1. **(25 pts)**
Answer the following questions.

a) (**8 pts**) An ideal gas is being expanded isothermally and then compressed through an isobaric process back to its original volume. Both expansion and compression are done reversibly. After the two-stage process,

i) Will $T_f < T_i, \quad T_f = T_i, \quad or \quad T_f > T_i$?
ii) Will $P_f < P_i, \quad P_f = P_i, \quad or \quad P_f > P_i$?
iii) Will $W_{net} < 0, \quad W_{net} = 0, \quad or \quad W_{net} > 0$?
iv) Will $\Delta U < 0, \quad \Delta U = 0, \quad or \quad \Delta U > 0$?

(Hint: Visualize the processes in a *P*-*V* diagram might help.)

b) (**4 pts**) You have two ideal gases with the same number of moles in your laboratory. You know that one of them is monatomic and the other is diatomic but you don't know which one is which. You are asked to distinguish them by heating the gases up and to observe the temperature changes. For the same amount of heat absorbed by the two gases, you observed that gas #1 results in a higher temperature rise than gas #2. Which one is the diatomic ideal gas (gas #1 or gas #2)?

c) (**4 pts**) An online advertisement claims that a heat engine operating between a high temperature reservoir at $120^\circ C$ and a low temperature reservoir at $30^\circ C$ can achieve a thermal efficiency of 50%. What is the theoretical maximum efficiency according to the Carnot Theorem for an heat engine working within this two temperature range? Is the claim in the advertisement theoretically possible?

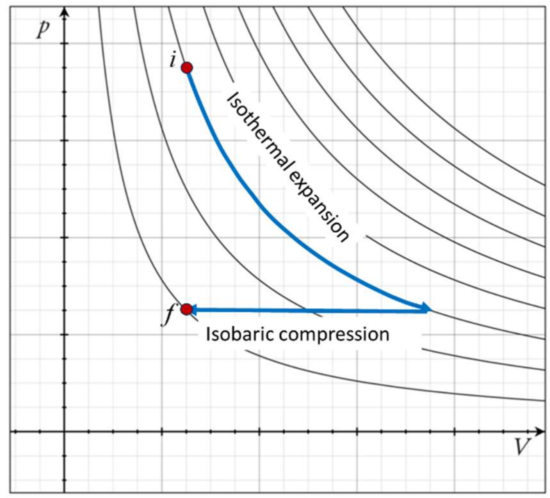
d) (**9 pts**) One mole of an ideal gas at an initial state *i* with (P_i, V_i, T_i) is allowed to double its volume to $2V_i$ through the following processes:

- Reversible isothermal expansion (**process a**)
- Reversible isobaric expansion (**process b**)
- Reversible adiabatic expansion (**process c**)

i) Which process (a, b, c) will result in the smallest entropy change ΔS?
ii) What is the value of the entropy change ΔS in part i)?
iii) Which process (a, b, c) will result in the largest entropy change ΔS?

SOLUTION:

a) Although the gas is compressed and expanded back to the same volume, it does NOT mean that the gas returns to the same initial state with the same *P* and *T*. These two-stage process is shown on a P-V graph to the right.



i) Temperature remains the same in an isothermal process and an isobaric compression will lower the gas' temperature. **So the final temperature after the two-stage process will be lower, $T_f < T_i$.**

ii) Pressure remains the same in an isobaric process and pressure will decreases in an isothermal expansion. **So the final pressure after the two-stage process will be lower, $P_f < P_i$.**

iii) Work done by the gas in an isothermal expansion is positive and the work done on the gas in an isobaric compression is negative. Since work done is the area under the graph, from the P-V graph, one can see that the work done by the isothermal expansion is larger than the work done on the gas during the isobaric compression. By the convention, work done by the gas is positive (isothermal expansion) and work done on the gas is negative (isobaric compression), **So, the net work done after the two-stage process will be positive,** $W_{net} > 0$.

