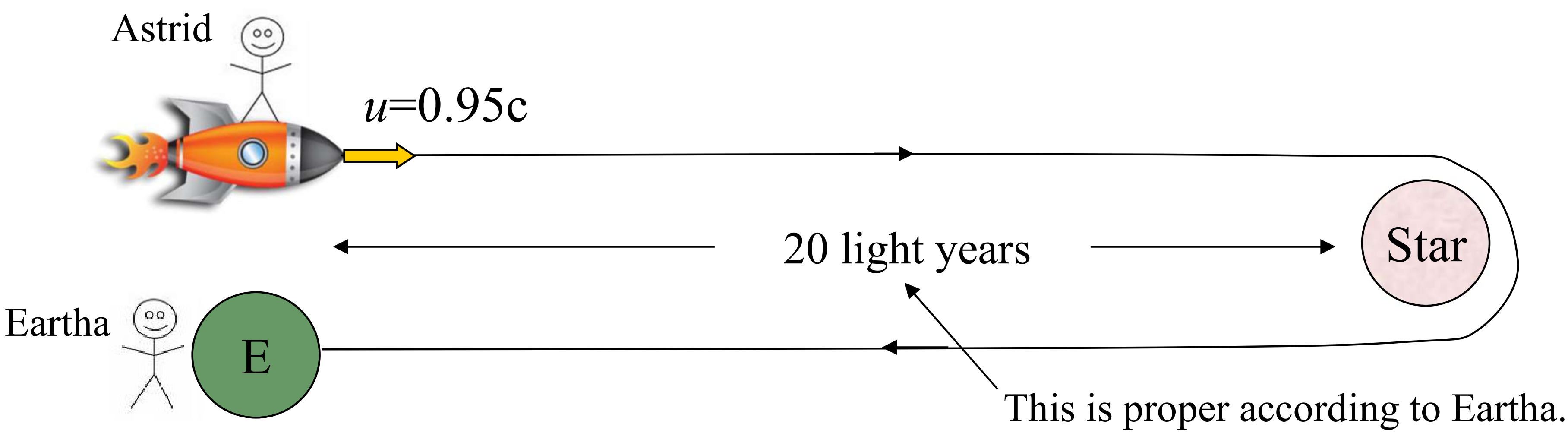
The Twin Paradox



First consider Time of Travel according to Astrid:

1st leg: distance from earth to star appears to be contracted according to Astrid.

$$\text{dist to star}_{\text{Astrid}} = \frac{20ly}{\gamma} = 20ly\sqrt{1-u^2/c^2} = 20ly\sqrt{1-0.95^2} = 6.25ly$$

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Time of travel = 6.25*ly* / 0.95c = 6.6 yrs

2nd leg: return trip distance also appeared to be contracted according to Astrid.

$$\text{dist to star}_{\text{Astrid}} = 6.25ly$$ And time of travel = 6.6 yrs

So, if Astrid started her journey at 20 years of age, when she comes back to earth, she will be 20+2*6.6 ~ 33 years old.

Now, we consider Time of Travel according to Eartha:

$$\text{dist to travel}_{\text{Eartha}} = 20ly \times 2 = 40ly$$ And time of travel = 40ly / 0.95c = 42 years

So, if Eartha said good bye to Astrid at 20 years of age, when they meet back, Eartha will be 20+42 ~ 62 years old.

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So, according to this description, Astrid will be almost *half* her twin sister's age after she comes back from her trip to the star !

Here is the apparent paradox:

Can Astrid claims that all *inertial* reference frames are equivalent and make exactly the same arguments with the spaceship being stationary and the earth moved so as to conclude that Eartha will be younger at the end of her journey?

The different *physical* descriptions cannot both be true but SR seems to suggest that they should or not? (→ apparent paradox).

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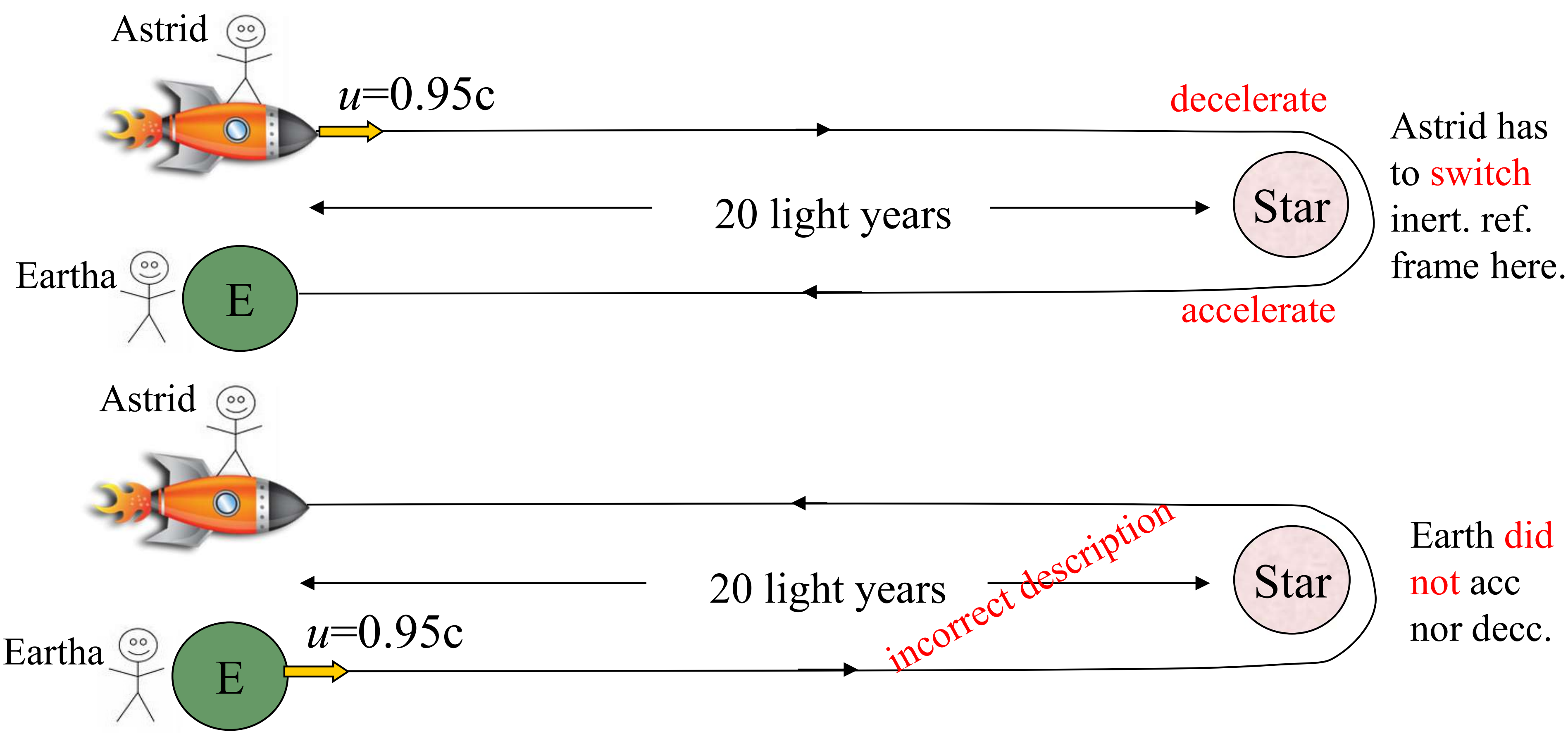
To resolve this apparent paradox, one need to realize that the following two situations are not *symmetric* !



The Twin Paradox

Additional Notes:

- Astrid was *not* in a *single* inertial reference frame. She switched inertial reference frame at midpoint.
- Eartha was in a single inertial reference frame throughout the entire journey.

