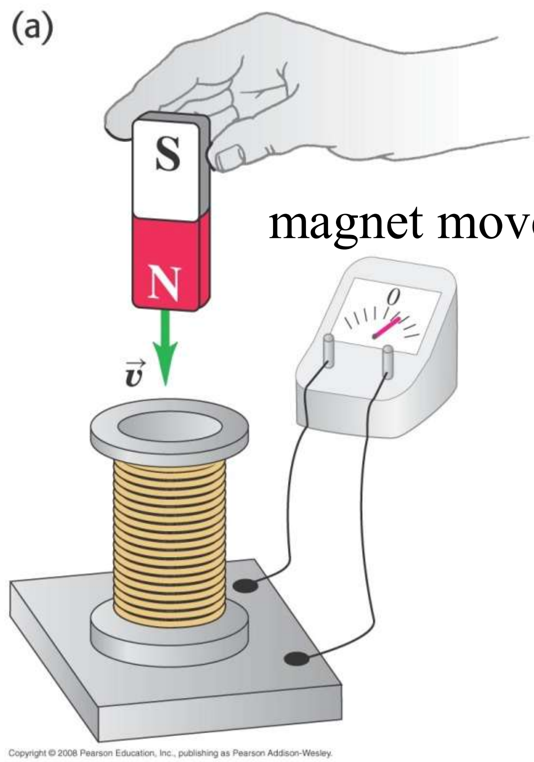
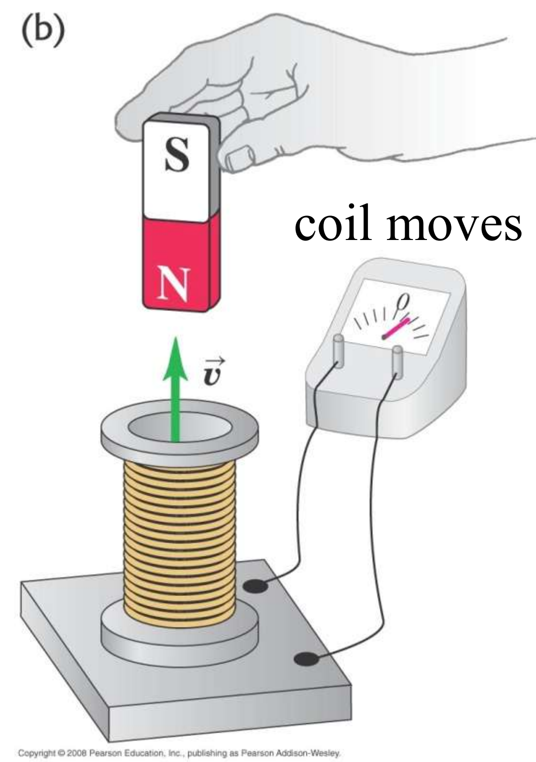
Einstein's Postulates for Special Relativity

1. All laws of physics must be the same in all inertial reference frames.
   - Specific observations might be different but the same phenomena must be described by the same physical law.
   - Not just the laws of mechanics (as in the Galilean viewpoint). *All* laws of physics include mechanics, EM, thermodynamics, QM, etc.



magnet moves down

SAME emf is
induced in the coil !



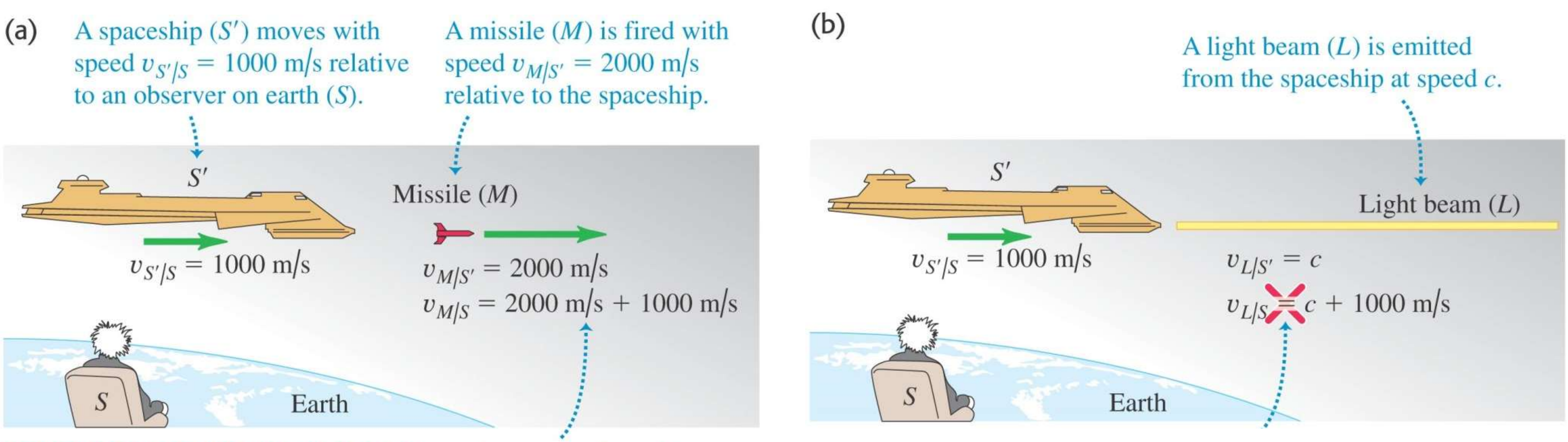
coil moves up

Einstein's Postulates

2. The speed of light *c in vacuum* is the same in all inertial reference frames and is independent of the observer or the source.

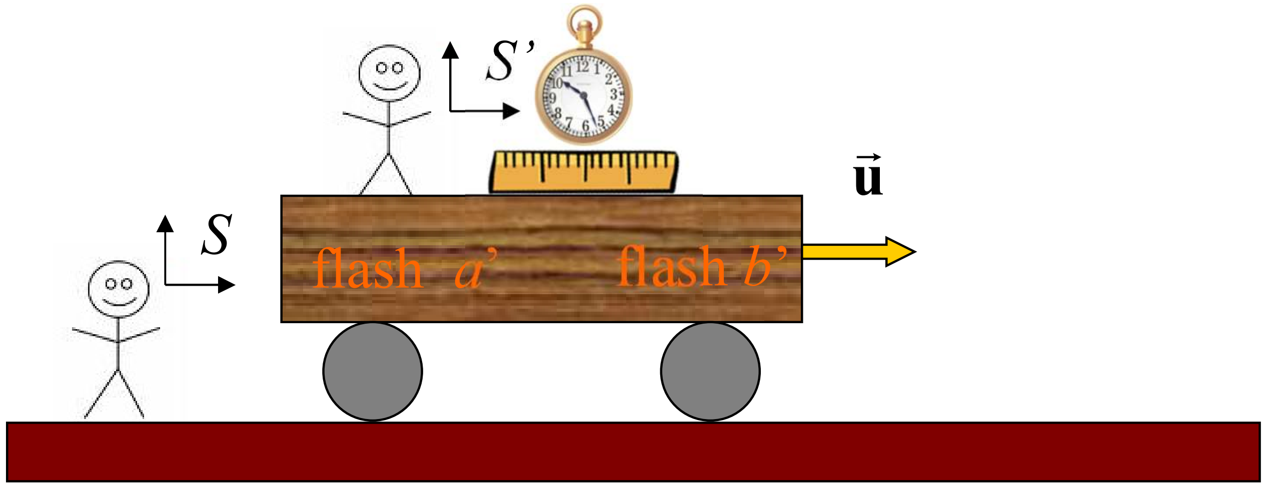
This is a revolutionary statement!

