# 1. (25 pts)

Answer the following questions. (Use space provided on the next page if needed.)

- a) (4 pts) Two ideal gases are in thermal equilibrium with each other.
  - i) Would the molecules inside the two gases necessarily have the same *average* translational kinetic energy?
  - ii) Would the two gases necessarily have the same internal energy?
- b) (9 pts) An ideal gas is being taken from temperature  $T_1$  to  $T_2$  through two different thermodynamic processes:  $i \rightarrow a$  (isochoric process) and  $i \rightarrow b$  (isobaric process). The two curves are isotherms. Please circle the correct relationship (<, =, or >) for the following thermodynamic quantities:



c) (12 pts) An ideal gas is being put through the following four thermodynamic processes, please indicate whether its entropy S will increase  $(\Delta S > 0)$ ,

decrease  $(\Delta S < 0)$ , or stay the same  $(\Delta S = 0)$ :

- i) A reversible isobaric expansion (increase, decrease, stay the same)
- ii) A reversible isothermal expansion (increase, decrease, stay the same)
- iii) A reversible adiabatic expansion (increase, decrease, stay the same)
- iv) A reversible isochoric process decreasing the pressure of the gas (increase, decrease, stay the same)

(use next blank page for answers to this question if needed)

#### SOLUTION:

- a)
- i. YES. Thermal equilibrium implies that the two ideal gases have the same temperature. Since temperature is directly proportional to the average translational kinetic energy of the molecules inside the gases, so the answer is yes. Note that while the *average* translational kinetic energy is the same for the gases, the individual molecules are random with different translational kinetic energy.

- ii. NO. The molar heat capacity and the number of moles *n* for the two gases might be different. Even *T* is the same,  $U = nC_vT$  will not necessarily be the same for the two gases.
- b)

$$\Delta U_{i \to a} < \equiv or > \Delta U_{i \to b} \text{ (same } \Delta T\text{)}$$

$$W_{i \to a} \leq or > W_{i \to b} \text{ (}W_{i \to a} = 0, W_{i \to b} > 0\text{)}$$

$$Q_{i \to a} \leq or > Q_{i \to b} \text{ (}Q = \Delta U + W \text{ so same relationship as } W\text{)}$$

c)

For an ideal gas, we have  $\Delta S = nC_V \ln(T_f/T_i) + nR \ln(V_f/V_i)$ 

- i. Increase. For an isobaric expansion, both V and T increase, so  $\Delta S > 0$ .
- ii. Increase. Temperature stay the same for an isothermal process but since it is an expansion, V increases, so  $\Delta S > 0$ .
- iii. Stay the same. For any adiabatic process, we have Q = 0. Since it is reversible, we have dS = dQ/T. With Q = 0,  $\Delta S = 0$ . Note that T decreases and V increases in a reversible adiabatic expansion in such a way that the two terms in the  $\Delta S$  equation above will cancel each other, i.e.,  $nC_v \ln(T_f/T_i) = -nR \ln(V_f/V_i)$ , giving  $\Delta S = 0$ .
- iv. **Decrease**. For this isochoric process, as pressure decreases, the final state will end up on an isotherm with a lower T. Since, volume stays the same (isochoric) and temperature decreases,  $\Delta S < 0$ .



### 2. (25 pts)

3.00g of steam at 100.°C is added to 15.0g of ice initially at  $-25.0^{\circ}C$ . If the final mixture is all liquid water, what will the final equilibrium temperature be?  $[L_f = 3.33 \times 10^2 J/g; L_v = 22.6 \times 10^2 J/g; c_{steam} = 2.080 J/g \cdot C, c_{water} = 4.186 J/g \cdot C;$  $c_{ice} = 2.100 J/g \cdot C$ 

[Note: There are two phase change points – boiling at 100°C and freezing at 0°C.]

# SOLUTION:

Let the final temperature be  $T_f$ ,

$$\sum Q = 0$$
 gives



This gives

$$-m_{s}L_{v} + m_{s}c_{w}(T_{f} - 100^{\circ}C) + m_{i}c_{i}(0^{\circ}C - (-25.0^{\circ}C)) + m_{i}L_{f} + m_{i}c_{w}(T_{f} - 0^{\circ}C) = 0$$

Rearranging terms, we then can solve for 
$$T_f$$
,  
 $[m_s c_w + m_i c_w] T_f = m_s (L_v + c_w 100^\circ C) - m_i (c_i 25^\circ C + L_f)$   
 $T_f = \frac{m_s (L_v + c_w 100^\circ C) - m_i (c_i 25^\circ C + L_f)}{[m_s c_w + m_i c_w]}$   
 $= \frac{[3.0g (22.6 \times 10^2 J/g + (4.186J/g \cdot C)(100^\circ C))]}{-15.0g[(2.1J/g \cdot C)(25^\circ C) + 3.33 \times 10^2 J/g]]}$   
 $= \frac{29.9^\circ C}{2}$ 

