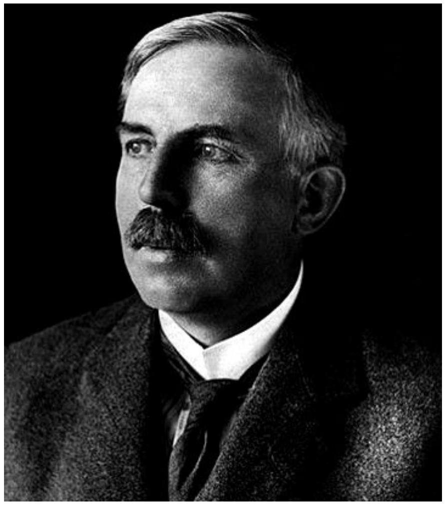
The Nuclear Atom (a bit of history)

What were known by 1910:

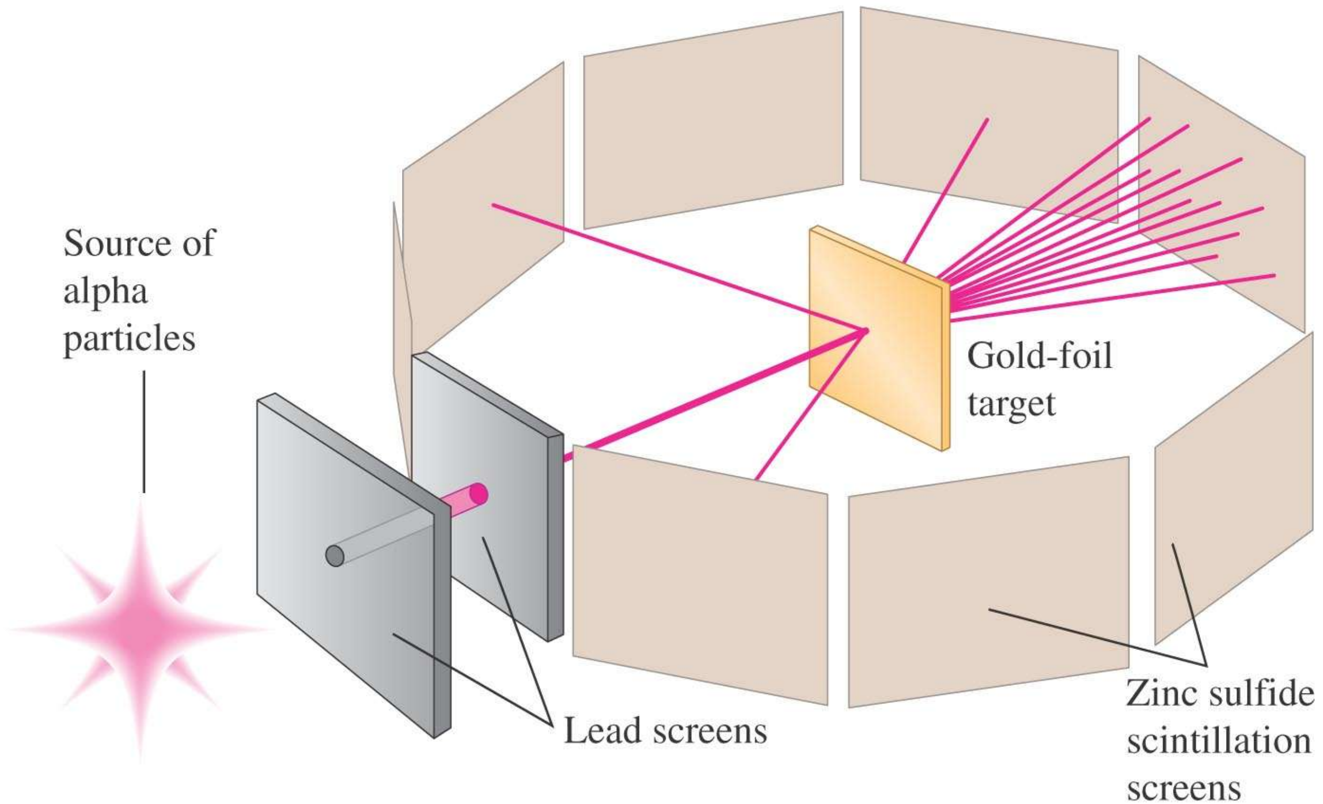
- 1897: J.J. Thomson discovered *e* and measured *e/m* ratio.
- 1909: Millikan completed the measurement of the electron charge -*e*.
- Size of atom is on the order of 10^{-10} m.
- Almost all mass of an atom is associated with the + charge.

But, the mass and charge distributions (+, -) inside the atom were not known. The leading assumption was by J.J. Thomson and an atom was modeled as a sphere with positive charge and the electrons were thought to be embedded within it like raisins in a muffin.

Ernest Rutherford designed the first experiment to probe the interior structure of an atom in 1910-1911. (together with Hans Geiger and Ernest Marsden)



Rutherford Scattering Experiments



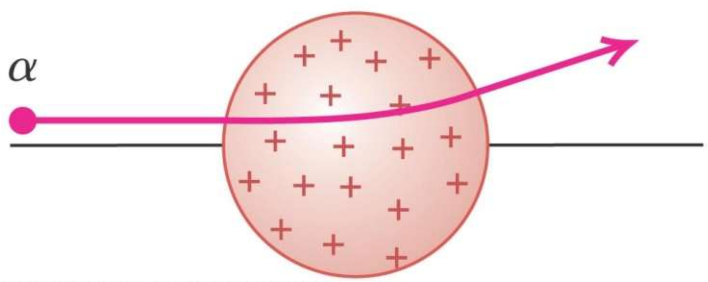
Alpha particles: high energy charged helium nuclei. It can travel several centimeter through air and ~0.1mm through solid matter.

Target: Thin gold, sliver, or copper foils.

The alpha particle is about 7300 times heavier than an electron. So, by momentum considerations alone, it will only minimally interact with the much *lighter* electron. Only the *positive* charge within the atom associated with the majority of its mass can produce significant deflections in scattering the alpha particles.

Rutherford Scattering Experiments

Thomson's model of the atom: An alpha particle is scattered through only a small angle.

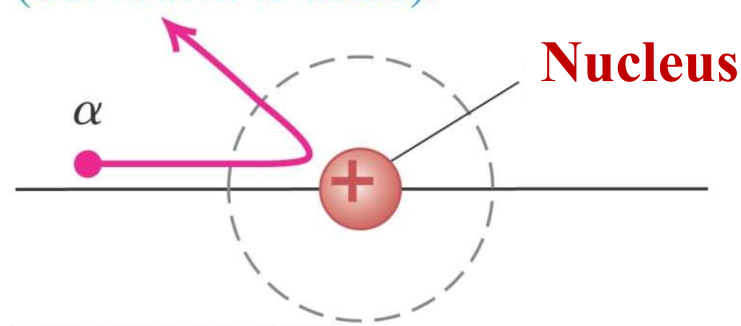


Positive charge inside atom is distributed uniformly throughout the volume so that electric field inside the atom is expected to be diffused (small) and the force that it exerts on the α will also be small → deflection angles are expected to be small.