iv) Since internal energy is a function of temperature only for an Ideal Gas, the change in internal energy during an isothermal process is zero, $\Delta U_{isothermal} = 0$. Since temperature decreases in the isobaric compression, so $\Delta U_{isobaric\ compression} < 0$. **So the net change in internal energy after the two-stage process is negative,** $\Delta U < 0$

b) According to Kinetic Theory of Gases, a diatomic ideal gas will have a larger molar heat capacity at constant volume so that for the same amount of heat absorbed and with the same *n*, it will result in a *lower* temperature raise $\left(dQ = nC_V dT\right)$. Thus, **gas #2 is the diatomic gas**.

c) The theoretical maximum efficiency of a heat engine is given by the efficiency of a Carnot Cycle, $e_c = 1 - \dfrac{T_C}{T_H}$. So for $T_H = 393.15K$ and $T_C = 303.15K$ (temp must be in K), the thermal efficiency for any heat engines cannot be larger than the

Carnot Cycle's efficiency, $e_c = 1 - \frac{303.15K}{393.15K} = 22.9\%$. **So, the claim of 50% thermal efficiency cannot be possible**.

d) For any ideal gas, the entropy for any reversible process is given by $\Delta S = nC_V \ln\left(\frac{T_f}{T_i}\right) + nR\ln\left(\frac{V_f}{V_i}\right)$.

Note that with $V_f = 2V_i$, the second term in the above equation will be $nR\ln 2 > 0$
Using the above equation with $n = 1$, we can explicitly calculate ΔS :

Process a (reversible isothermal expansion): $\Delta S_a = R\ln 2 > 0$

Process b (reversible isobaric expansion): $\Delta S_b = C_V \ln\left(\frac{T_f}{T_i}\right) + R\ln 2$. For an isobaric expansion $T_f > T_i$ so, while we don't have the numeric value for $C_V \ln\left(\frac{T_f}{T_i}\right)$ but it will be positive and most importantly , $\Delta S_b > \Delta S_a$.

Process c (reversible adiabatic expansion): $\Delta S_c = 0$ since by definition $dS = \frac{dQ}{T}$ for any reversible process and $dQ = 0$ for an adiabatic process so ΔS will also be zero for a *reversible* adiabatic process.

Putting all the above calculations together, we have

i) **Process c (reversible adiabatic expansion) will result in the smallest entropy change $\Delta S_c = 0$.**
ii) $\Delta S_c = 0$
iii) **Process b (reversible isobaric expansion) will result in the largest entropy change $\Delta S_b = C_V \ln\left(\frac{T_f}{T_i}\right) + R\ln 2$**

2.(**25 pts**)

In making a matcha latte, 2.50g of hot steam at a temperature of $125.^{\circ}C$ is delivered into 325g of milk initially at a temperature of $20.0^{\circ}C$. After the system comes to a thermal equilibrium, the combined liquid (milk and cooled-and-condensed steam) has an equilibrium temperature of $65.0^{\circ}C$. Assuming that heat is only exchanged between the hot steam and the milk and the milk does not go through a phase change, calculate the milk's specific heat c_{milk}. (Your answer needs to have at least 3 significant figures.)

[$L_v(water) = 2.256 \times 10^4\ J/g$; $c_{steam} = 1.996 J/g \cdot K$; $c_{water} = 4.186 J/g \cdot K$]

SOLUTION:

While the milk does not go through a phase change, the hot steam will condensed into liquid water at its boiling point at $100.^{\circ}C$. Writing out all the individual heat exchange processes,

$$\sum |Q_{gain}| = \sum |Q_{loss}| \quad \text{gives}$$

$$\begin{vmatrix}\text{heat absorbed,}\\ \text{milk warming}\\ 20^{\circ}C \to 65^{\circ}C\end{vmatrix} = \begin{vmatrix}\text{heat released,}\\ \text{steam}\\ \text{cooling}\\ 125^{\circ}C \to 100^{\circ}C\end{vmatrix} + \begin{vmatrix}\text{heat released,}\\ \text{steam}\\ \text{condensing}\\ at\ 100^{\circ}C\end{vmatrix} + \begin{vmatrix}\text{heat released,}\\ \text{condensed}\\ \text{steam (water)}\\ \text{cooling}\\ 100^{\circ}C \to 65^{\circ}C\end{vmatrix}$$

