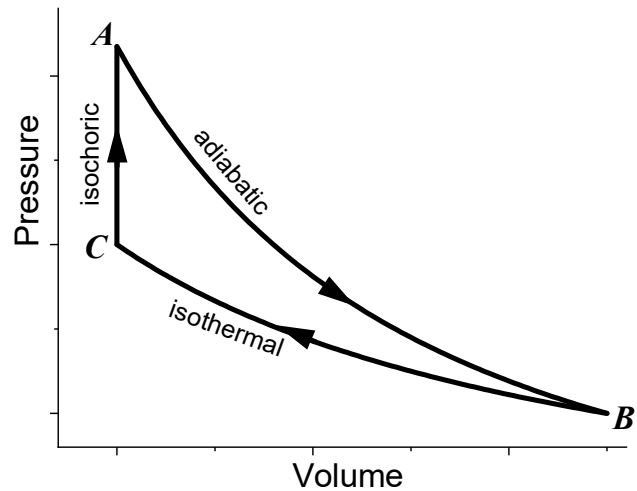
$$c_{liq} = 2290\text{J/kg} \cdot \text{C}$$

3. (25 pts)

A heat engine with one mole of a monatomic ideal gas is operating in a cycle as shown to the right.

$$[1L = 10^{-3} m^3; 1 atm = 1.103 \times 10^5 Pa]$$

The gas starting from an initial state **A**: ($T_A = 270.K, V_A = 15.0L$) is first being expanded to an intermediate state **B** with twice its original volume $V_B = 30.0L$ through a reversible adiabatic process.



- What is the temperature T_B of the gas after the expansion at state **B**?
- Calculate the work done by the gas W_{AB} and heat exchange Q_{AB} as the gas adiabatically expands from **A** to **B**.

From **B**, the ideal gas is then isothermally and reversibly compressed back to its original volume at state **C**, i.e., $V_C = V_A$.

- What is the temperature T_C of the gas at state **C**?
- Calculate the work done by the gas W_{BC} and heat exchange Q_{BC} as the gas being isothermally compressed from **B** to **C**.

From **C**, the ideal gas is then being brought back to its initial state **A** through a reversible isochoric process.

- Calculate the work done by the gas W_{CA} and heat exchange Q_{CA} for this isochoric process.
- Calculate the efficiency of this heat engine.

SOLUTION:

- Since $A \rightarrow B$ is a reversible **adiabatic** process, we can use the following relation,

$$T_B = T_A \left(\frac{V_A}{V_B} \right)^{\gamma-1} = 270.K \left(\frac{15}{30} \right)^{\frac{2}{3}} = 170.K$$

- The work done by the gas for the adiabatic expansion is

$$W_{AB} = -\Delta U_{AB} = -nC_v(T_B - T_A) = -(1 \text{ mole}) \left(\frac{3R}{2} \right) (170.K - 270.K) = 1.25kJ$$

By definition, we also have $Q_{ab} = 0$ for an adiabatic process.

c) By definition, temperature in an isothermal process stays the same so that we have $T_C = T_B = 170.K$.

d) Work done in an isothermal process is given by

$$W_{BC} = nRT_B \ln\left(\frac{V_C}{V_B}\right) = (1\text{mole})R(170.K) \ln\left(\frac{15}{30}\right) = -980.J$$

For an isothermal process, $\Delta U = 0$, so by the 1st law of thermodynamics, $Q_{BC} = W_{BC} = -980.J$ (*heat released*)

e) For the last branch of the cycle, work done by the ideal gas in an isochoric process is zero, i.e., $W_{CA} = 0J$ since $dV = 0$.

Since this is an isochoric process, we can calculate the heat absorbed by the gas Q_{CA} directly in terms of C_v ,

$$Q_{CA} = nC_v(T_A - T_C) = (1\text{mole})\left(\frac{3R}{2}\right)(270.K - 170.K) = +1.25kJ \quad (\text{heat absorbed})$$

f) Lastly, the efficiency of an heat engine is given by $e = \frac{W_{net}}{Q_H}$, where

$$W_{net} = W_{ab} + W_{bc} + W_{ca} = 1250J - 980.J + 0J = 270.J$$

$$\text{and } Q_H = Q_{CA} = 1250J \quad (\text{heat absorbed only})$$

So, we have

$$e = \frac{W_{net}}{Q_H} = \frac{270.J}{1250J} = 21.6\%$$

If this is a Carnot cycle, the efficiency is $e_{carnot} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{170.K}{270.K} = 37.0\%$. As it should be, the efficiency of this non-Carnot cycle is less than the theoretical maximum given by the Carnot cycle.

4. (25 pts)

One mole of a monatomic ideal gas in a container is allowed to cool from 50.0 °C to room temperature at 20.0 °C at constant atmospheric pressure in the lab.

- Calculate the heat released by the gas Q_{gas} as the gas cools to room temperature.
- Calculate the entropy change ΔS_{gas} of the gas as the gas cools to room temperature.
- Assuming that all heat released by the gas is absorbed by the air in the room and the temperature of the air in the room remains at 20.0 °C, calculate the entropy change ΔS_{room} for the room air.
- Assuming the ideal gas together with the surrounding room air as an isolated system, what is the total entropy change for the system: ideal gas + air?

[Please pay attention to the sign of all Q and ΔS .]

SOLUTION:

- For a constant pressure process, the heat released by the ideal gas as it cools is given by,

$$Q_{gas} = nC_p(T_f - T_i)$$

$$Q_{gas} = (1\text{mol})\left(\frac{5R}{2}\right)(293.15\text{K} - 323.15\text{K}) = -623.55\text{J} = -624\text{J} \quad (\text{heat released})$$

- Working from definition, we have $dS = \frac{dQ}{T}$. With the gas cooling at constant pressure, we have,

$$\begin{aligned} \Delta S_{gas} &= \int_{T_i}^{T_f} \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{nC_p dT}{T} = nC_p \ln\left(\frac{T_f}{T_i}\right) \\ &= (1\text{mol})\left(\frac{5R}{2}\right) \ln\left(\frac{293.15\text{K}}{323.15\text{K}}\right) \\ &= (1\text{mol})\left(\frac{5 \times 8.314\text{J} / \text{K} \cdot \text{mol}}{2}\right)(-0.097432) = -2.03\text{J} / \text{K} \end{aligned}$$

Note the negative sign for ΔS_{gas} (entropy decreases) since the gas is cooling.

- Assuming that there is a large amount of air in the room so that the heat that it absorbed will not change the room temperature by any measurable amount. Then, the entropy change for the room air can be calculated through an isothermal process,

$$\Delta S_{room} = \frac{Q_{room}}{T_{room}}$$

Since $Q_{room} = +|Q_{gas}| = +624J$ (with explicit sign) and $T_{room} = 293.15K$, we have,

$$\Delta S_{room} = \frac{623.55J}{293.15K} = +2.13J / K$$

The entropy change for the room air ΔS_{room} is positive (entropy increases) as the room air absorbs the heat.

- d) The total entropy for the system with the ideal gas and the surrounding air in the room is then,

$$\Delta S_{tot} = \Delta S_{gas} + \Delta S_{room} = -2.03J / K + 2.13J / K = +0.10J / K$$

$\Delta S_{tot} > 0$ as it should be according to the 2nd Law of Thermodynamics.