BUT, there were large scattering angles (~ 180°, almost backward) observed !

Indicating that Thomson's model was not correct.

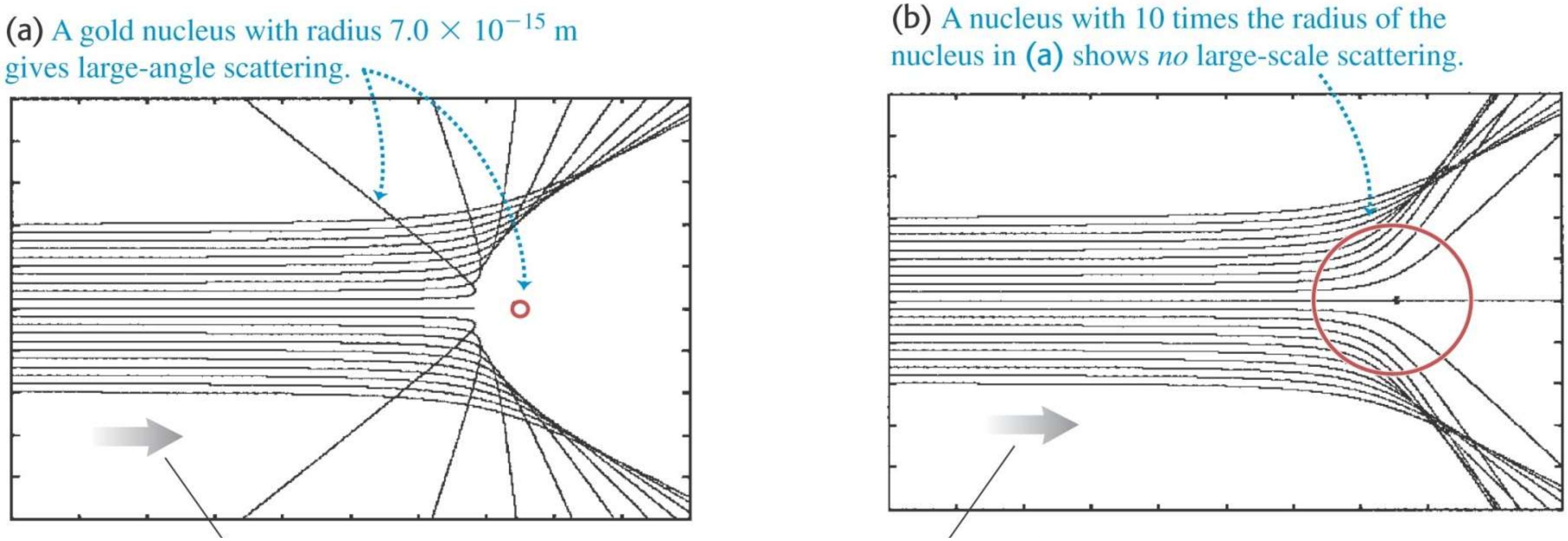
Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).



Rutherford Scattering Experiments

These large angle deflections can only be possible if the *positive* charge within an atom is concentrated in a *small* and *dense* space called the **nucleus**.

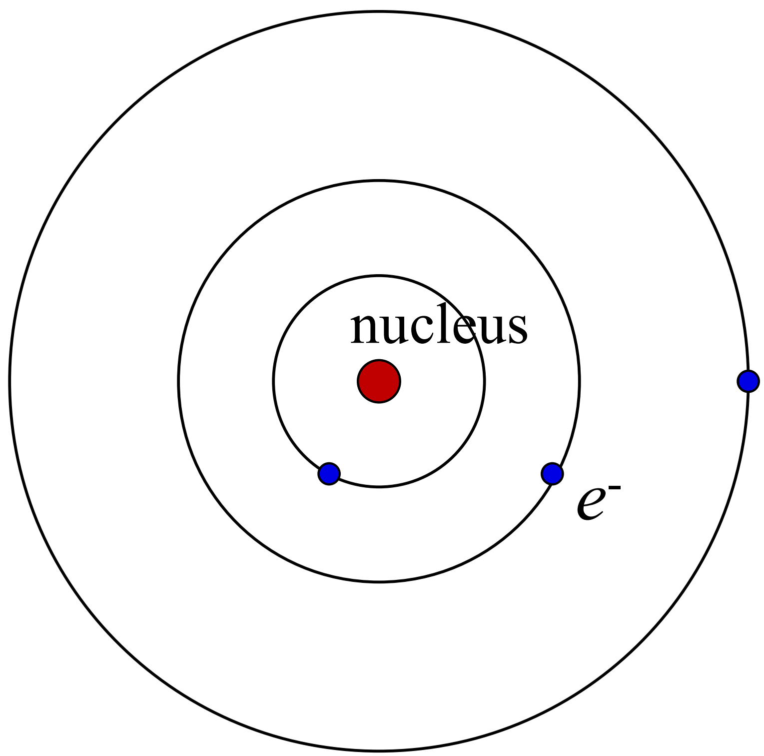
The following is a computer simulation of the deflection angles with two different nucleus size (done much later). The experimental results agreed with simulation data using small (~10^{-15} m) nucleus sizes.



The nucleus occupies only about 10^{-12} of the total volume but it contains 99.95% of the total mass.

Rutherford's Atomic Model

Following his discovery of the atomic **nucleus**, Rutherford also suggested that the negatively charged electron might revolve around the positively charged nucleus similar to a planet orbiting the sun in classical physics.



Failure of Classical Physics

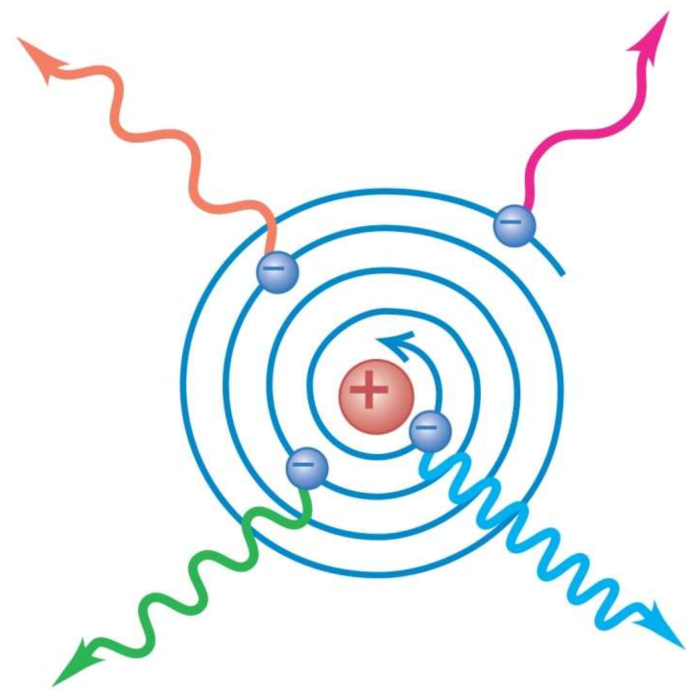
Following his discovery of the atomic **nucleus**, Rutherford also suggested that the negatively charged electron might revolve around the positively charged nucleus similar to a planet orbiting the sun in classical physics.