One of the immediate non-intuitive consequence → $\cancel{c = c' + u}$



Together with #1, SR requires us to rethink how time and space are measured!

Stating the Results First



Time Dilation: (moving clock runs slow)

$$\Delta t = \gamma \, \Delta t_0, \quad \gamma = 1 \Big/ \sqrt{1 - u^2/c^2}$$

Measured by S

Measured by S'

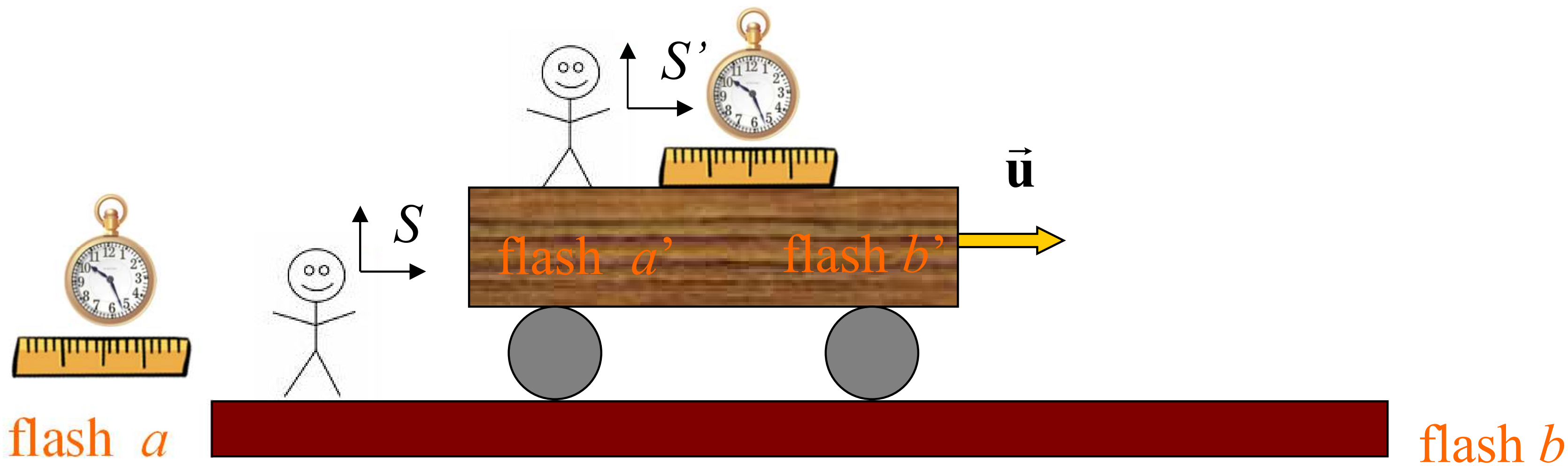
$$L = L_0 / \gamma$$

Length Contraction: (moving ruler get shorter)

Simultaneity:

Two flashes simultaneous in S' but not in S.

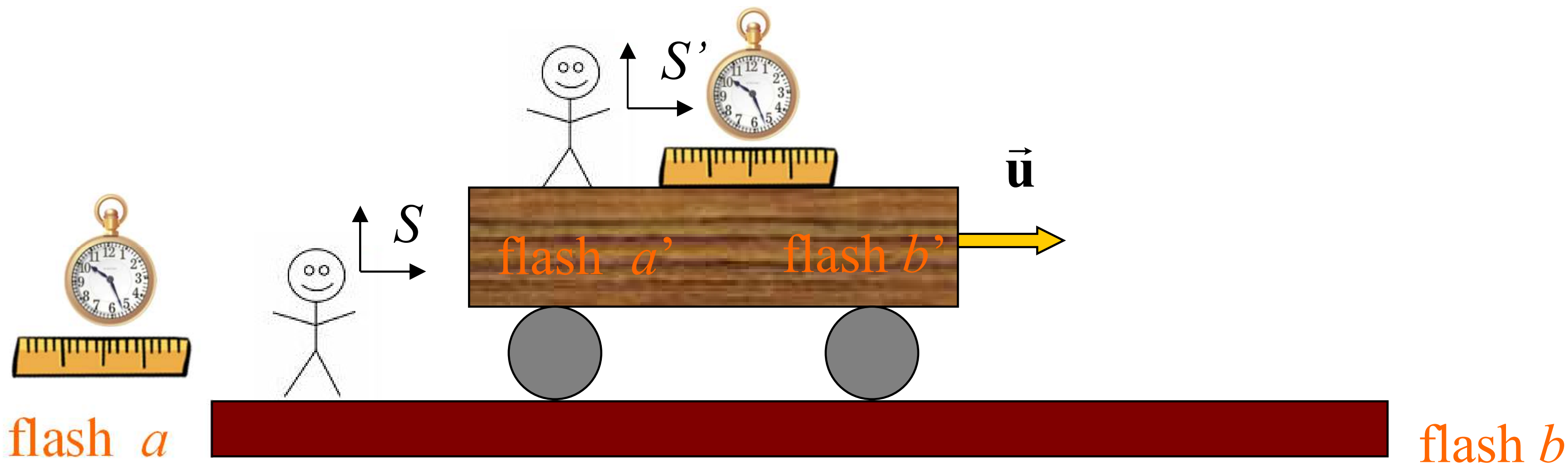
Notes on Relative Motion



- Both observers in *S* and *S*' have their own measurement devices and they can also measure his/her partners devices and compare with his/her own.

- Both S and S' will respectively measure time dilations and length contractions from the moving clock and ruler from his/her partner.