Conclusion:

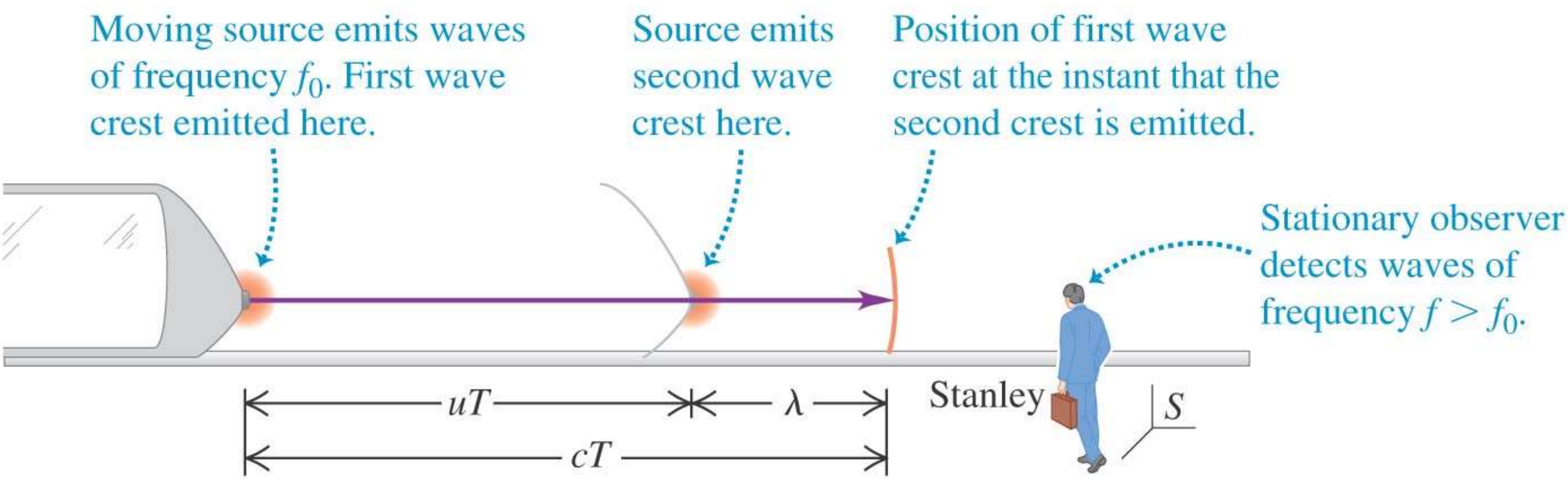
The seemingly symmetric descriptions between Astrid and Eartha are *not* symmetric at all and their actual physical experience needs *not* be the same.

And, they were not!

SR remains correct but one needs to be careful in making claims which are consistent with SR's assumptions.

Relativistic Doppler Effect

Statement of the problem: A source of light is *moving* at constant speed u toward a stationary observer (Stanley). The source emits EM waves with $f_0 = 1/T_0$ in its *rest frame*. What is the frequency measured by Stanley?



Relativistic Doppler Effect

From Stanley point of view, $\lambda = cT - uT = (c-u)T$ (dist. between wave crests)

Then, since $\lambda f = c$, we have $f = \dfrac{c}{(c-u)T}$

As we have seen, time intervals are measured differently by different observers.

T_0 is measured in the rest frame of the source so that it is the *proper* measurement and T is not and in fact, T is *dilated*, i.e.,

$$T = \gamma T_0 = \frac{T_0}{\sqrt{1-u^2/c^2}} = \frac{cT_0}{\sqrt{c^2-u^2}}$$

Rearranging, $\dfrac{1}{T} = \dfrac{\sqrt{c^2-u^2}}{c}\dfrac{1}{T_0} = \dfrac{\sqrt{c^2-u^2}}{c} f_0$

Note:

In the S (rest fr),

$$\left.\begin{aligned} \lambda_0 &= cT_0 \\ \lambda_0 f_0 &= c \end{aligned}\right\} \rightarrow f_0 = \frac{1}{T_0}$$

Relativistic Doppler Effect

Now substitute the expression for $1/T$ into $f = \frac{c}{(c-u)}\frac{1}{T}$

$$f = \frac{\not{c}}{(c-u)}\frac{\sqrt{c^2-u^2}}{\not{c}} f_0$$

$$f = \frac{\sqrt{(c-u)(c+u)}}{(c-u)} f_0$$

$$f = \sqrt{\frac{c+u}{c-u}}\, f_0$$

(Doppler Shift for an approaching source)

higher freq → blue shifted

For source receding, the only difference is the sign of the relative speed u → $-u$,

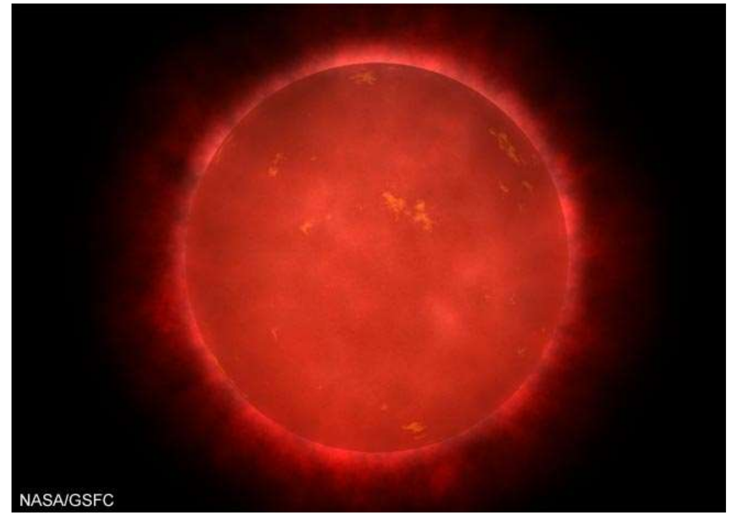
$$f = \sqrt{\frac{c-u}{c+u}}\, f_0$$

(Doppler Shift for a receding source)

lower freq → red shifted

Example: Red Shifted Red Dwarf Star

From **Wein's Displacement Law**: A red dwarf star at 3000K has a peak in its emittance at 1000nm but observing from Earth, the red dwarf appears red (650 nm). Estimate its receding speed with respect to Earth. (see Q38.19)



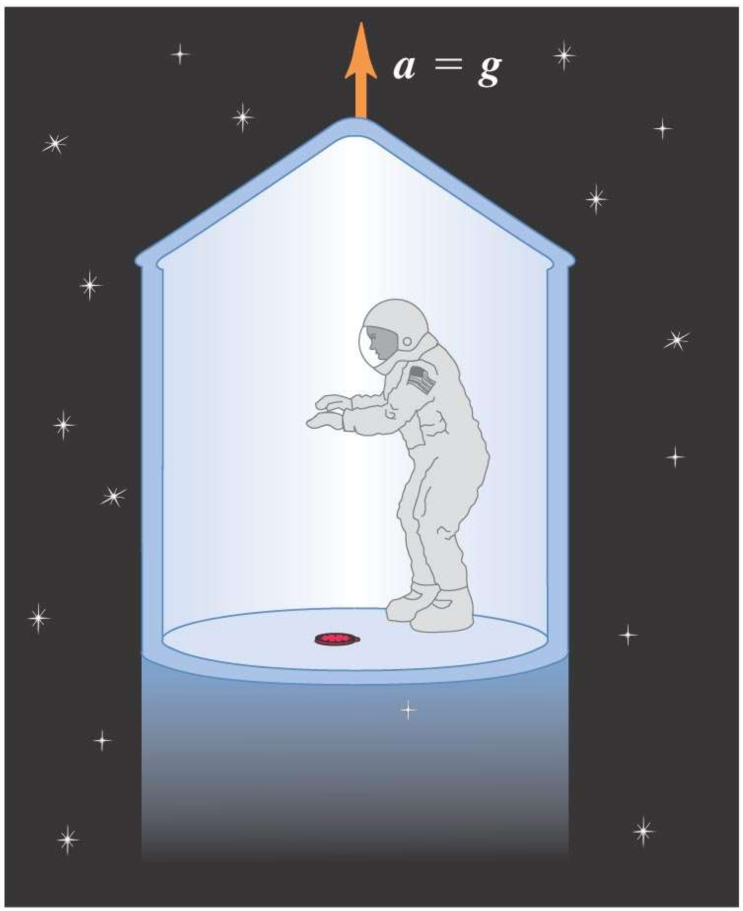
$$\frac{f}{f_0} = \frac{\lambda_0}{\lambda} = \sqrt{\frac{c-u}{c+u}}$$ (Dopper Shift for Receding Object)