However, there are conceptual problems with the classical circular orbits idea:

→According to classical EM, an *accelerating* electron in circular orbit around the nucleus will *radiate* EM radiations.

→Thus, its energy will *decrease* continuously and spiral rapidly toward the nucleus.

→The radiated frequency will also depend on the frequency of revolution and since its orbit is expected to decay continuously, the radiated spectrum will also be *continuous* with a mixture of frequencies.



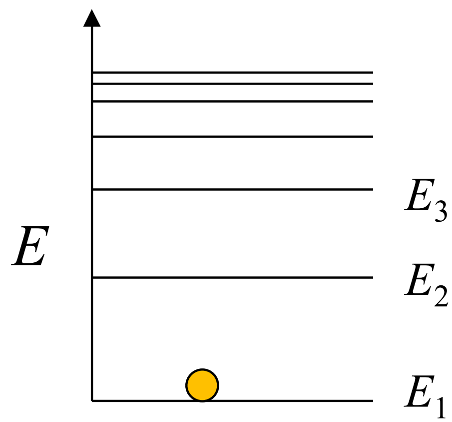
Photon Emission by Atoms

In order to explain the observed *discrete* spectra lines from atomic emissions, Niels Bohr in 1913 combined the following two central ideas in his model:

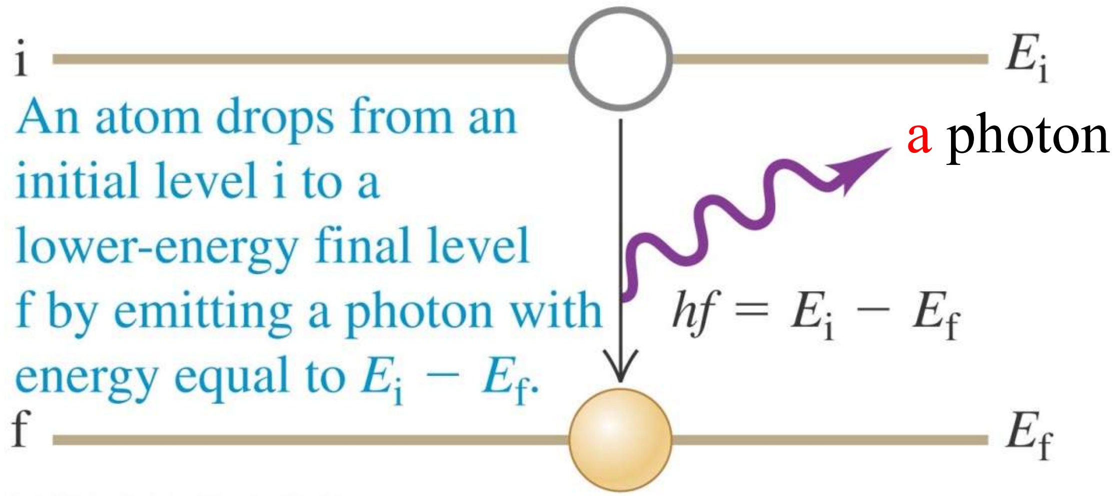
electrons as waves & **light as packets of energy**

(energy levels in atoms) ***(photons)***

Energy Levels in a typical atom

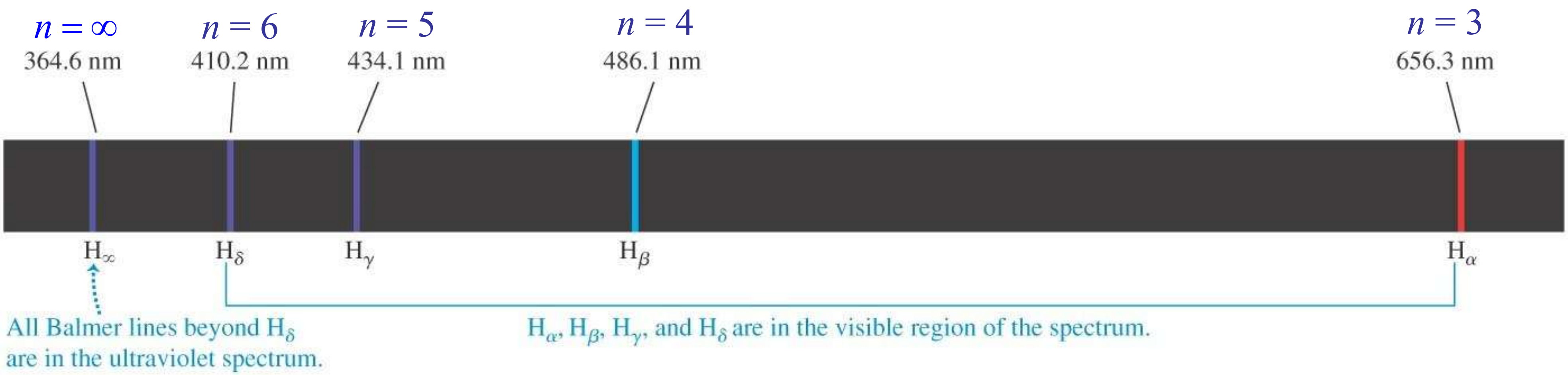


Each atom has a *specific set* of possible internal energy states. An atom can possess any one of these levels but cannot take on any *intermediate* values.



$$hf = \frac{hc}{\lambda} = E_i - E_f$$

The Hydrogen Spectrum



Hydrogen is the simplest atom and it also has the easiest spectrum to analyze.

In 1885, Johann Balmer first analyzed its spectrum and derived an *empirical* relation to accurately describe the wavelengths in the spectrum.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5, \cdots$$

$R = 1.097 \times 10^7 \, m^{-1}$ is called the **Rydberg constant** which was experimentally found to match with observed data.

The Hydrogen Spectrum

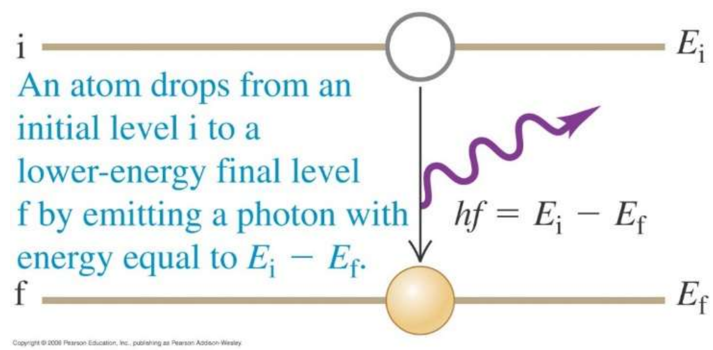
The formula for the Balmer series can be interpreted in terms of Bohr's hypothesis.