Notes on Relative Motion



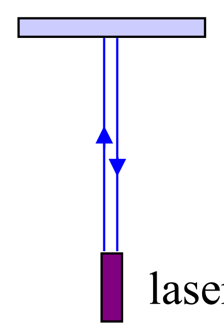
- Although time and length measurement will depend on the observer's inertial frame, they will *agree* on the following three items:

- *c* is the same in all frames
- all physical laws apply equally
- their relative speed *u* is the same

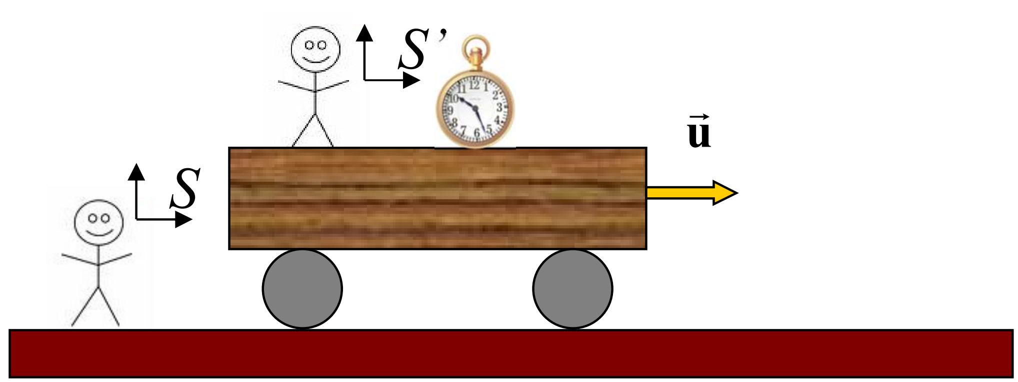
Relativity of Time Intervals

Measuring Time Intervals with a light "clock":

One time unit is measured by the duration of *two events*:
- laser light leaving (tic)
- laser light return (toc)



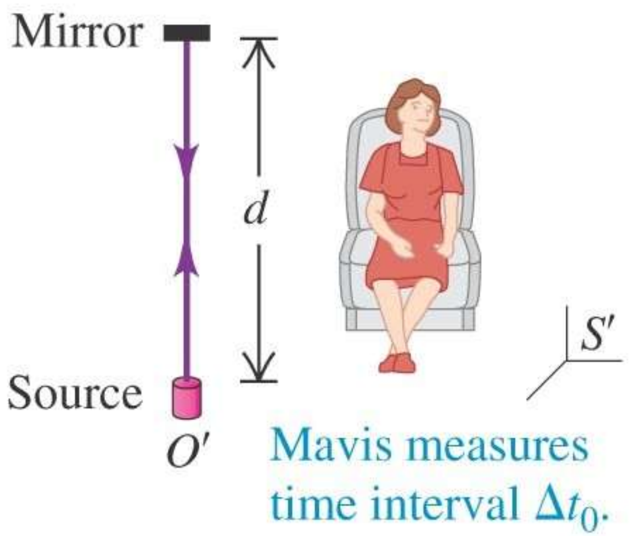
Consider a boxcar moving with respect to the ground and we are interested in the measurement of an interval of time by both *S* and *S*' from a clock placed in the boxcar.



Relativity of Time Intervals

In the S' – frame:

- Mavis O' is moving with the boxcar
- the clock is stationary with Mavis O'



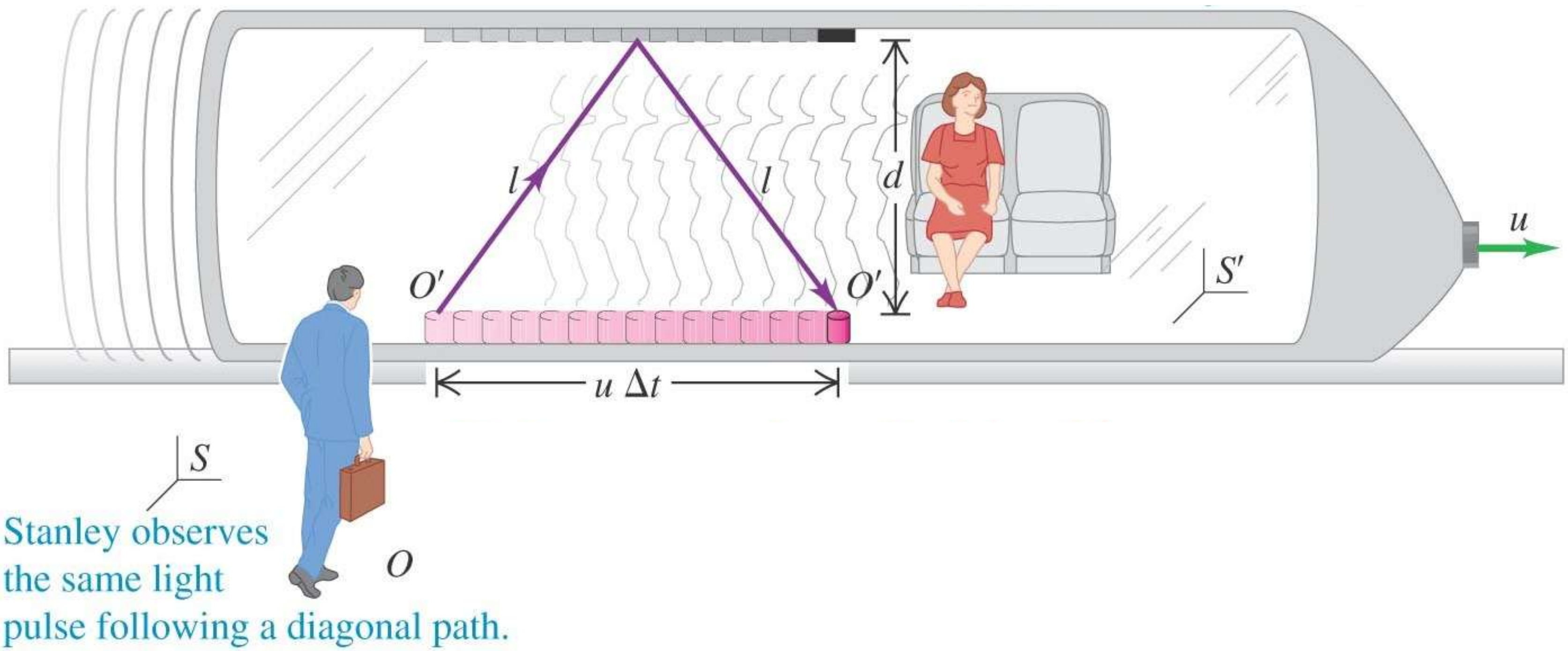
distance travelled by light = $2d$
speed of light = c