$$\left(\frac{\lambda_0}{\lambda}\right)^2 = \frac{c-u}{c+u} \quad \rightarrow \quad u = \left(\frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2}\right)c = 0.406c$$

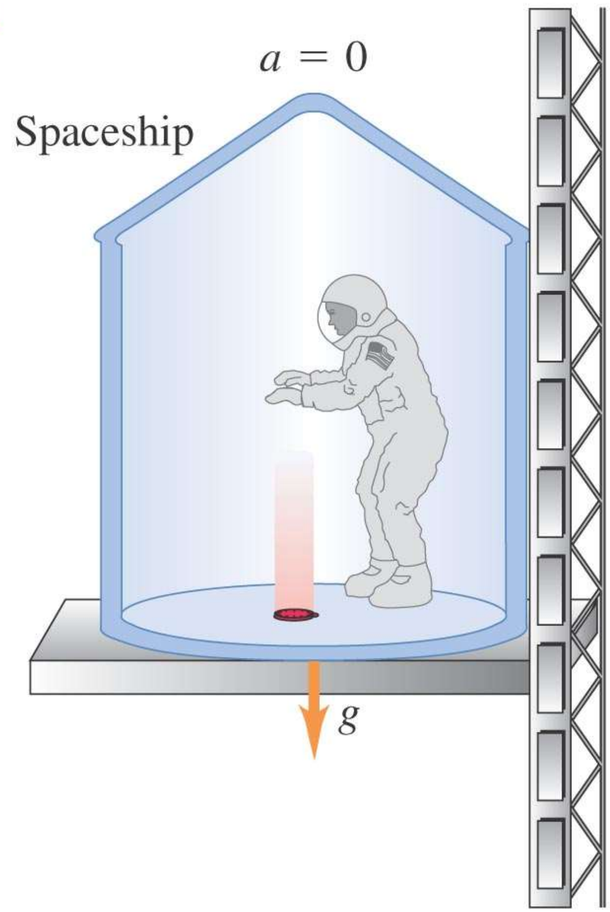
General Relativity and Gravity

Equivalence of gravity and acceleration (locally)

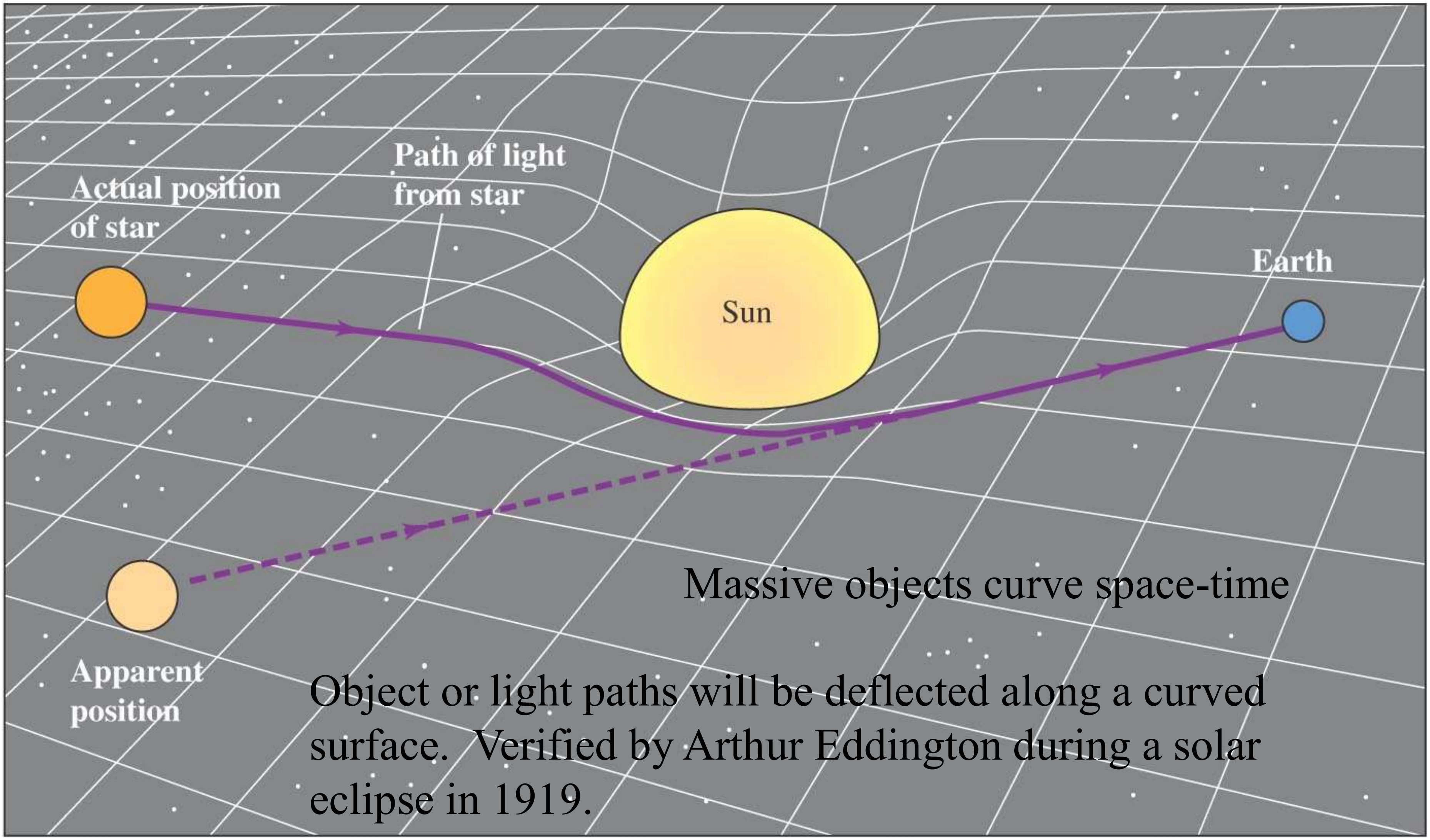
(b) In gravity-free space, the floor accelerates upward at $a = g$ and hits the watch.



(c) On the earth's surface, the watch accelerates downward at $a = g$ and hits the floor.