Multiplying Balmer's empirical equation by *hc*, we have

$$hc\frac{1}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \Delta E, \quad n = 3, 4, \cdots$$

Comparing this with Bohr's equation for photon emission by an excited atom, we can identify the term on the **right** as the difference between two energy levels *i* and *f* and the **left** as the energy of the emitted photon,

$$hf = \frac{hc}{\lambda} = E_i - E_f$$



$$E_i = -\frac{hcR}{n^2}$$

$$E_f = -\frac{hcR}{2^2}$$

The Hydrogen Spectrum

This suggests that the hydrogen atom has a series of *discrete* **energy levels**,

$$E_n = -\frac{hcR}{n^2} \qquad n = 1, 2, 3, 4, 5, \cdots$$

Then, according to Bohr's explanation, the various lines from the Balmer series corresponds to the *transition* of an *excited* atom from n = 3 or above to the n = 2 level.

The constant hcR has the following numerical value: $hcR = 2.19 \times 10^{-18}\,J = 13.6\,eV$

For a hydrogen atom, the lowest possible energy level is given by E_1 = -13.6 eV. n =1 called the **ground level (ground state)** and all the higher levels (n >1) are called the **excited levels (excited states)**.

The Hydrogen Spectrum

Other special series corresponding to different transitions have also been described:

$$\textit{Lyman Series} \quad \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \quad (n = 2, 3, 4, \cdots)$$

$$\textit{Balmer Series} \quad \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad (n = 3, 4, 5, \cdots)$$

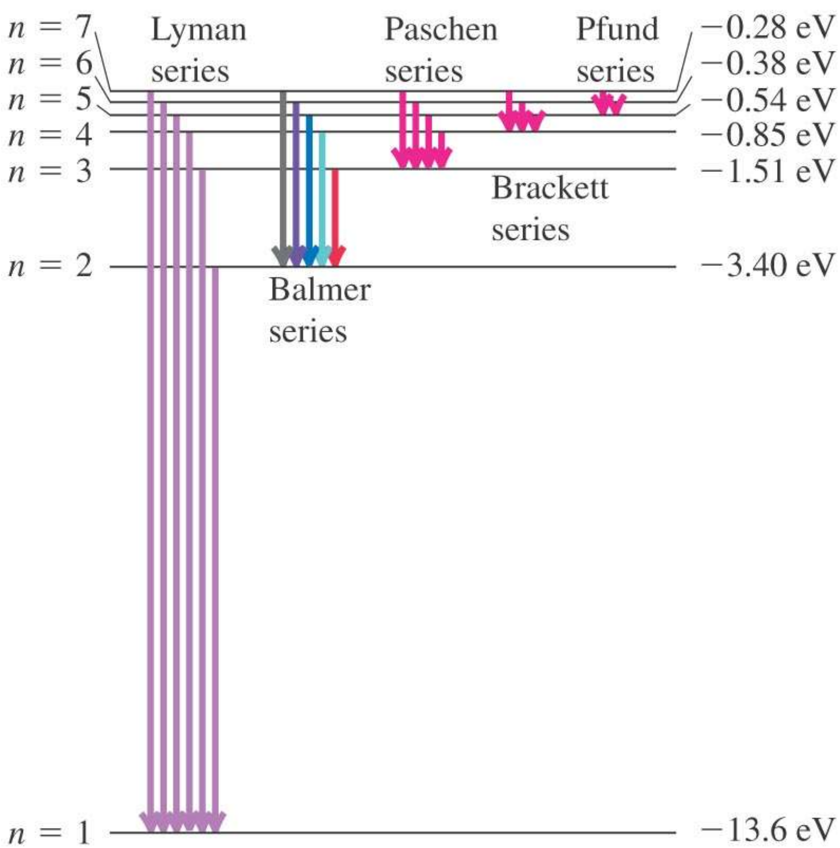
$$\textit{Paschen Series} \quad \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad (n = 4, 5, 6, \cdots)$$

$$\textit{Brackett Series} \quad \frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right) \quad (n = 5, 6, 7, \cdots)$$

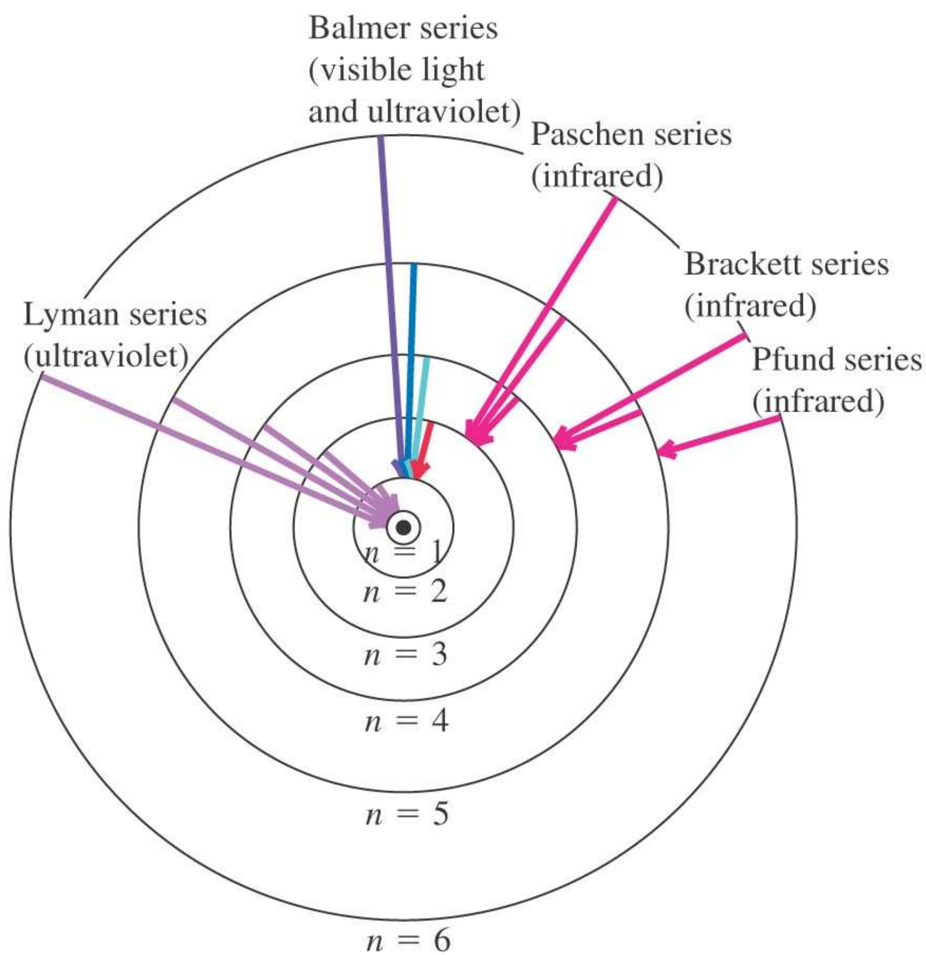
$$\textit{Pfund Series} \quad \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) \quad (n = 6, 7, 8, \cdots)$$