According to Mavis O',

$$\Delta t_0 = \frac{2d}{c} \qquad \text{(measured in } S'\text{)}$$

Relativity of Time Intervals

Now, consider the observation from Stanley's S – frame (stationary frame),



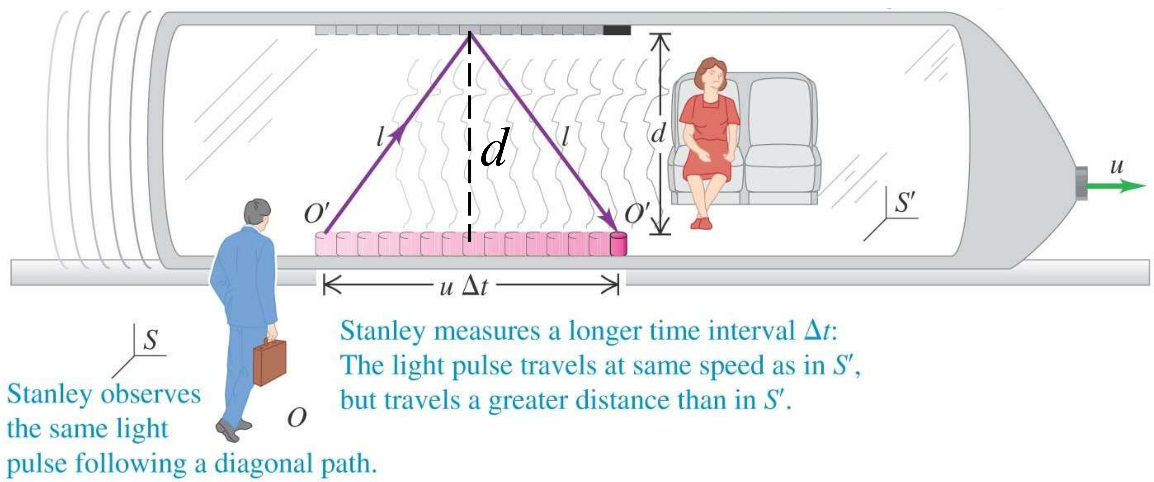
Note: speed of light is still c in this frame but Stanley will see it travels on a *longer* path!

Relativity of Time Intervals

Thus, if Δt is the time between the bounces of the laser light in S – frame

$\Delta t = 2l/c$ (light travels at the same speed c in S !)

then it must be *longer* than Δt_0



From the given geometry, we can explicitly calculate Δt:

$$d^2 + \left(\frac{u\Delta t}{2}\right)^2 = l^2 = \left(\frac{c\Delta t}{2}\right)^2$$

$$d^2 = \left(\frac{c\Delta t}{2}\right)^2 - \left(\frac{u\Delta t}{2}\right)^2 = \left(c^2 - u^2\right)\frac{\Delta t^2}{4}$$

$$\Delta t^2 = \frac{4d^2}{\left(c^2 - u^2\right)}$$

$$\Delta t = \frac{2d}{c\sqrt{1 - u^2/c^2}}$$

$$\Delta t = \gamma\, \Delta t_0$$

where $\gamma = 1\big/\sqrt{1 - u^2/c^2} \quad > 1$

Time Dilation

$$\Delta t = \gamma \Delta t_0 = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$$

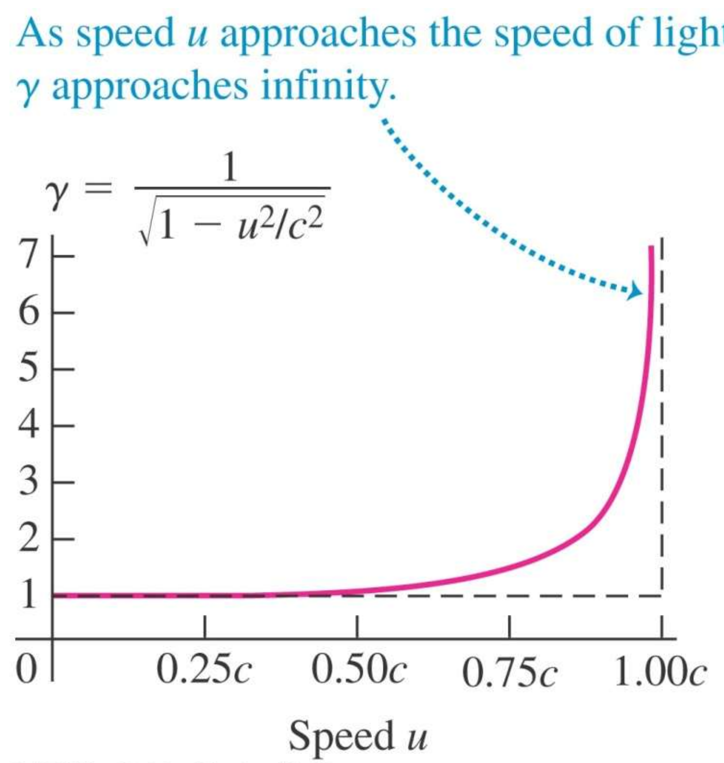
This is the **time dilation** formula in SR.

(Note: both observers S and S' will agree on this relationship between time intervals as long as they are both looking at the same clock in S'.)

Since u is strictly less than c,

$$\gamma = 1\Big/\sqrt{1-u^2/c^2} > 1$$

and $\Delta t > \Delta t_0$ always !



Proper Time

$$\Delta t = \gamma \, \Delta t_0$$

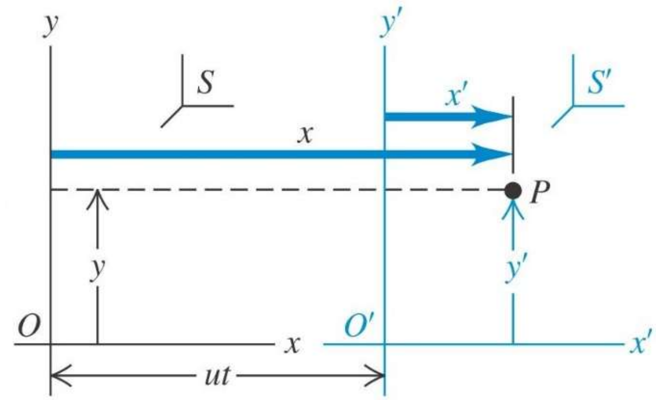
Δt_0 is called the **proper time** and it is a "special" (or "proper") time interval since it is the time interval of the clock measured by an observer *stationary* with respect to that clock, i.e., the two events (tic & toc) occur at the *same* location.

Δt is the measurement of this same pair of tic-toc events by another observer in *relative motion* with respect to the clock.

All observers have his/her own proper time and all other observers measuring other observer's clocks will *not* necessary be *proper*.

The **proper time** will always be the *shortest* time interval among all observers.