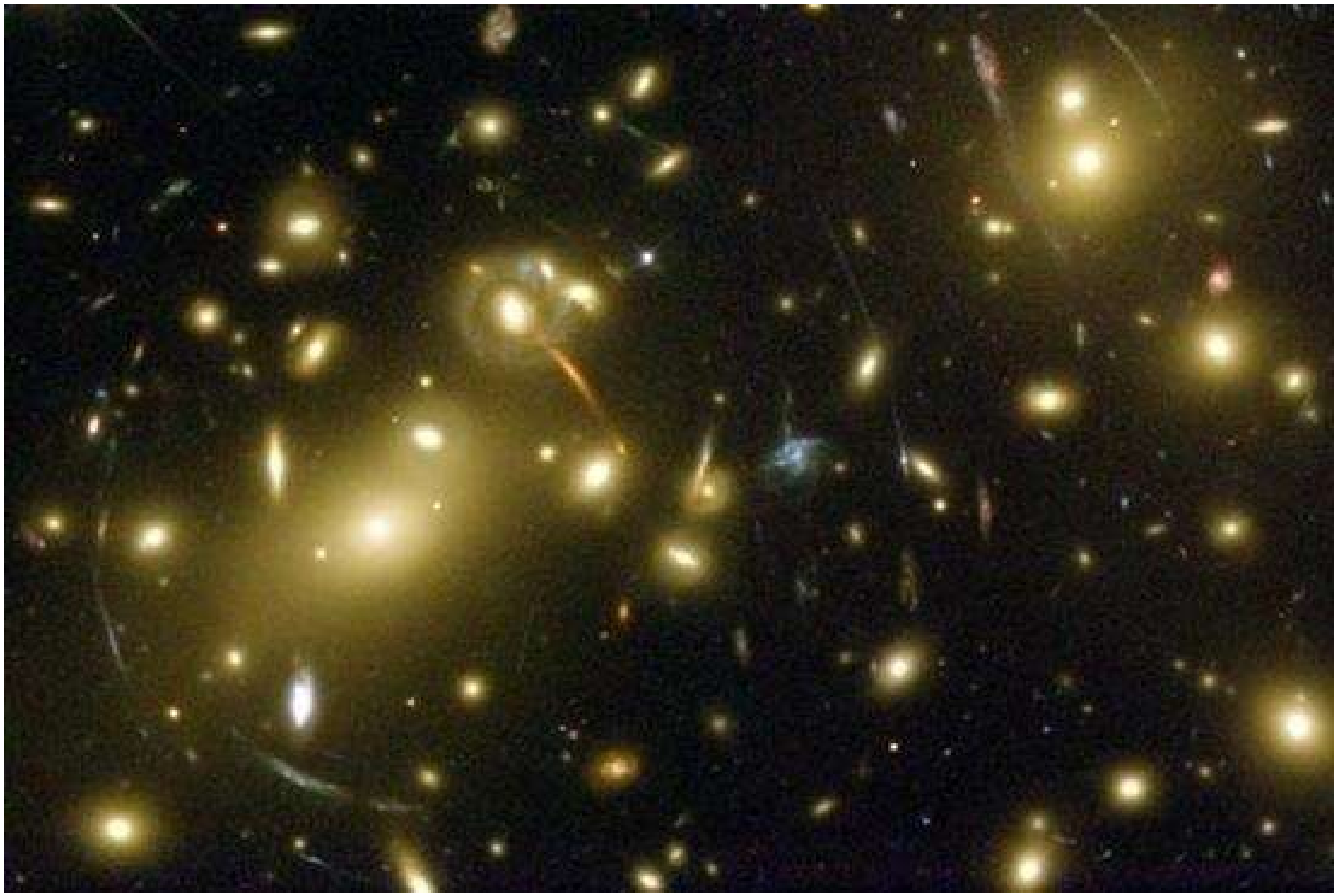
General Relativity and Space-Time



Massive objects curve space-time

Object or light paths will be deflected along a curved surface. Verified by Arthur Eddington during a solar eclipse in 1919.

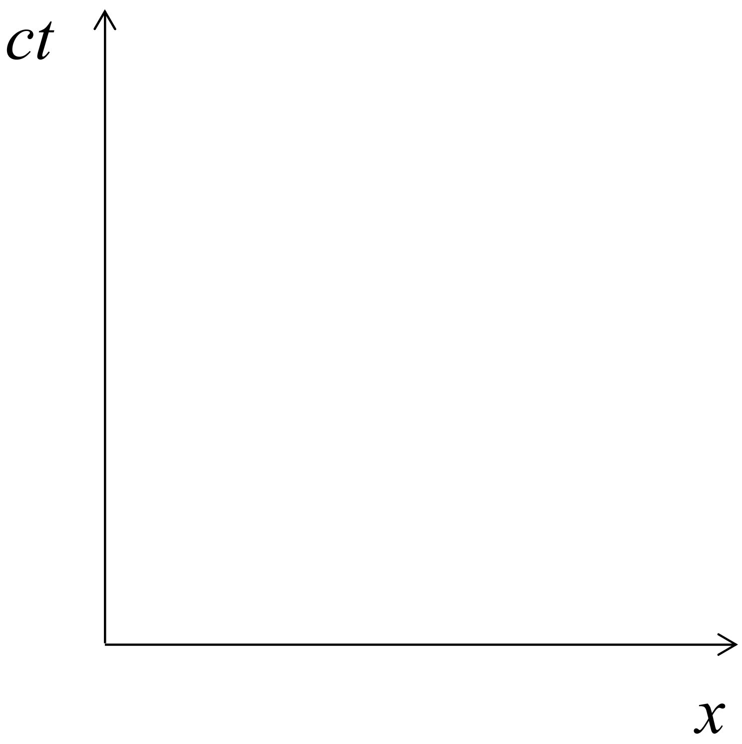
Gravity Lens



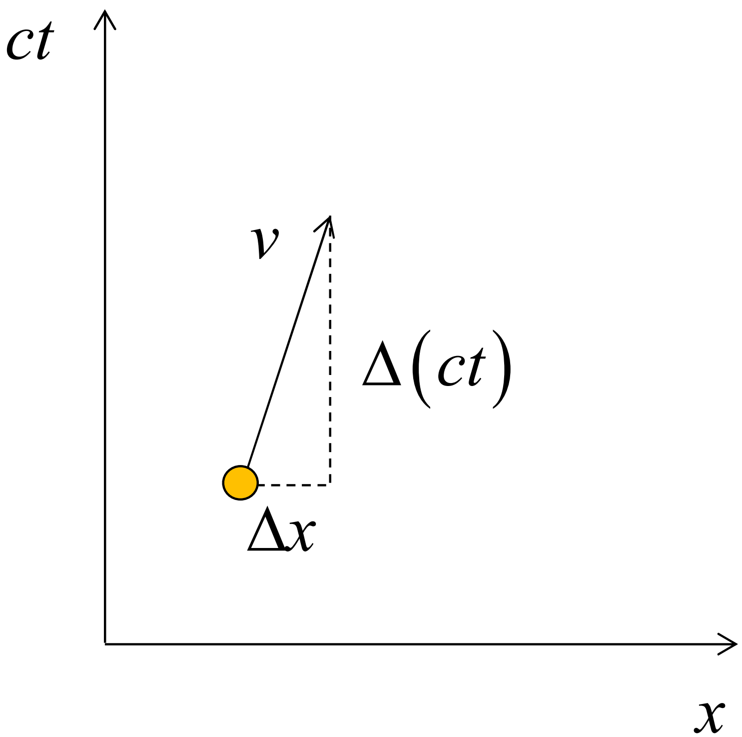
Distant galaxy lensed by Cluster Abell 2218
J-P Kneib and R. Ellis (Caltech) 2004

Space-Time Diagram

The time axis is measured in units of *ct*.