The Hydrogen Spectrum

Energy-level diagram for hydrogen, showing some transitions corresponding to the various series

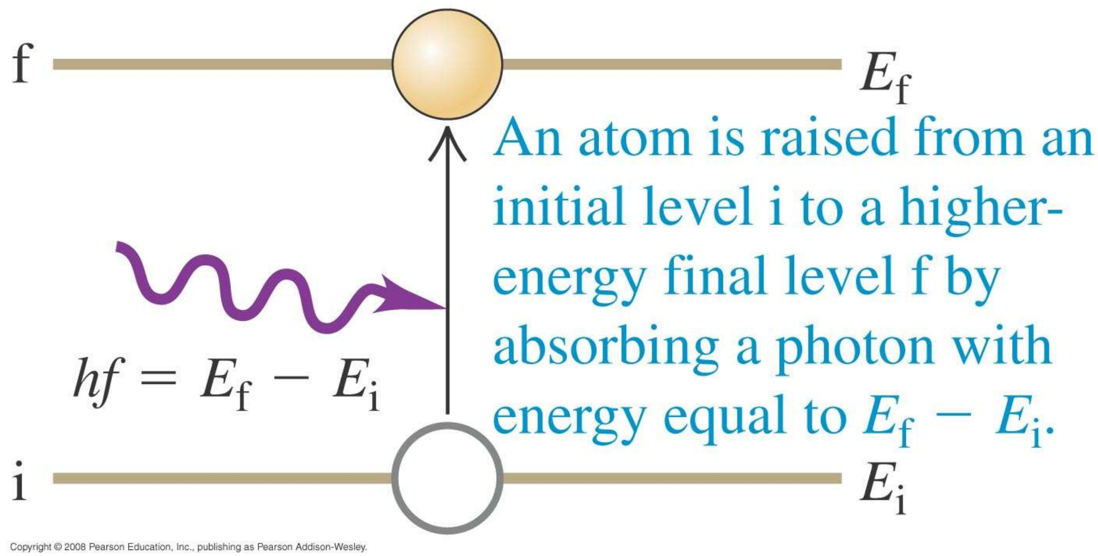


"Permitted" orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.



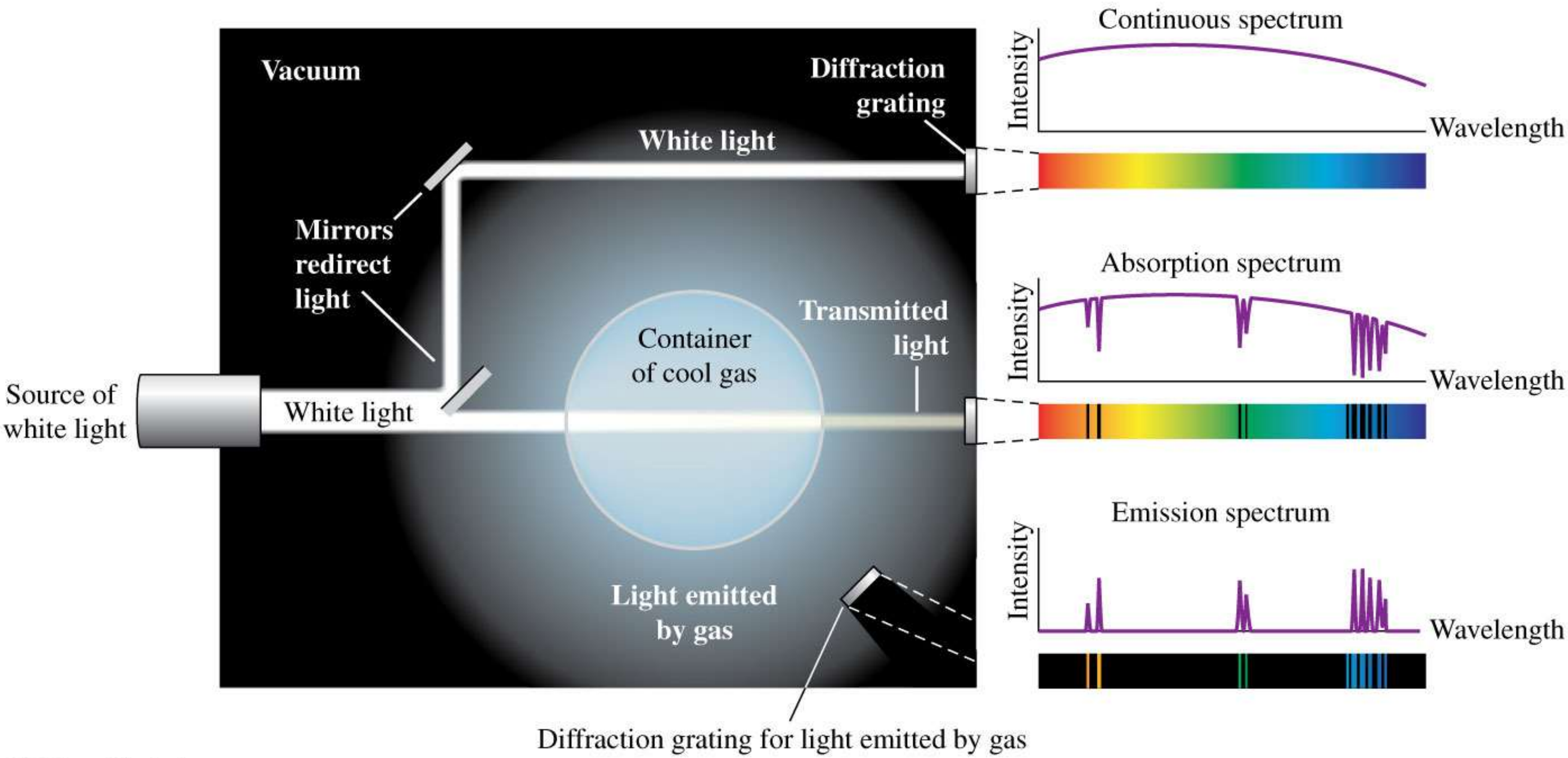
Photon Absorption by Atoms

In general, a photon, *emitted* when an *excited* atom makes a transition from a higher level to a lower level, can also be *absorbed* by a similar atom that is initially in the lower level.