Notes on Subscript & Labeling



$$\Delta t \text{ and } \Delta t'$$

Δt - time interval measured by observer *S*

$\Delta t'$ - time interval measured by observer *S'*

$$\Delta t_{np} \text{ and } \Delta t_0 \quad \left(\text{in } \Delta t_{np} = \gamma\, \Delta t_0\right)$$

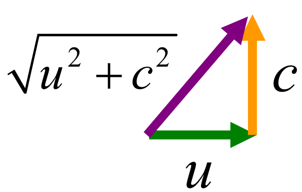
Δt_0 - **proper time** (time interval of the clock measured by an observer *stationary* with respect to that clock)

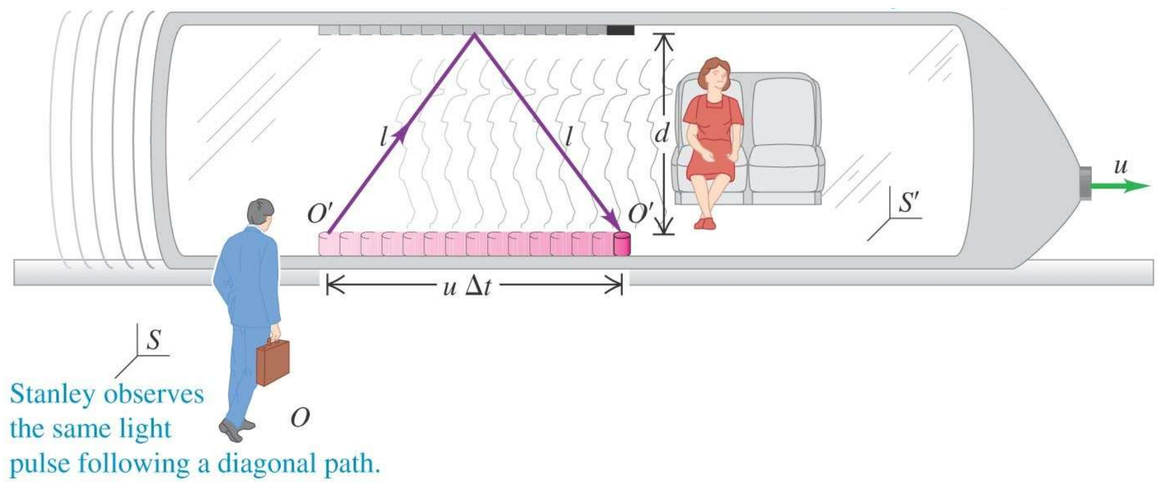
Δt_{np} - NOT proper time (time interval of the same clock above as measured by *other* observers not at rest with the clock)

Both Δt *and* $\Delta t'$ depending on whose clock (on boxcar or platform) we are measuring can either be Δt_0 *and* Δt_{np} .

Unreality Check

If speed of light changes according to Galilean Velocity Transformation, then





and, $\Delta t = 2l \big/ \sqrt{u^2+c^2}$

Following the same calculation as previously, we have

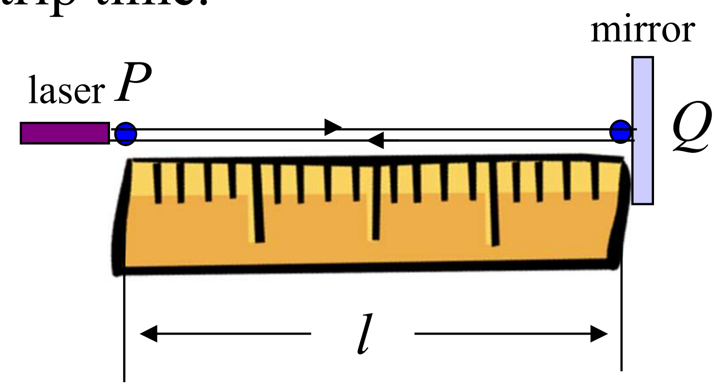
$$d^2 + \left(\frac{u\Delta t}{2}\right)^2 = l^2 = \left(\sqrt{u^2+c^2}\,\frac{\Delta t}{2}\right)^2$$

$$d^2 = \left(\cancel{u^2} + c^2\right)\left(\frac{\Delta t}{2}\right)^2 \cancel{-u^2\left(\frac{\Delta t}{2}\right)^2} = \frac{c^2 \Delta t^2}{4} \Rightarrow \Delta t^2 = \frac{4d^2}{c^2} = \Delta t_0^2$$

no time dilation: all inertial observers measure the same time interval

Relativity of Length

→ Distance between two points on a rigid body *P* & *Q* can be measured by a light signal's round-trip time.

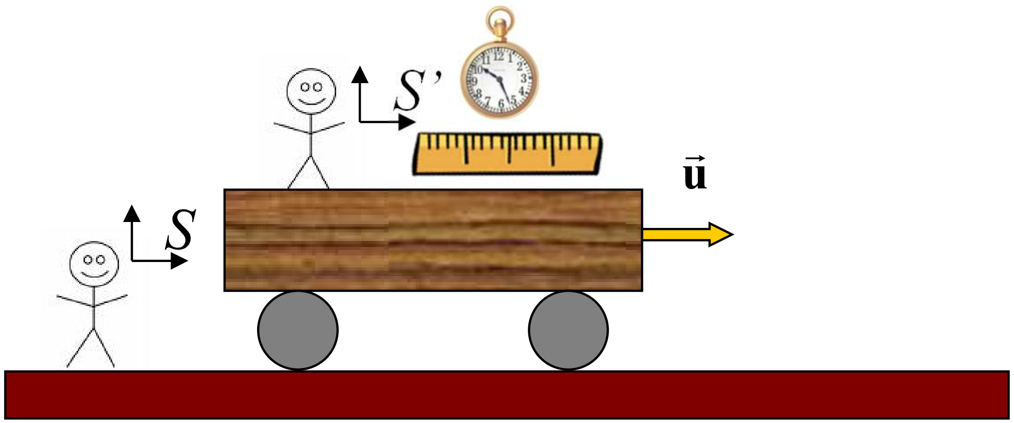


l can be measured by the time interval: $t_2 - t_1$, $2l = c(t_2 - t_1) \quad \rightarrow \quad l = \frac{c}{2}(t_2 - t_1)$

→ As we have seen, Δt will be different for different inertial observers, l will also !

Proper Length

Similar to the concept of **proper time** Δt_0 which is the measured time interval of a clock which is *at rest* with the observer, **proper length** l_0 is the measured length of an object *at rest* with the observer.