Writing them into equation form gives

$$m_{milk}c_{milk}\left(\left(65.0-20.0\right)K\right) = m_{steam}c_{steam}\left(\left(125-100\right)K\right) + m_{steam}L_v + m_{steam}c_{water}\left(\left(100-65\right)K\right)$$

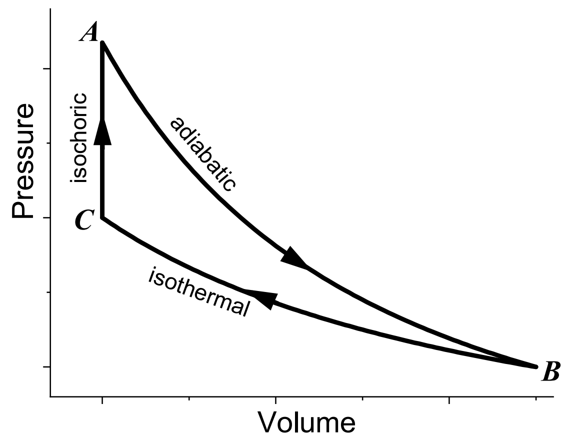
Rearranging terms, we then can solve for c_{milk},

$$c_{milk} = \frac{m_{steam}\left(c_{steam}\left(25K\right) + L_v + c_{water}\left(35K\right)\right)}{m_{milk}\left(45K\right)}$$

$$c_{milk} = \frac{\left(2.50g\right)\left[\left(1.996 J/g \cdot K\right)(25.0K) + \left(2.256 \times 10^4\ J/g\right) + \left(4.186 J/g \cdot K\right)\left(35.0^{\circ}C\right)\right]}{\left(325g\right)\left(45.0K\right)}$$

$$c_{milk} = 3.89 J/g \cdot K$$

3.**(25 pts)**
A heat engine with one mole of a monatomic ideal gas is operating in a cycle as shown to the right.
$\left[1L = 10^{-3}m^3; 1\,atm = 1.103\times10^5\,Pa\right]$



The gas starting from an initial state **A**: $(T_A = 270.K, V_A = 15.0L)$ is first being expanded to an intermediate state **B** with twice its original volume $V_B = 30.0L$ through a reversible adiabatic process.

a) What is the temperature T_B of the gas after the expansion at state **B**?
b) Calculate the work done by the gas W_{AB} and heat exchange Q_{AB} as the gas adiabatically expands from **A** to **B**.

From **B**, the ideal gas is then isothermally and reversibly compressed back to its original volume at state **C**, i.e., $V_C = V_A$,.

c) What is the temperate T_C of the gas at state **C**?
d) Calculate the work done by the gas W_{BC} and heat exchange Q_{BC} as the gas being isothermally compressed from **B** to **C**.

From **C**, the ideal gas is then being brought back to its initial state **A** through a reversible isochoric process.

e) Calculate the work done by the gas W_{CA} and heat exchange Q_{CA} for this isochoric process.
f) Calculate the efficiency of this heat engine.

SOLUTION:

a) Since $A \rightarrow B$ is a reversible **adiabatic** process, we can use the following relation,

$$T_B = T_A\left(\frac{V_A}{V_B}\right)^{\gamma-1} = 270.K\left(\frac{15}{30}\right)^{\frac{2}{3}} = 170.K$$

b) The work done by the gas for the adiabatic expansion is

$$W_{AB} = -\Delta U_{AB} = -nC_v\left(T_B - T_A\right) = -(1\,mole)\left(\frac{3R}{2}\right)\left(170.K - 270.K\right) = 1.25kJ$$

By definition, we also have $Q_{ab} = 0$ for an adiabatic process.

c) By definition, temperature in an isothermal process stays the same so that we have $T_C = T_B = 170.K$.

d) Work done in an isothermal process is given by

$$W_{BC} = nRT_B \ln\left(\frac{V_C}{V_B}\right) = (1mole)R\left(170.K\right)\ln\left(\frac{15}{30}\right) = -980.J$$

For an isothermal process, $\Delta U = 0$, so by the 1st law of thermodynamics,
$Q_{BC} = W_{BC} = -980.J$ (*heat released*)

e) For the last branch of the cycle, work done by the ideal gas in an isochoric process is zero, i.e., $W_{CA} = 0J$ since $dV = 0$.