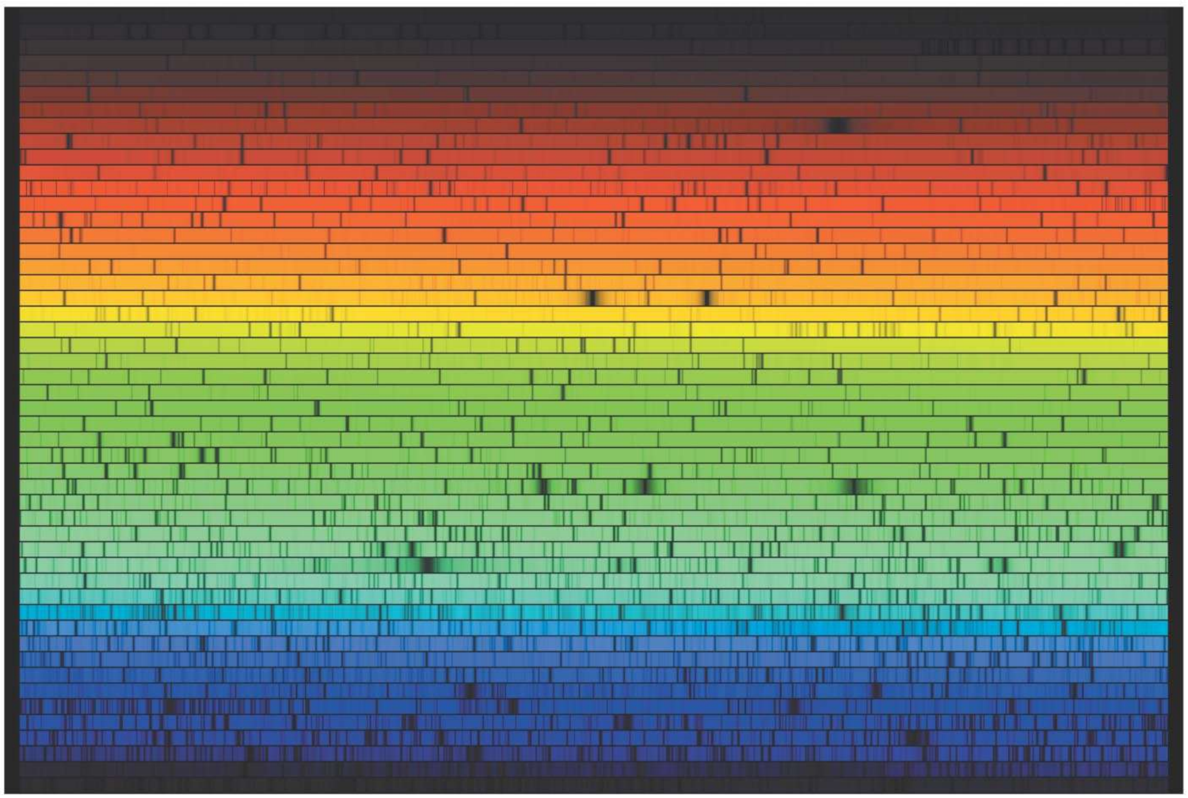
After the atom has been excited by absorbing the photon, it typically relax back to the lower energy levels within a short **lifetime** characteristic of the excited level. An excited level is called **metastable** if it has a relatively long lifetime.

Photon Absorption by Atoms



Absorption Spectrum

Astronomers can analyze the atomic components in inter-stellar gases by studying these absorption spectra.

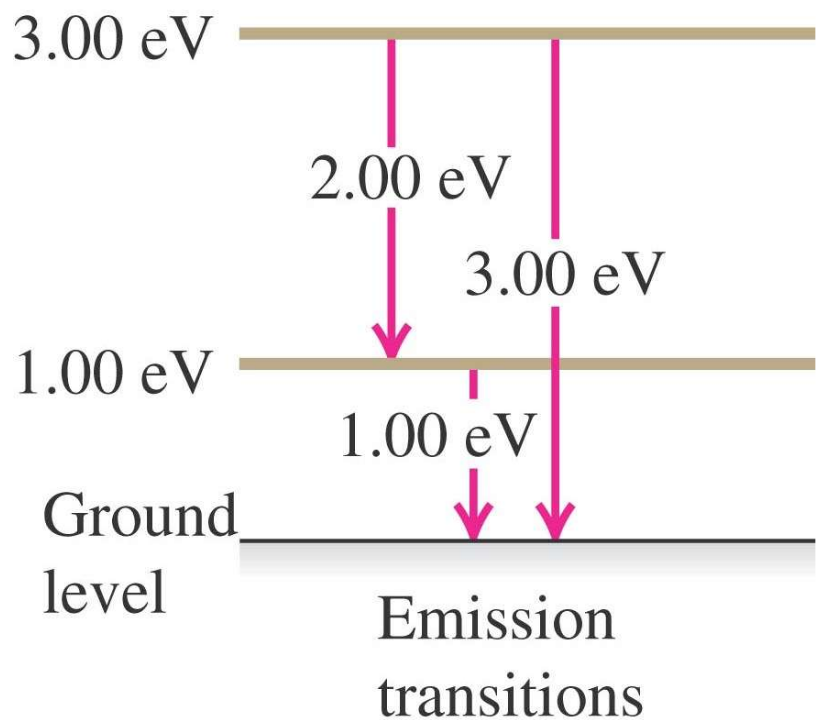


Absorption line spectrum (black lines) of the sun. The continuous emission spectrum (colors) is produced deep within the sun. The absorption lines (dark) indicate the kind of atoms that the emitted photons pass thru in the solar atmosphere.

Example 38.5: Emission and Absorption Spectra

Hypothetical atom with three energy levels.

a) λ can be emitted when an excited atom relax back to its ground state.



$$\Delta E = hf = \frac{hc}{\lambda} \quad \rightarrow \quad \lambda = \frac{hc}{\Delta E}$$

