

1. (25 pts)

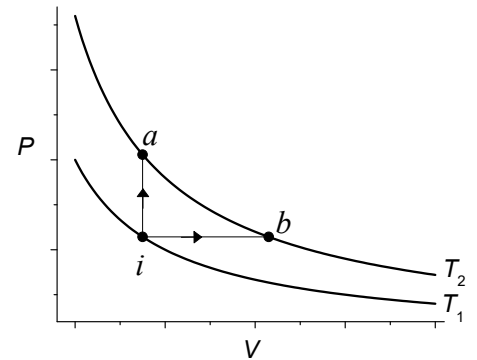
Answer the following questions.

- a) (3 pts) A Carnot engine operates between a high temperature reservoir at T_H and a low temperature reservoir at T_L . One can increase the efficiency of the engine by either increasing T_H by ΔT or decreasing T_L by ΔT . Which change ($T_H + \Delta T$) or ($T_L - \Delta T$) will result in a larger efficiency for the engine?

b) (4 pts)

- i) To raise the temperature of an object, must you add heat to it?
- ii) If you heat an object, must you raise its temperature?

- c) (6 pts) An ideal gas is being taken from an initial temperature T_1 at state i to a final temperature T_2 in two separate processes: (a) isochoric and (b) isobaric, as shown in the diagram to the right.



- i) Will the entropy of the system decrease or increase in process $i \rightarrow a$?
- ii) Will the entropy of the system decrease or increase in $i \rightarrow b$?
- iii) Which process will result in a larger entropy change?

- d) (12 pts) An ideal gas is being put through the following four thermodynamic processes, please indicate whether its internal energy U will **increase** ($\Delta U > 0$), **decrease** ($\Delta U < 0$), or **stay the same** ($\Delta U = 0$):

- i) A reversible isochoric process with heat being absorbed by the gas
(**increase, decrease, stay the same**)
- ii) A reversible isobaric expansion
(**increase, decrease, stay the same**)
- iii) A reversible isothermal expansion
(**increase, decrease, stay the same**)
- iv) A reversal adiabatic expansion
(**increase, decrease, stay the same**)

SOLUTION:

- a) For a Carnot engine, the efficiency is given by $e = 1 - \frac{T_L}{T_H}$ so that both decreasing T_L and/or increasing T_H will give a smaller T_L/T_H and makes e closer to 1. As

for a fixed change in temperature, ΔT , the ratio $\frac{T_L - \Delta T}{T_H}$ will generally be smaller than $\frac{T_L}{T_H + \Delta T}$ so that $1 - \frac{T_L - \Delta T}{T_H} > 1 - \frac{T_L}{T_H + \Delta T}$ and

$$\boxed{e(T_L \rightarrow T_L - \Delta T) > e(T_H \rightarrow T_H + \Delta T)}.$$

To see that, we can consider the following ratio:

$$\frac{(T_L - \Delta T)/T_H}{T_L/(T_H + \Delta T)} = \frac{T_L T_H - (T_H - T_L)\Delta T - \Delta T^2}{T_L T_H} = 1 - \frac{T_H - T_L}{T_L T_H} \Delta T - \frac{1}{T_L T_H} \Delta T^2 < 1$$

Both blue terms are positive so that this ratio will always be less than 1 indicating that,

$$\frac{T_L - \Delta T}{T_H} \text{ will always be smaller than } \frac{T_L}{T_H + \Delta T}.$$

- b) i) **No**. One can perform work on the system. ii) **No**. Examples: isothermal processes or during phase changes.
- c) i) and ii) In both processes (isobaric and isochoric), heat enters the system as the temperature increases. Since $dS = dQ/T$, the change in entropy for both processes will also be positive (an **increase**).
- iii) For this problem, the change in temperature for both processes is the same. Since $C_p > C_v$, we have $dQ_p > dQ_v$ and $\Delta S_p > \Delta S_v$.

$$\left[\text{note: } \Delta S_{p,v} = \int_{T_i}^{T_f} \frac{dQ_{p,v}}{T} = nC_{p,v} \int_{T_i}^{T_f} \frac{dT}{T} = nC_{p,v} \ln\left(\frac{T_f}{T_i}\right) \right]$$

Thus, the isobaric process will result in a larger entropy increase.