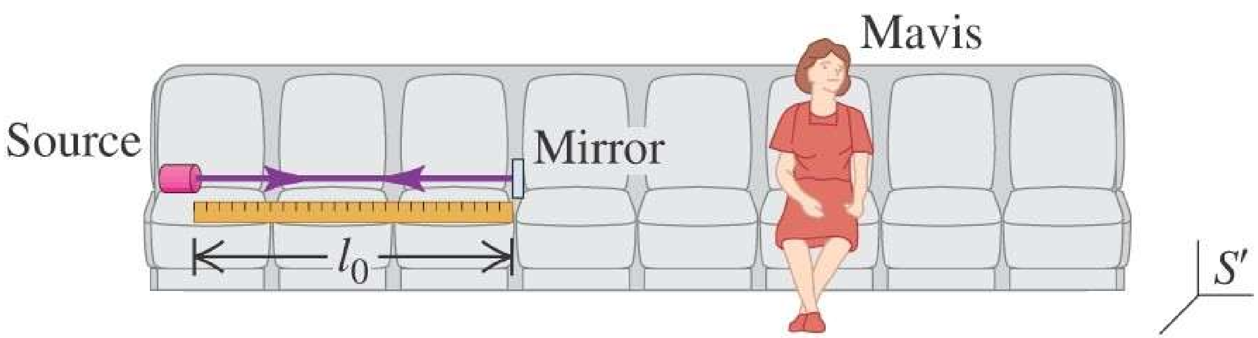


In this case, Observer *O'* in *S'*- frame will measure proper time and proper length for the clock and ruler shown.

Length Contraction (parallel to **u**)

Let consider a ruler *at rest* in the moving frame (*S*') and it lays *parallel* to the direction of the relative motion between *S* and *S*' (as shown previously).

Within *S*'- frame, the ruler is at rest with Mavis.



The length of the ruler is measured using a light-clock by measuring the time interval between *two events* (light leaving the laser and light arrives back to the source). In this measurement process, light pulse travels a distance of a total of $2l_0$ within a time interval of Δt_0.

Length Contraction (parallel to **u**)

Since both measurement events are *at rest* (same location) within *S*'-frame, Δt_0 is the *proper time* measurement and we have,

$$\Delta t_0 = \frac{2l_0}{c}$$

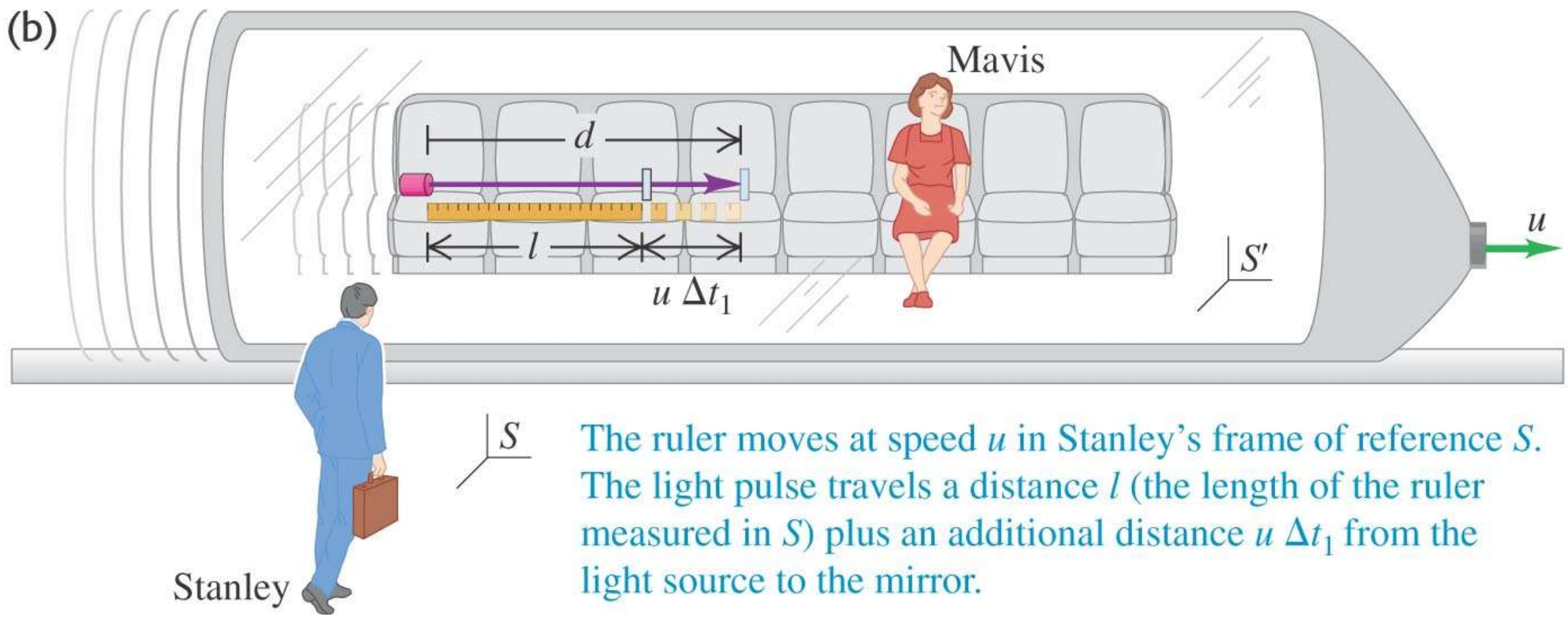
and l_0 is the proper length of the ruler.

Now, let consider the description according to Stanley in *S*-frame.

The ruler according to Stanley will have a length of l and let the time of travel for the light pulse from the source to the mirror be Δt_1.

(note: Since Stanley is measuring these from afar and he is in relative motion with respect to Mavis, his measurement of Δt_1 will NOT be proper.)

Length Contraction (parallel to **u**)



Within the time interval Δt_1, the mirror will have moved a distance of $u\,\Delta t_1$ Stanley has to take that into account in his measurement of the length l and his measurement of the distance that the laser light has to travel is,

$$d = l + u\Delta t_1$$

Since the speed of light is also c in Stanley frame, we can also write,

$$d = c\Delta t_1$$

Length Contraction (parallel to **u**)

Combing these two equations and eliminating *d*, we have

$$c\Delta t_1 = l + u\Delta t_1$$

$$\Delta t_1 = \frac{l}{c-u}$$

Length Contraction (parallel to **u**)