Space-Time Diagram



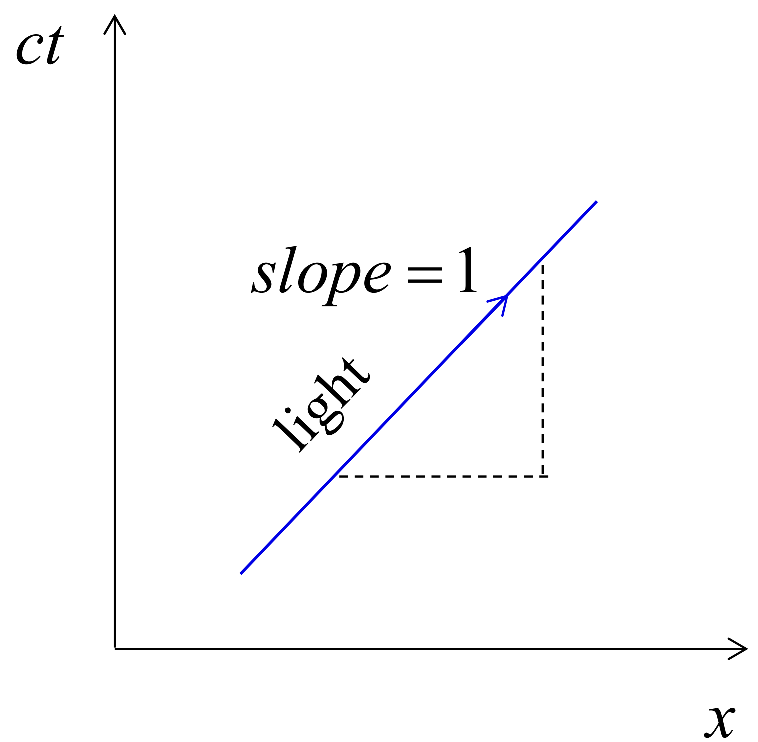
The time axis is measured in units of *ct*.

An object moving with speed *v* in the *x* direction will trace out a line with slope *c*/*v*

$$slope = \frac{\Delta(ct)}{\Delta x} = \frac{c\Delta t}{v\Delta t} = \frac{c}{v}$$

An stationary object (*v*=0) will trace out a vertical line (slope = ∞).

Space-Time Diagram



The time axis is measured in units of *ct*.

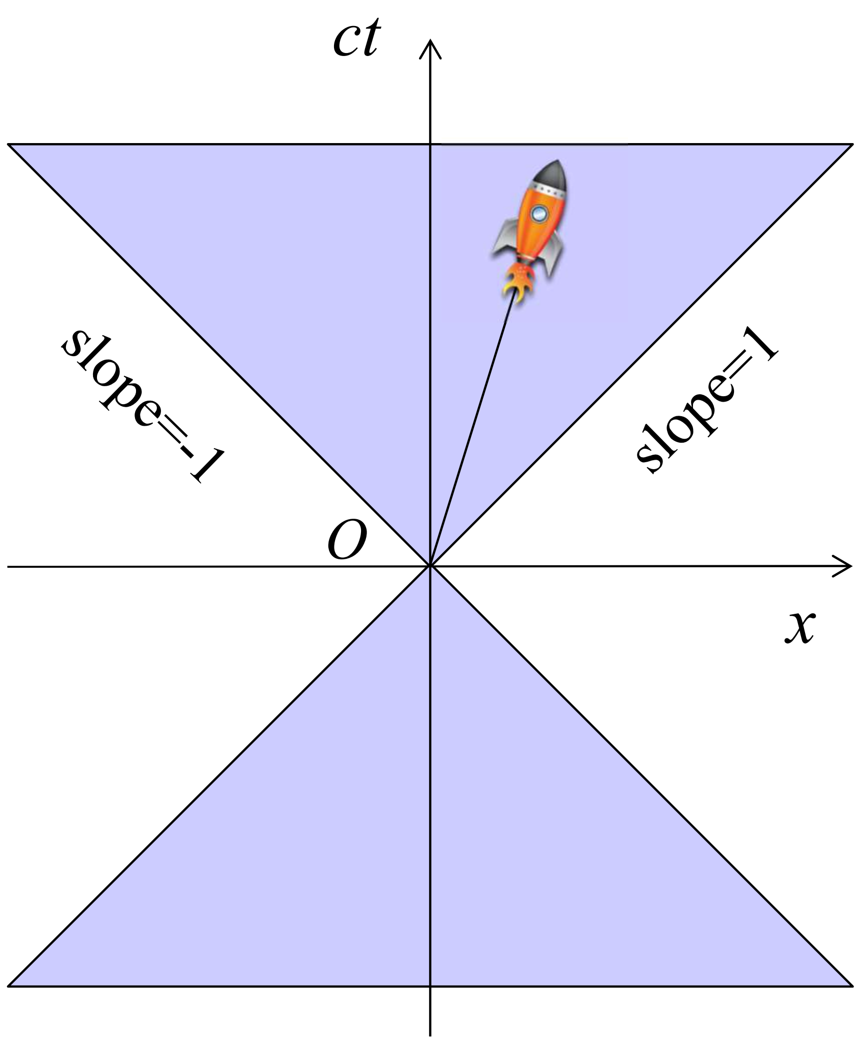
An object moving with speed *v* in the *x* direction will trace out a line with slope *c*/*v*

$$slope = \frac{\Delta(ct)}{\Delta x} = \frac{c\Delta t}{v\Delta t} = \frac{c}{v}$$

An stationary object (v=0) will trace out a vertical line (slope = ∞).

A light beam will trace out a 45° line with slope = 1 or -1.