- d)
- Increase**. Since there is no volume change, work done for an isochoric process is zero. Then from 1st law, $\Delta U = Q$. With heat being absorbed by the ideal gas, i.e., $Q > 0$, $\Delta U > 0$.
 - Increase**. For an isobaric process, we have P being a constant. Then from the ideal gas law, we have ΔT proportional to ΔV . For an expansion, $\Delta V > 0$ so that $\Delta T > 0$. Since internal energy for an ideal gas is directly proportion to its temperature, $\Delta T > 0$ implies that $\Delta U > 0$.
 - Stay the same**. By definition, temperature stay the same for an isothermal process and since the internal energy of an ideal gas is a function of temperature only, $\Delta U = 0$.

- d. **Decrease.** In a reversible adiabatic expansion, we have $Q = 0$ (adiabatic) and $W > 0$ (work done by gas), so by 1st law $\Delta U = Q - W$, $\Delta U < 0$.

2. (25 pts)

A piece of hot iron with unknown mass initially at $550.^{\circ}\text{C}$ was placed in 1.60g of water at 22.0°C . After the system has come to equilibrium, all water was vaporized and the final temperature for the steam and hot iron was measured to be at 115°C ? What was the mass of the hot iron?

$$[L_v = 22.6 \times 10^2 \text{ J/g}; c_{\text{steam}} = 2.080 \text{ J/g} \cdot ^{\circ}\text{C}, c_{\text{water}} = 4.186 \text{ J/g} \cdot ^{\circ}\text{C}, c_{\text{iron}} = 0.450 \text{ J/g} \cdot ^{\circ}\text{C}]$$

SOLUTION:

Let the mass of the iron be m_i .

$$\sum Q = 0 \text{ gives}$$

$$\left(\begin{array}{l} \text{heat released,} \\ \text{hot iron} \\ \text{cooling} \\ 550^{\circ}\text{C} \rightarrow 115^{\circ}\text{C} \end{array} \right) + \left(\begin{array}{l} \text{heat absorbed,} \\ \text{water warming} \\ 22^{\circ}\text{C} \rightarrow 100^{\circ}\text{C} \end{array} \right) + \left(\begin{array}{l} \text{latent heat absorbed} \\ \text{to vaporize at } 100^{\circ}\text{C} \end{array} \right) + \left(\begin{array}{l} \text{heat absorbed,} \\ \text{steam warming} \\ 100^{\circ}\text{C} \rightarrow 115^{\circ}\text{C} \end{array} \right) = 0$$

This gives

$$m_i c_i (115^{\circ}\text{C} - 550^{\circ}\text{C}) + m_w c_w (100^{\circ}\text{C} - 22^{\circ}\text{C}) + m_w L_v + m_w c_s (115^{\circ}\text{C} - 100^{\circ}\text{C}) = 0$$

Rearranging terms, we then can solve for m_i ,

$$m_i c_i (550^{\circ}\text{C} - 115^{\circ}\text{C}) = m_w c_w (100^{\circ}\text{C} - 22^{\circ}\text{C}) + m_w L_v + m_w c_s (115^{\circ}\text{C} - 100^{\circ}\text{C})$$

$$m_i = \frac{m_w [c_w (78^{\circ}\text{C}) + L_v + c_s (15^{\circ}\text{C})]}{c_i (435^{\circ}\text{C})}$$

$$m_i = \frac{(1.60\text{g}) [(4.186 \text{ J/g} \cdot ^{\circ}\text{C})(78.0^{\circ}\text{C}) + (22.6 \times 10^2 \text{ J/g}) + (2.08 \text{ J/g} \cdot ^{\circ}\text{C})(15.0^{\circ}\text{C})]}{(0.450 \text{ J/g} \cdot ^{\circ}\text{C})(435^{\circ}\text{C})}$$

$$m_i = 21.4\text{g}$$

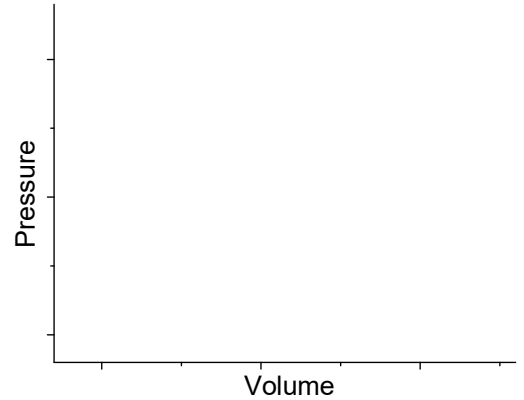
3.(25 pts)