Combing these two equations by eliminating d, we have

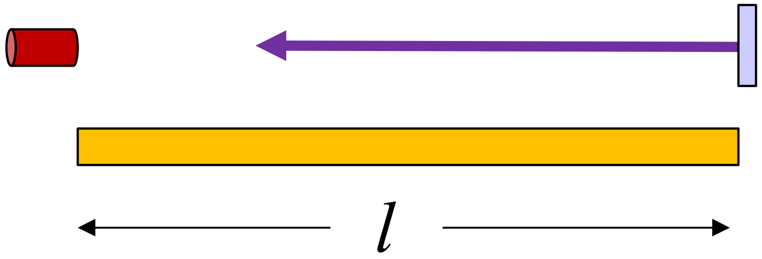
$$c\Delta t_1 = l + u\Delta t_1$$

$$\Delta t_1 = \frac{l}{c-u}$$

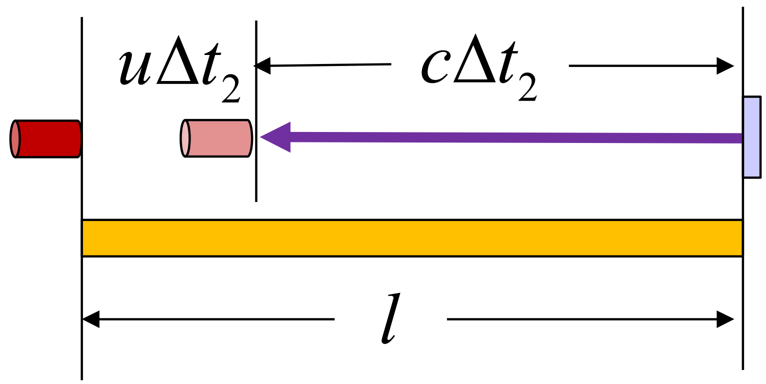
Now, let consider the return trip of the laser light…

Let Δt_2 be the time measured by Stanley for the light to travel back from the mirror back to the source.

Note that as the train moves forward, the source also moves forward to meet the laser light



Length Contraction (parallel to **u**)



Analogous to the outgoing trip, the return trip of the laser light will be *shortened* by $u\Delta t_2$ and

$$c\Delta t_2 = l - u\Delta t_2 \quad \rightarrow \quad \Delta t_2 = \frac{l}{c+u}$$

The total travel time for the laser pulse is $\Delta t = \Delta t_1 + \Delta t_2$

$$\Delta t = \frac{l}{c-u} + \frac{l}{c+u} = \frac{l(c \cancel{+u} + c \cancel{-u})}{c^2 - u^2} = \frac{2lc}{c^2\left(1-u^2/c^2\right)} = \frac{2l}{c\left(1-u^2/c^2\right)}$$

Length Contraction (parallel to **u**)

From the time dilation formula, we also have

$$\Delta t = \gamma \, \Delta t_0$$

Combing these two equations for Δt, we have

$$\frac{2l}{c\left(1-u^2/c^2\right)} = \frac{1}{\left(1-u^2/c^2\right)^{1/2}} \frac{2l_0}{c}$$

$$\frac{l}{\left(1-u^2/c^2\right)^{1/2}} = l_0$$

or $$l = \frac{l_0}{\gamma}$$ (length contraction)

Recall: The two timing events for the length measurement are *at rest* in S'-frame so that Mavis measure the *proper time* Δt_0 and *proper length* l_0.

On the other hand, Stanley's measurements (Δt and l) of the same two events are *not proper*.

Note: The proper length l_0 is always the *longest* among all inertial observers.

Unreality Check

According to Galilean Velocity Transformation, in *S*-frame, we have

On the out going trip, $c_1 = c + u$ so that $\Delta t_1 = \dfrac{l}{c_1 - u} = \dfrac{l}{c + u - u} = \dfrac{l}{c}$

On the returning trip, $c_2 = c - u$ so that $\Delta t_2 = \dfrac{l}{c_2 + u} = \dfrac{l}{c - u + u} = \dfrac{l}{c}$

Now, the total time for the whole trip is now,

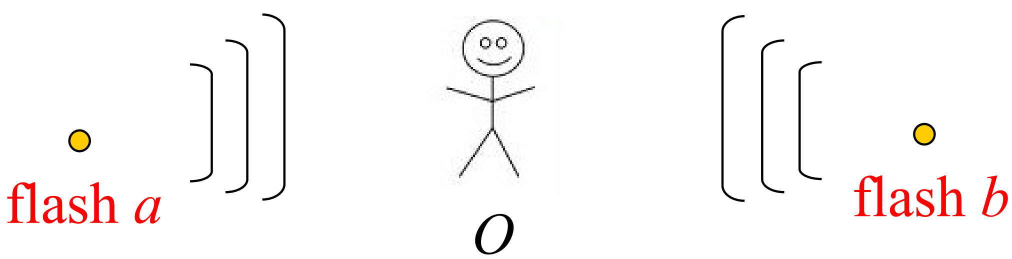
$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{2l}{c}$$

From our previous "unreality check", $\Delta t = \Delta t_0$ (all clocks clicks at the same rate)

$$\Rightarrow \quad \Delta t = \frac{2l}{c} \Leftrightarrow \frac{2l_0}{c} = \Delta t_0 \quad \rightarrow \quad l = l_o$$