#### 3.(25 pts)

A heat engine operating between a high temperature reservoir at  $T_H = 293.K$  and a low temperature reservoir at  $T_L = 195.K$  is shown at right. Starting from A, one mole of a monatomic ideal gas with volume  $V_1 = 10.0L$  is isothermally expanded at temperature  $T_H$  to B with volume  $V_2 = 20.0L$ . Then, the gas is isochorically cooled to a lower temperature  $T_L$  from B to C. At C, the gas is then isothermally compressed back to  $V_1$  from C to D. The



cycle is closed when the gas is isochorically heated back to  $T_H$  again from D to A.

- a) Calculate the net work done  $W_{net}$  by the ideal gas over one cycle.
- b) What is the total amount of heat absorbed  $Q_{abs}$  by the gas in one cycle?
- c) What is the efficiency *e* of this cycle?  $[1L = 10^{-3}m^3, 1 \text{ atm} = 1.013 \times 10^5 Pa]$

SOLUTION:

For the two isothermal processes, we have

$$W_{AB} = nRT_H \ln\left(\frac{V_2}{V_1}\right)$$
 and  $W_{CD} = nRT_L \ln\left(\frac{V_1}{V_2}\right)$ 

There is NO work done during the isochoric processes, so we have

$$W_{net} = W_{AB} + W_{CD} = nR(T_H - T_L)\ln\left(\frac{V_2}{V_1}\right) \text{ (since } \ln\left(\frac{V_1}{V_2}\right) = -\ln\left(\frac{V_2}{V_1}\right) \text{ )}$$
$$W_{net} = 1mol(8.314J / mol \cdot K)(293K - 195K)\ln\left(\frac{20.0L}{10.0L}\right) = 565J$$

Now, for the heat exchanges, all four branches will involve heat exchanges,

Isothermal: 
$$Q_{AB} = W_{AB} = nRT_H \ln\left(\frac{V_2}{V_1}\right)$$
 (expansion  $\rightarrow$  positive since  $V_2 > V_1$ )  
 $Q_{CD} = W_{CD} = nRT_L \ln\left(\frac{V_1}{V_2}\right)$  (compression  $\rightarrow$  negative since  $V_1 < V_2$ )

Isochoric:  $Q_{BC} = \Delta U_{BC} = nC_V (T_L - T_H)$  (Since TL < TH, this is negative)  $Q_{DA} = \Delta U_{DA} = nC_V (T_H - T_L)$  (This is positive.)

Now, for total heat absorbed in one cycle, we sum up only the "+" heats, we have

$$Q_{abs} = Q_{AB} + Q_{DA} = nRT_H \ln\left(\frac{V_2}{V_1}\right) + nC_V \left(T_H - T_L\right) = 2910J$$

Then, the efficiency of the heat engine is given as

$$e = \frac{W_{net}}{Q_{abs}} = \frac{nR(T_H - T_L)\ln\left(\frac{V_2}{V_1}\right)}{nRT_H \ln\left(\frac{V_2}{V_1}\right) + nC_V (T_H - T_L)} = \frac{564.757J}{2910.67J} = 0.19403 = 19.4\%$$

# 4. (25 pts)

The volume of one mole of a monoatomic ideal gas is being expanding from V to 5V. a) Calculate the entropy change if the process is an isothermal expansion.

b) Calculate the change in entropy if the process is a *reversible* adiabatic expansion?

c) Calculate the change in entropy if the process is a *non-reversible* adiabatic free expansion?

### SOLUTION:

a) For an isothermal process, we have by definition,

$$dU = dQ - dW = 0$$
$$dQ = dW = pdV = nRT \frac{dV}{V}$$

Then for a reversible process, we have dS = dQ/T and,

$$\Delta S = \int_{i}^{f} \frac{dQ}{T} = \int_{i}^{f} \frac{nRT}{TV} dV = nR \ln(V_{f} / V_{i})$$

 $\Delta S = (1.00 \, mol)(8.314 \, J \, / \, mol \cdot K) \ln 5 = 13.4 \, J \, / \, K$ 

b) For any reversible processes, we have dS = dQ/T. Since dQ = 0 for an adiabatic process, we will have no entropy change for this process, i.e.,

$$\Delta S = 0$$

c) Entropy is a state variable so that the change in entropy between two states is path independent. Although we can't use dS = dQ/T to calculate the entropy change for the adiabatic free expansion directly, we can use a surrogate reversible process that links the same initial and final states. In an adiabatic free expansion, temperature of the gas remains the same so that a good surrogate reversible process is an isothermal process that expands the gas from (T, V) to (T, 5V). We have calculated this in part a) already and it is given by,

 $\Delta S = 13.4 J / K .$