Space-Time Diagram

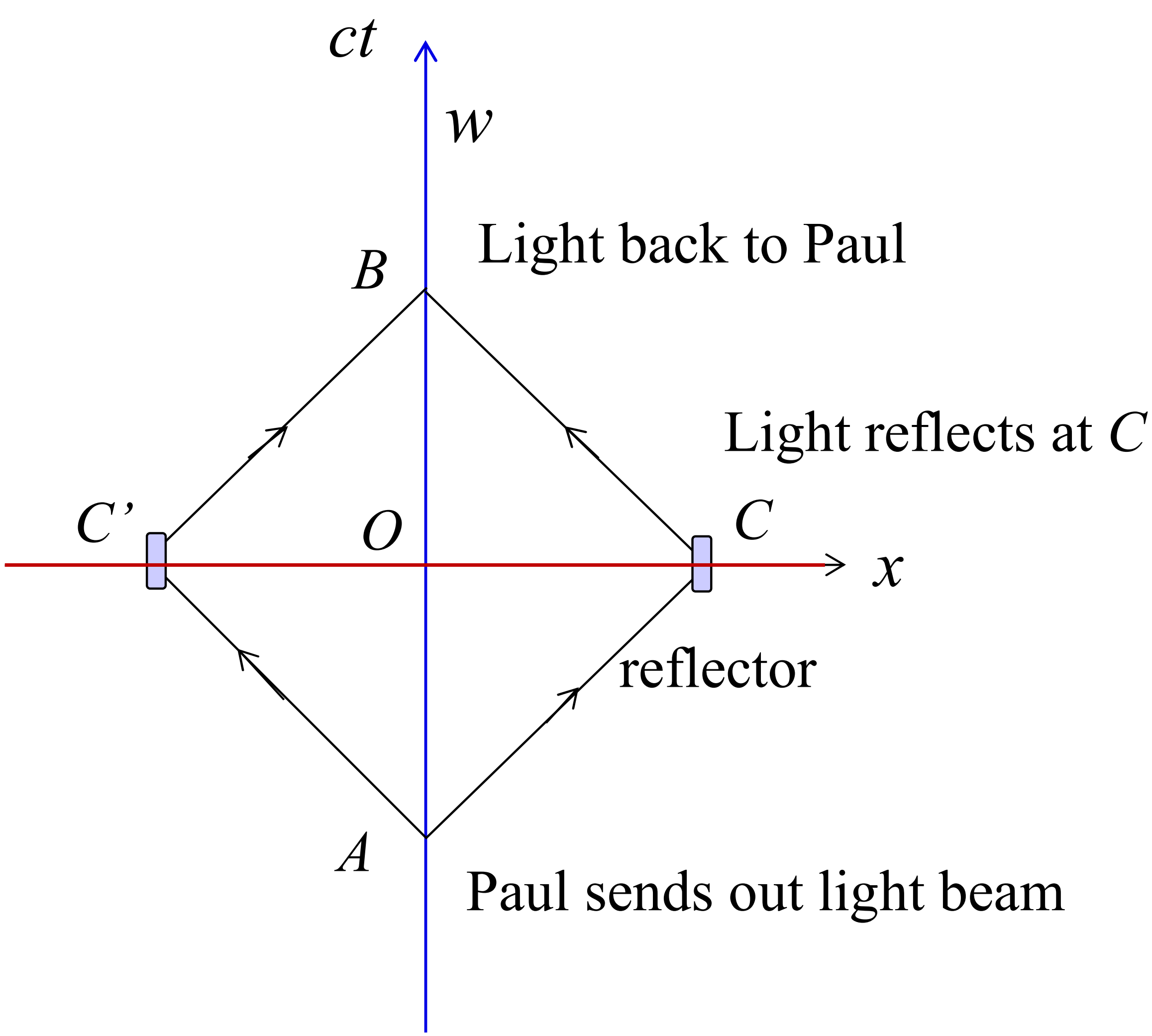


The path an object traced out in Space-Time is called a **World Line**.

Since nothing can go faster than the speed of light →

All world lines starting from *O* must stay within the shaded “light cone”.

Space-Time Diagram & Simultaneity



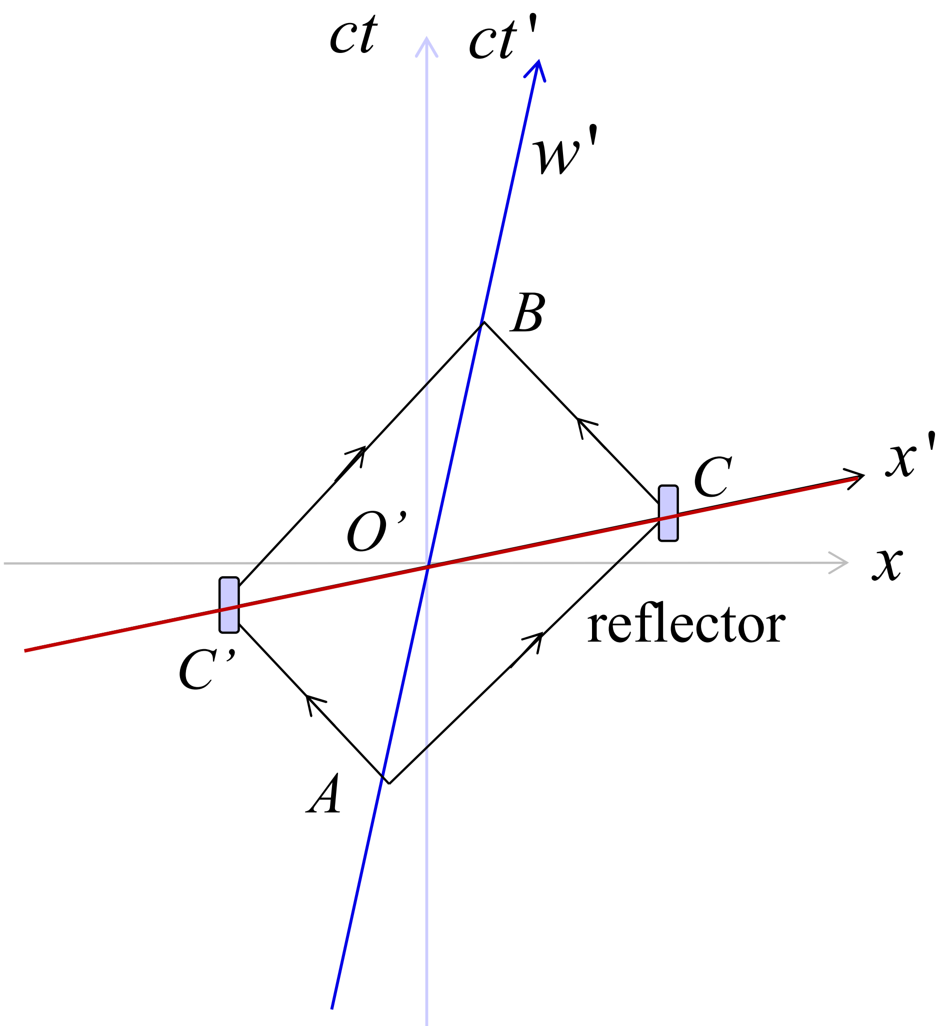
Since event O is exactly half way between events A and B,

Paul will consider event C to be simultaneous with O.

Similarly, C' and all points on the x-axis can be considered to simultaneous with O !

The x-axis (red) is also referred to the **line of simultaneity** with respect to the observer along the world line W at O.

Space-Time Diagram & Simultaneity

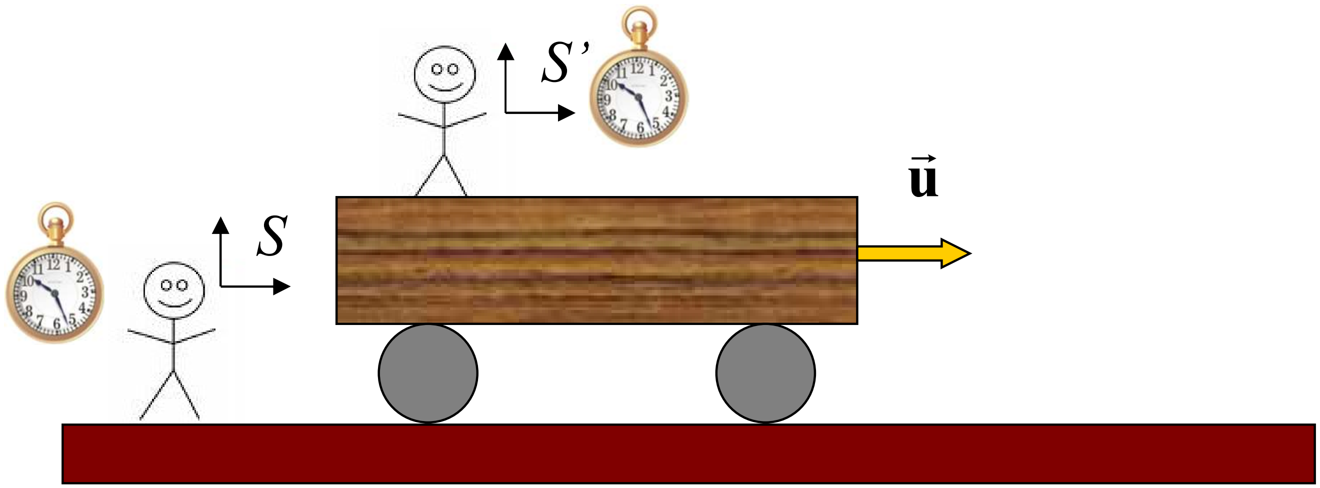


One can also construct the **line of simultaneity** for another observer *S*' who is moving relative to *S* along the world line w.'

Similarly, *S*' sends out a light signal to a reflector at *C* and *C'*, one can infer the midway point *O*' to be simultaneous with *C* and *C'*.

The line joining *O*' and C will be a segment of the **line of simultaneity** (red) for observer *S*' at *O*'.