Example 38.5

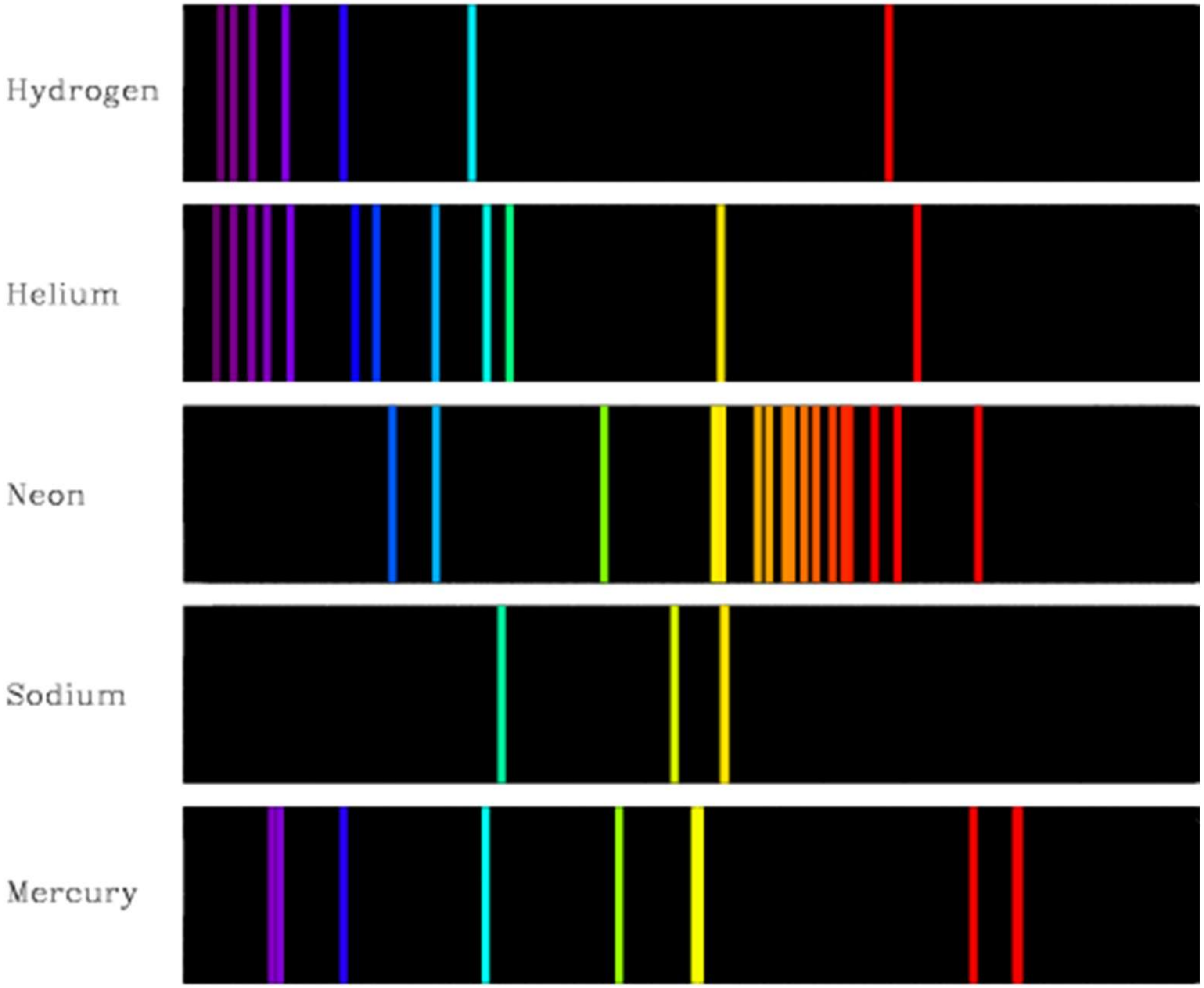
a) Three possible transitions: $\Delta E = 1eV$, $2eV$, and $3eV$ (red downward arrows)

$$\lambda = \frac{hc}{\Delta E} = \frac{4.136\times10^{-15}\,\mathrm{eV\cdot s}\left(3.00\times10^{8}m/s\right)}{(\mathrm{1eV,\ 2eV,\ or\ 3eV})} = 1240nm,\ 620nm,\ \text{or}\ 414nm$$

b) Two possible transitions: $\Delta E = 1eV$, and $3eV$ (blue upward arrows)

$$\lambda = \frac{hc}{\Delta E} = \frac{4.136\times10^{-15}\,\mathrm{eV\cdot s}\left(3.00\times10^{8}m/s\right)}{(\mathrm{1eV\ or\ 3eV})} = 1240nm\ \text{or}\ 414nm$$

Emission Line Spectra



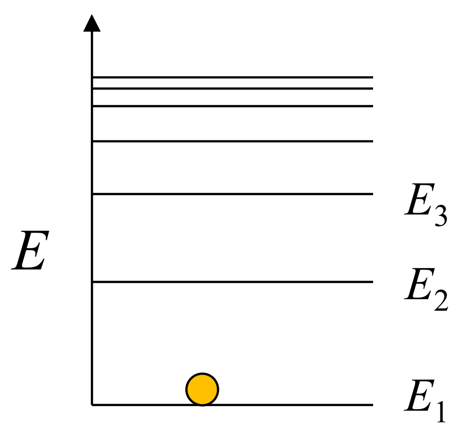
The Bohr Model

In order to explain the observed *discrete* spectra lines from atomic emissions, Niels Bohr in 1913 combined the following two central ideas in his model:

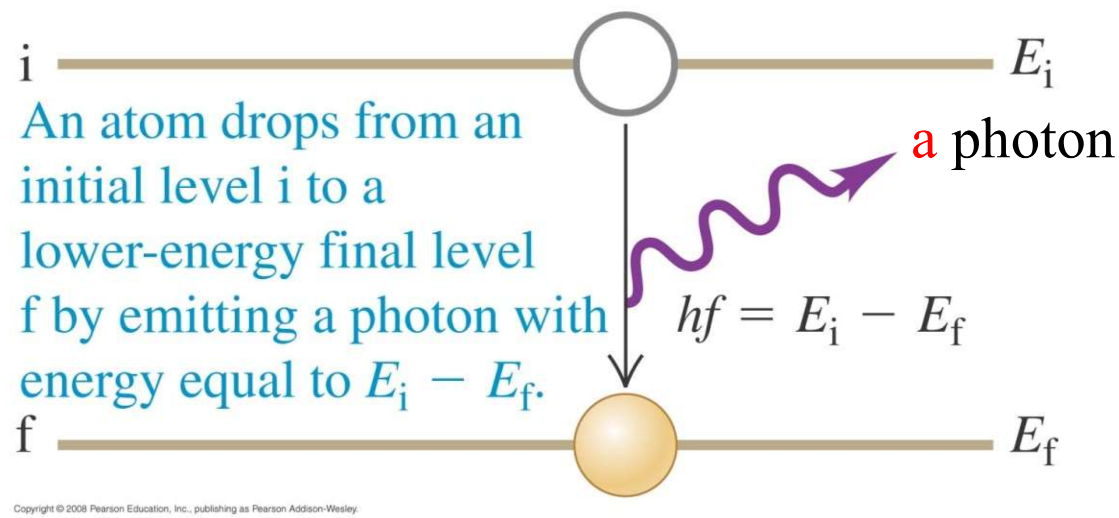
electrons as waves & **light as packets of energy**

(energy levels in atoms) ***(photons)***

Energy Levels in a typical atom



Each atom has a *specific set* of possible internal energy states. An atom can possess any one of these levels but cannot take on any *intermediate* values.



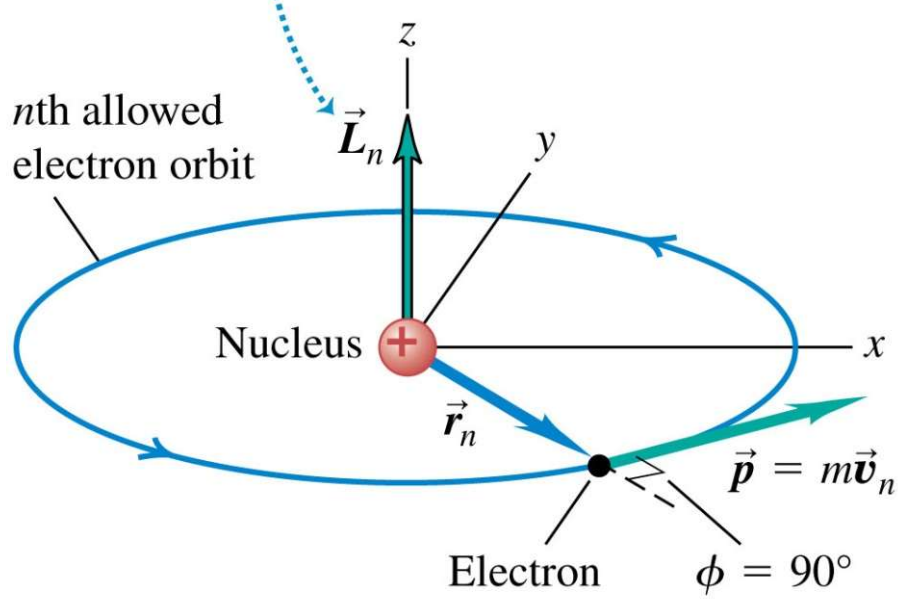
$$hf = \frac{hc}{\lambda} = E_i - E_f$$

The Bohr Model

It is a mixture of classical and new quantum ideas (**semi-classical**) in trying to theoretically calculate the energy levels of a hydrogen or hydrogen-like atom.