Five moles of a monatomic ideal gas starting in an initial state **A** ($P_A = 1.80 \times 10^6 \text{ Pa}$, $T_A = 325 \text{ K}$) are taken through the following three-step thermodynamic cycle:

Step 1: A reversible isothermal process to state **B**
 ($P_B = 1.80 \times 10^5 \text{ Pa}$, $T_B = 325 \text{ K}$)

Step 2: A reversible isobaric process to state **C** with the same volume as state **A**

Step 3: A reversible isochoric process back to state **A**, i.e. $V_C = V_A$



$$[1L = 10^{-3} m^3, 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}]$$

- Calculate the volume of the idea gas at state **A** and **B**.
- Schematically draw the thermodynamic cycle (states as dots and processes as connecting curves) in the PV graph above. Qualitatively is ok. Just make sure the direction of the process is correct.
- Calculate the net work done on or by the system through one cycle? Is the work done on the system or by the system? Is the cycle a heat engine or a refrigerator?
- Calculate the heat exchange by the sytem along the reversible isochoric process (Step 3)? Is the heat absorbed by the system or released by system during Step 3?

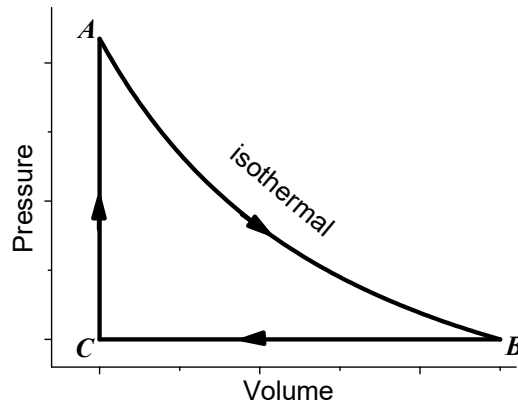
SOLUTION:

a) Using the Ideal Gas Law,

$$V_A = \frac{nRT_A}{P_A} = \frac{(5 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(325 \text{ K})}{1.80 \times 10^6 \text{ Pa}} = 7.506 \times 10^{-3} m^3 = 7.51 \text{ L}$$

$$\text{And } V_B = \frac{V_A}{0.1} = 75.1 \text{ L}$$

b)



c) Step 1 is an isothermal expansion and the work done is give by

$$W_{AB} = nRT_A \ln\left(\frac{V_B}{V_A}\right) = (5\text{mol})(8.314\text{J} / \text{mol} \cdot \text{K})(325\text{K}) \ln\left(\frac{75.1\text{L}}{7.51\text{L}}\right) = 31.109\text{kJ}$$

Step 2 is an isobaric compression and the work done is,

$$W_{BC} = P_B (V_C - V_B) = (1.80 \times 10^5 \text{ Pa})(7.51 \times 10^{-3} \text{ m}^3 - 7.51 \times 10^{-2} \text{ m}^3) = -12.166\text{kJ}$$

Step 3 is an isochoric process so NO work is done, i.e., $W_{CA} = 0\text{J}$

$$W_{net} = W_{AB} + W_{BC} + W_{CA} = 31.109\text{kJ} - 12.168\text{kJ} + 0\text{J} = +18.9\text{kJ}$$

$W_{net} > 0$ so, work is DONE BY the system. It is an heat engine.

d) To calculate Q for Step 3, we need to calculate the temperature at Step C first. Since state C and A have the same volume, we can use the Ideal Gas Law to calculate

$$\frac{P_C}{T_C} = \frac{P_A}{T_A}$$

$$T_C = \frac{P_C}{P_A} T_A = \frac{1}{10} 325\text{K} = 32.5\text{K}$$

For an isochoric process, heat exchange is given by

$$Q_{CA} = nC_V (T_A - T_C) = (5\text{mol}) \frac{3}{2} (8.314\text{J} / \text{mol} \cdot \text{K})(325\text{K} - 32.5\text{K}) = +18.2\text{kJ}$$

Since $Q_{CA} > 0$, heat is being ABSORBED by the gas in Step 3.

4. (25 pts)

The same amount of work $W = 1.50kJ$ is being done on one mole of a monatomic gas initially at $(V = 25.0L, T = 293K)$ through two different reversible compression processes: **isothermal compression** and **adiabatic compression**.

- Calculate the final temperature of the gas for the two compression processes.
- Calculate the final volume of the gas for the two compression processes?
- Calculate the entropy change resulted from the two compression processes?