Since this is an isochoric process, we can calculate the heat absorbed by the gas Q_{CA} directly in terms of C_v,

$$Q_{CA} = nC_v\left(T_A - T_C\right) = (1mole)\left(\frac{3R}{2}\right)\left(270.K - 170.K\right) = +1.25kJ \quad (heat\ absorbed)$$

f) Lastly, the efficiency of an heat engine is given by $e = \frac{W_{net}}{Q_H}$, where

$$W_{net} = W_{ab} + W_{bc} + W_{ca} = 1250J - 980.J + 0J = 270.J$$

and $Q_H = Q_{CA} = 1250J \quad (heat\ absorbed\ only)$

So, we have

$$e = \frac{W_{net}}{Q_H} = \frac{270.J}{1250J} = 21.6\%$$

If this is a Carnot cycle, the efficiency is $e_{carnot} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{170.K}{270.K} = 37.0\%$. As it should be, the efficiency of this non-Carnot cycle is less than the theoretical maximum given by the Carnot cycle.

4.**(25 pts)**
A cup with 80.0*g* of water is allowed to cool from 90.0 ºC to room temperature at 20.0 ºC on your kitchen tabletop. Assume that the mass of the cup is negligible and take the specific heat for water to be $c_{water} = 4.186\, J/g \cdot K$.

a) Calculate the heat released by the water Q_{water} into the room as it cools to room temperature.
b) Calculate the entropy change ΔS_{water} of the water as the water cools to room temperature.
c) Assume that all heat released by the water is absorbed by the air in the room and the room is large so that the temperature of the air in the room changes negligibly. Taking the room air's temperature to be at a constant 20.0 ºC, calculate the entropy change ΔS_{room} for the room air.
d) Considering the water together with the surrounding room air as one isolated system, what is the total entropy change for the combined system: water + room air?

[Please play attention to the sign of all Q and ΔS.]

SOLUTION:

a) The heat released by the water as it cools is given by,

$$Q_{water} = mc_{water}\left(T_f - T_i\right)$$
$$Q_{water} = (80.0g)(4.186\, J/g \cdot K)(-70K) = -2.3442 \times 10^4 J = -2.34 \times 10^4 J \quad \text{(heat released)}$$

b) Working from definition, we have $dS = \dfrac{dQ}{T}$. With the water cooling, we have,

$$\Delta S_{water} = \int_{T_i}^{T_f} \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc_{water}dT}{T} = mc_{water} \ln\left(\frac{T_f}{T_i}\right)$$
$$= (80.0g)(4.186\, J/g \cdot K)\ln\left(\frac{293.15K}{363.15K}\right)$$
$$= (80.0g)(4.186\, J/g \cdot K)(-0.21413) = -71.7\, J/K$$

Note the negative sign for ΔS_{gas} (entropy decreases) since the gas is cooling.

c) Assuming that there is a large amount of air in the room so that the heat that it absorbed will not change the room temperature by any measurable amount. Then, the entropy change for the room air can be calculated through an isothermal process,

$$\Delta S_{room} = \frac{Q_{room}}{T_{room}}$$

Since $Q_{room} = +\left|Q_{gas}\right| = +2.3442\times10^{4} J$ (with explicit sign) and $T_{room} = 293.15K$, we have,

$$\Delta S_{room} = \frac{23442J}{293.15K} = +79.966\, J/K = +80.0\, J/K$$

The entropy change for the room air ΔS_{room} is positive (entropy increases) as the room air absorbs the heat.

d) The total entropy for the combined system with the water and the surrounding air in the room is then,

$$\Delta S_{tot} = \Delta S_{water} + \Delta S_{room} = -71.708\, J/K + 79.966\, J/K = +8.26\, J/K$$

$\Delta S_{tot} > 0$ as it should be according to the 2nd Law of Thermodynamics.