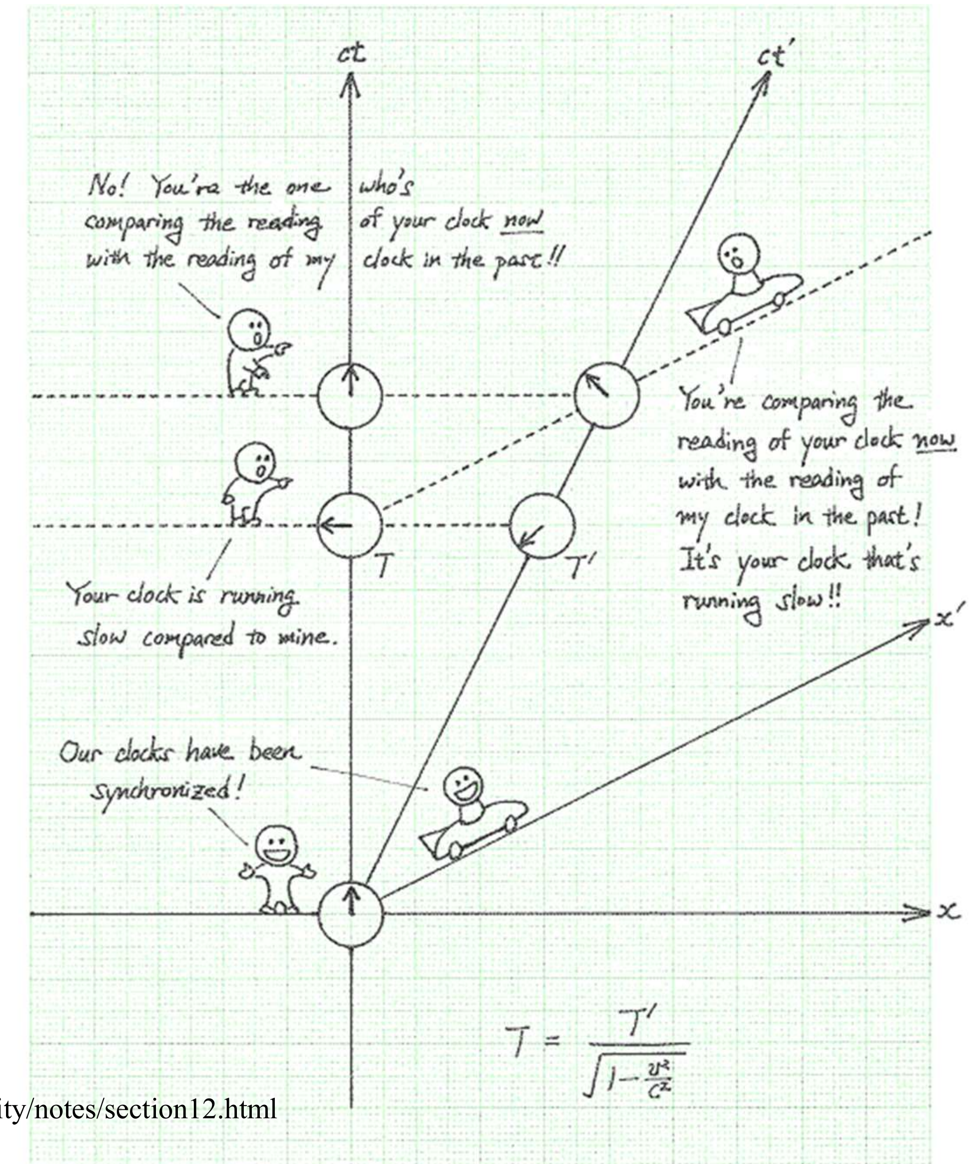
Symmetry of Time Dilation & Simultaneity



- Both observers in *S* and *S*' have their own measurement devices and they can also measure his/her partners devices and compare with his/her own.

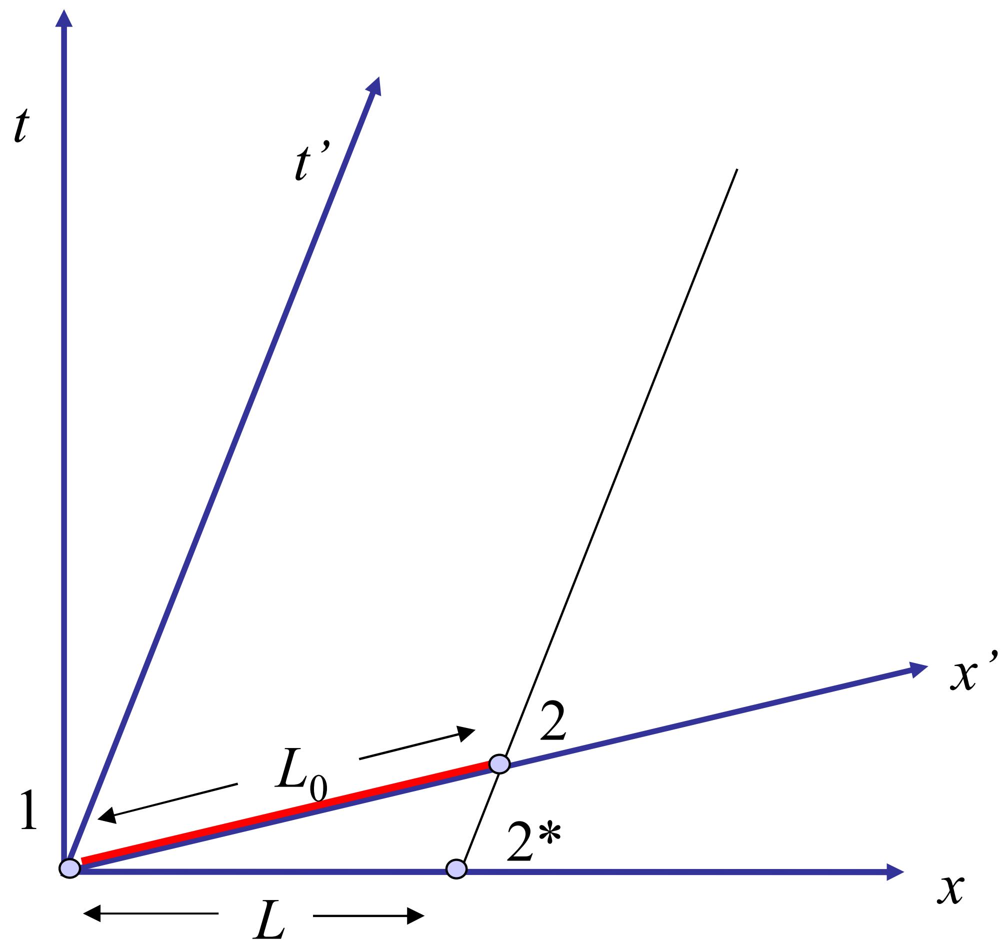
- Both S and S' will respectively measure time dilations from the moving clock from his/her partner.