SOLUTION:

- a) Temperature of the final state:

1. Isothermal

By definition, temperature of the gas will stay same in an isothermal process so $T_{f, isothermal} = T_i = 293K$

2. Adiabatic

By definition, $Q = 0$ for an adiabatic process, so from 1st Law, we have $\Delta U_{adiabatic} = -W$

For any process, we can use $\Delta U = nC_V\Delta T$ so, we have,

$$\Delta U_{adiabatic} = nC_V(T_{f, adiabatic} - T_i) = -W$$

$$T_{f, adiabatic} = \frac{-W}{nC_V} + T_i = \frac{-(-1.50 \times 10^3 J)}{(1mol)3(8.314J / mol \cdot K)/2} + 293K = 413.28K = 413K$$

NOTE: work is done ON the gas so $W < 0 = -1.50 \times 10^3 J$

- b) Volume of the final state:

1. Isothermal

Work for an isothermal process is given by $W = nRT \ln\left(\frac{V_{f, isothermal}}{V_i}\right)$.

Solving for $V_{f, isothermal}$, we have (work is done ON the gas so $W < 0$),

$$\begin{aligned} V_{f, isothermal} &= V_i \exp\left(\frac{W}{nRT}\right) = (25.0L) \exp\left(\frac{-1.5 \times 10^3 J}{1mol(8.314J / mol \cdot K)(293K)}\right) \\ &= 13.5057L = 13.5L \end{aligned}$$

2. Adiabatic

For an adiabatic process, we can use $T_i V_i^{\gamma-1} = \text{const}$ so, we have,

$$T_{f,adiabatic} V_{f,adiabatic}^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$\left(\frac{V_{f,adiabatic}}{V_i} \right)^{\gamma-1} = \left(\frac{T_i}{T_{f,adiabatic}} \right)$$

$$V_{f,adiabatic} = \left(\frac{T_i}{T_{f,adiabatic}} \right)^{\frac{1}{\gamma-1}} V_i = \left(\frac{293K}{413.28K} \right)^{\frac{3}{2}} (25.0L) = 14.923L = 14.9L$$

$$\text{Note: } \gamma = \frac{C_p}{C_v} = \frac{5/2}{3/2} = \frac{5}{3} \text{ and } \gamma-1 = \frac{2}{3}.$$

- c) In general, we can use the following formula to calculate the entropy change for any reversible processes for an Ideal Gas,

$$\Delta S = \int_i^f \frac{dQ}{T} = nC_v \ln \left(\frac{T_f}{T_i} \right) + nR \ln \left(\frac{V_f}{V_i} \right)$$

For an isothermal process, $T_f = T_i$ so the first term is zero and we have,

$$\Delta S = 1 \text{ mol} (8.314 \text{ J} / \text{mol} \cdot \text{K}) \ln \left(\frac{V_{f,isothermal}}{V_i} \right) = (8.314 \text{ J} / \text{K}) \ln \left(\frac{13.5057L}{25L} \right)$$

$$= -5.12 \text{ J} / \text{K}$$

For a reversible adiabatic process, $dQ = 0$ so $\Delta S = 0 \text{ J} / \text{K}$

One can also explicitly calculate it using the general formula,

$$\Delta S = nC_v \ln \left(\frac{T_f}{T_i} \right) + nR \ln \left(\frac{V_f}{V_i} \right)$$

$$= \left(\frac{3}{2} \right) 8.314 \text{ J} / \text{K} \ln \left(\frac{T_{f,adiabatic}}{T_i} \right) + 8.314 \text{ J} / \text{K} \ln \left(\frac{V_{f,adiabatic}}{V_i} \right)$$

$$= 8.314 \text{ J} / \text{K} \ln \left(\frac{T_{f,adiabatic}}{T_i} \right)^{\frac{3}{2}} + 8.314 \text{ J} / \text{K} \ln \left(\frac{V_{f,adiabatic}}{V_i} \right)$$

From part b, we have gotten the following relation in our calculation,

$$\frac{V_{f,adiabatic}}{V_i} = \left(\frac{T_i}{T_{f,adiabatic}} \right)^{\frac{3}{2}}$$

$$\text{So, } \Delta S = 8.314 J / K \ln \left(\frac{V_i}{V_{f,adiabatic}} \right) + 8.314 J / K \ln \left(\frac{V_{f,adiabatic}}{V_i} \right) = 0$$