Assumptions of the model:

1. e^- moves in **stable** circular orbits around the nucleus under the influence of Coulomb Force.
2. Atom *only* radiates when e^- jumps from a higher energy orbit to a lower one, i.e., $hf = E_i - E_f$.
3. Only certain circular orbits are allowed: angular momentum of e^- around nucleus are **quantized**, i.e., $|\vec{\mathbf{L}}|$ must be multiples of $\hbar = h/2\pi$.



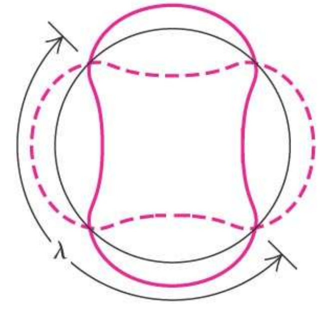
Electron Waves and the Bohr's Model

In the Bohr's model, angular momentum of the electron in a particular Bohr's orbit is **quantized**.

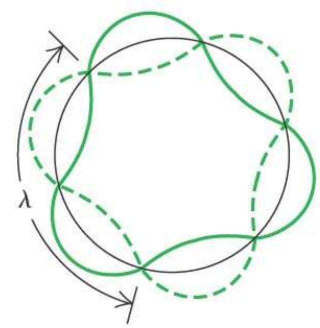
Using the de Broglie wave hypothesis, one can imagine an electron as a **standing wave** in a given energy state *n*.

In order for the standing wave to fit around a given Bohr's orbit, it must satisfy a **periodic boundary condition** (the wave must match back onto itself).

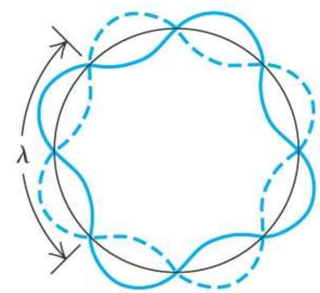
$$\Rightarrow \quad 2\pi r_n = n\lambda, \quad n = 1, 2, 3, \cdots$$



$n = 2$



$n = 3$



$n = 4$

Electron Waves and the Bohr's Model

With $2\pi r_n = n\lambda, \quad n = 1, 2, 3, \cdots$

Then, from de Broglie's relation, we have $\lambda = h / mv$

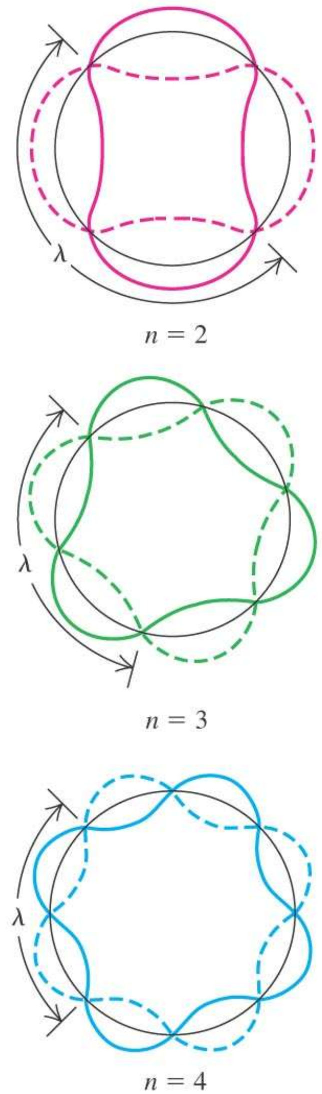
Putting these together, we have

$$2\pi r_n = n\frac{h}{mv}$$

$$\boxed{L_n = mv_n r_n = n\frac{h}{2\pi} = n\hbar, \quad n = 1, 2, 3, \cdots}$$

Therefore, the stable **energy levels** in the Bohr's Model are *quantized* with discrete angular momentum.

n is called the **principal quantum number** for the orbit.



Bohr's Model (mathematical details)

From a classical calculation, for a Hydrogen atom, the electron is bounded to the nucleus by the Coulomb's Force:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

For an electron in circular orbit around a nucleus with radius r, we have

$$F = ma \quad \rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^{\cancel{2}}} = \frac{mv^2}{\cancel{r}}$$

Solving for r, we have, $\boxed{r = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m}\left(\frac{1}{v^2}\right)}$

From the angular momentum **quantization**, we have, $v_n = \frac{n\hbar}{mr_n}, n = 1, 2, \cdots$

Bohr's Model (mathematical details)

Substituting this expression for v into the radius equation (boxed equation), we have,

$$r = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m}\left(\frac{1}{v^2}\right) \quad \rightarrow \quad r_n = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m}\left(\frac{mr_n}{n\hbar}\right)^2$$

$$\cancel{r_n} = \frac{e^2}{4\pi\varepsilon_0}\frac{mr_n^{\cancel{2}}}{n^2\hbar^2}$$

$$\boxed{r_n = \frac{\varepsilon_0 h^2}{\pi m e^2} n^2}$$ (orbital radii for a Bohr atom)

and, $$\boxed{v_n = \frac{e^2}{2\varepsilon_0 h}\frac{1}{n}}$$ (orbital speeds for a Bohr atom)

Bohr's Model (mathematical details)

Writing this orbital speeds in the unit of c, we have

$$v_n = \frac{e^2}{2\varepsilon_0 h}\frac{1}{n} = \left(\frac{e^2}{2\varepsilon_0 hc}\right)\frac{1}{n}c$$

where $\alpha = \dfrac{e^2}{2\varepsilon_0 hc}$ $\quad or \quad$ $\alpha = \dfrac{e^2}{4\pi\varepsilon_0 \hbar c} \simeq \dfrac{1}{137}$

is called he **Fine Structure Constant**.

This gives $v_1 \simeq \dfrac{1}{137}c$ with $n = 1$ at the **ground state** and it justifies to treat the orbital speed being nonrelativistic.

Bohr's Model (mathematical details)

The smallest radius for the Bohr atom corresponds to $n = 1$ and this minimum radius for the Bohr atom is called the **Bohr's radius**,

$$a_0 = r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2} = 5.29 \times 10^{-11} m$$

With this fundamental length scale for an atom, the other radii can be written as,

$$r_n = a_0 n^2, \quad n = 1, 2, 3, \cdots$$