Time Dilation (Space-Time Diagram)



http://www.phys.vt.edu/~takeuchi/relativity/notes/section12.html

Length Contraction



Ruler is *stationary* in *S*'

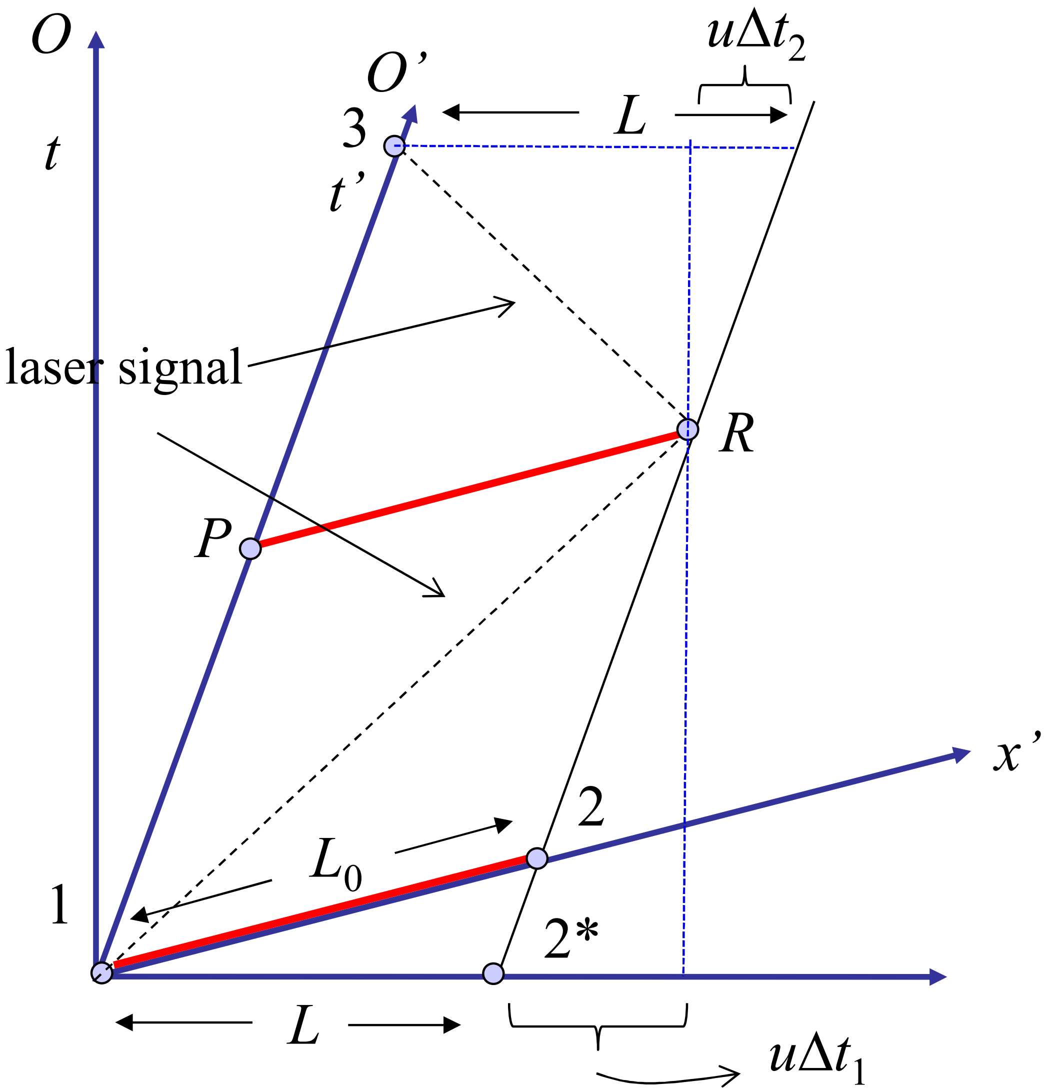
Both *O* and *O*' must measure the two ends of the ruler *at the same time* according to their own clocks.

O' measure L_0 bet 1 and 2

O measure *L* bet 1 and 2*

Length measurement involves simultaneity and L/L_0 are measured from two different sets of events.

Length Measurement using Laser



In *O*' frame, laser reflects from the mirror at *R* and since *P* is half way between the leaving event (1) and arriving-back event (3) of the laser, *O*' interprets *P* being simultaneous with *R*.

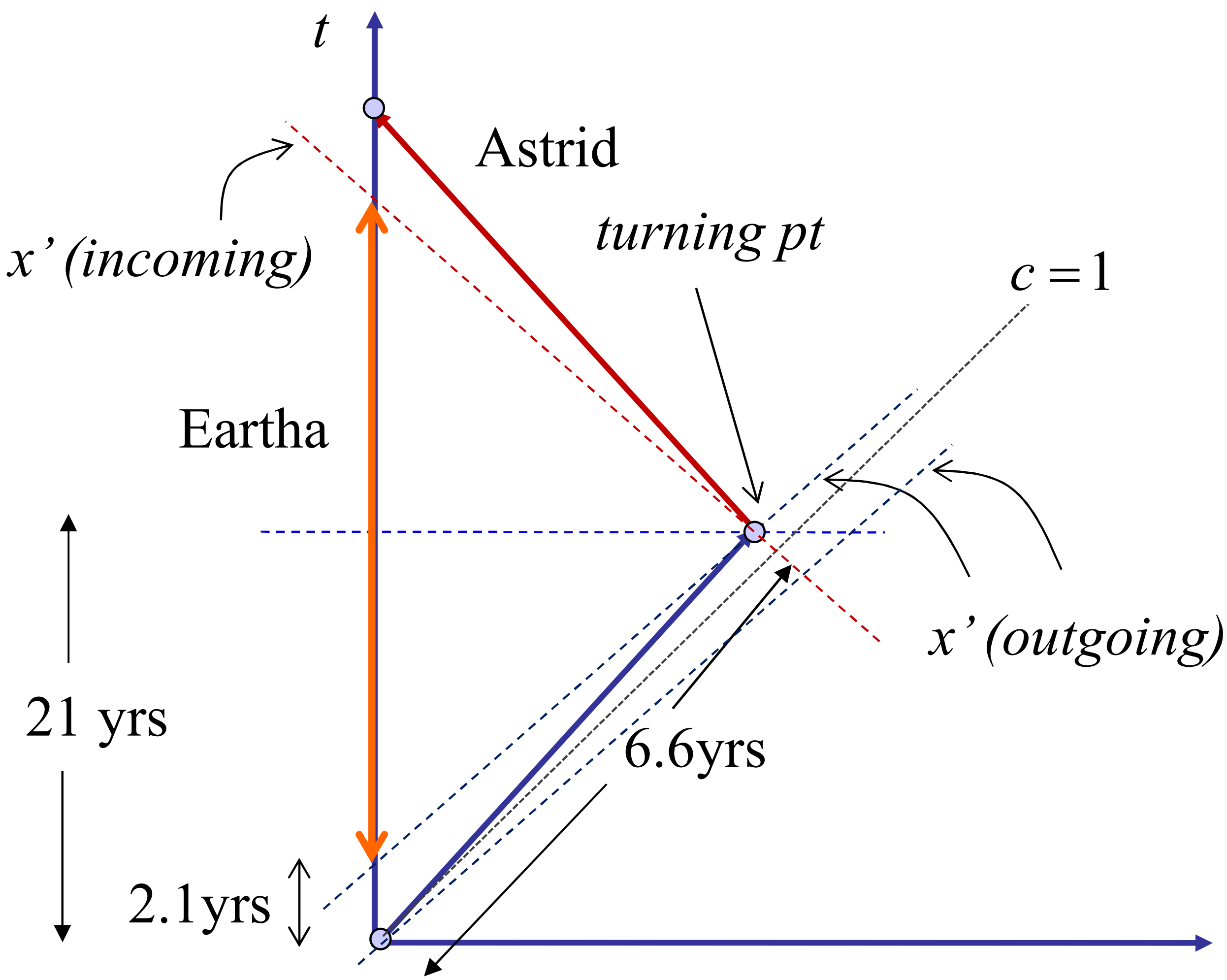
In *O* frame, the distance for laser light to travel to mirror is

$$L + u\Delta t_1$$

and the distance for laser light to return is

$$L - u\Delta t_2$$

The Twin Paradox



Note: Astrid changes his inertial reference frame at turning pt

1. Time dilation is symmetric before Astrid switches his ref frame:
   - -Eartha will see Astrid's clock dilates from 6.6yrs to 21 yrs
   - -Astrid will equivalently see Eartha' clock dilates from 2.1yrs to 6.6yrs (with the same γ factor)
2. Eartha's ages significantly according to Astrid as she switches her reference frame (here is the deceleration-acceleration effect)