Relativity of Simultaneity

Simultaneity

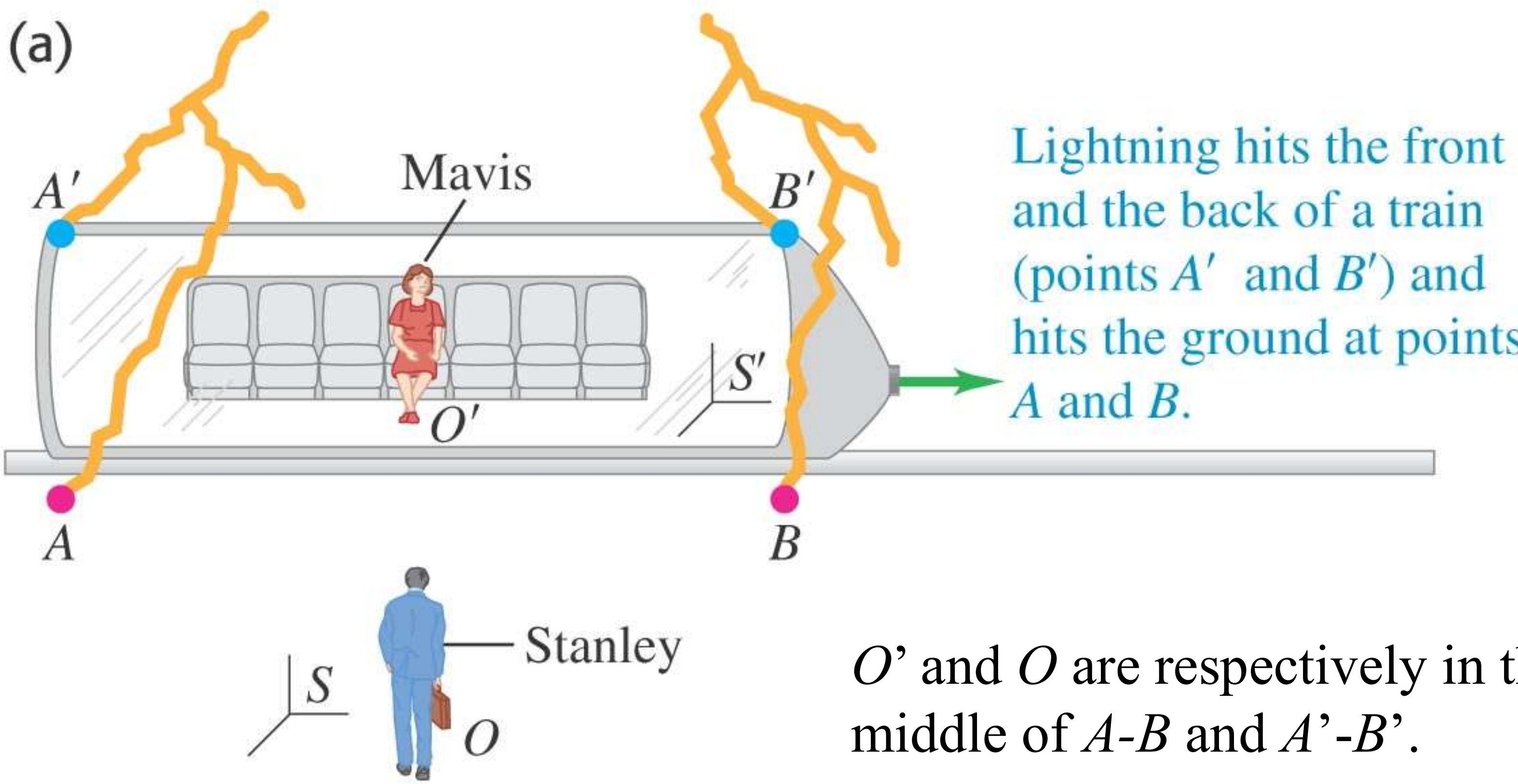


Definition: Two flashes (event *a* and *b*) are considered to be simultaneous with respect to observer *O* if light from *a* & *b* (equal distance to observer *O* in the middle) arrive at the observer at the same time.

Relativity of Simultaneity

We will analyze the situation in Stanley's *S*-frame in the following slides.

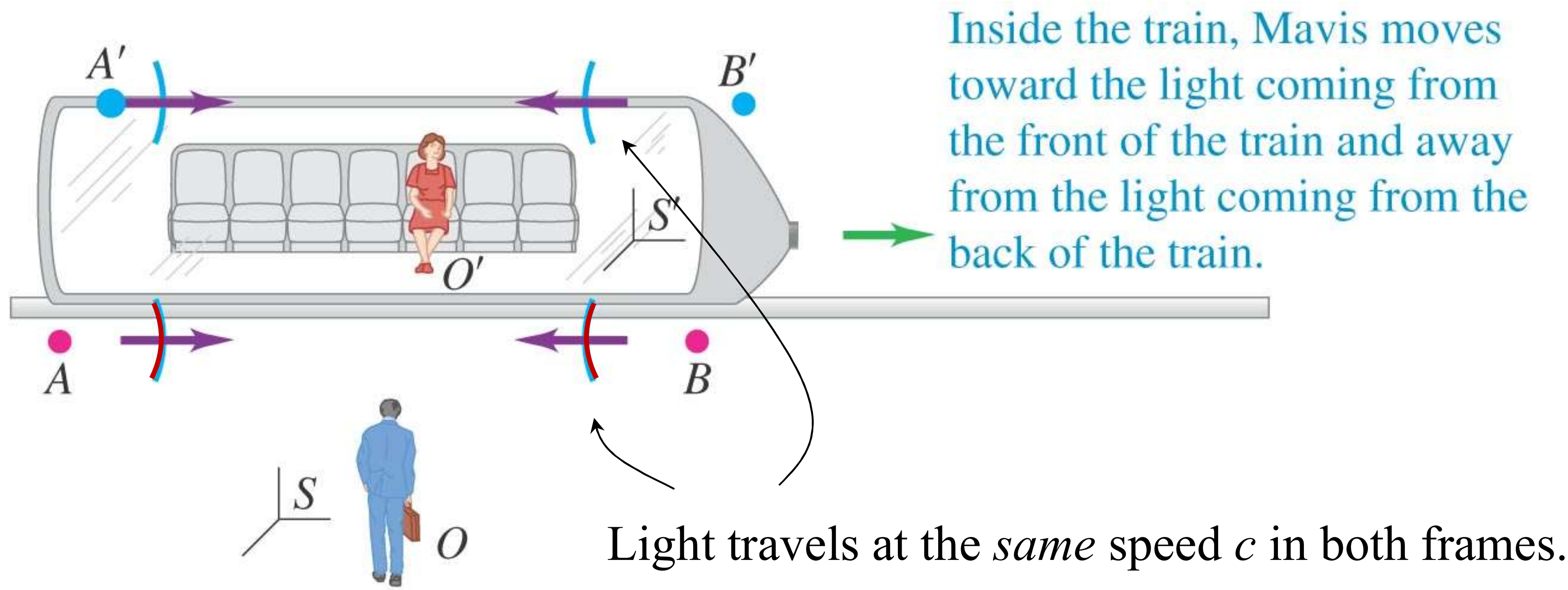
At time *t* = 0:



O' and *O* are respectively in the middle of *A*-*B* and *A*'-*B*'.

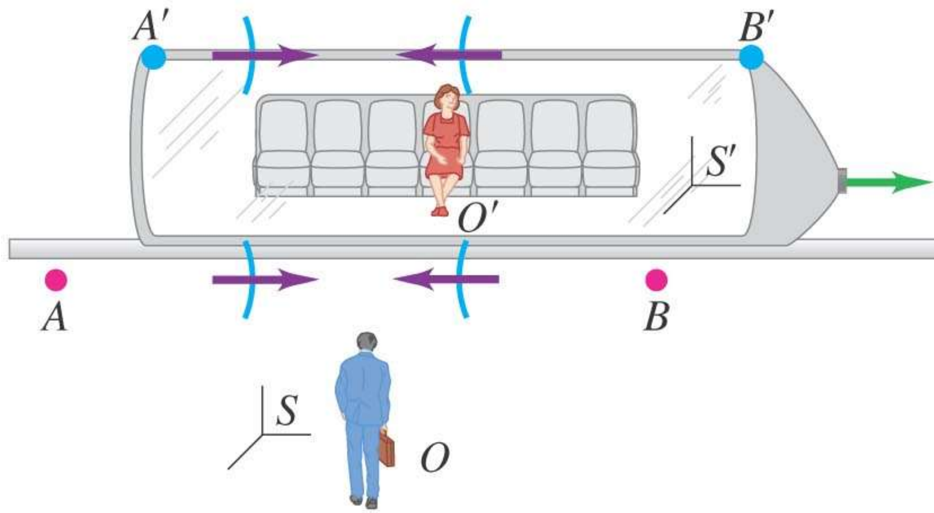
Relativity of Simultaneity

On the ground (*S*-frame) after some time *t*,

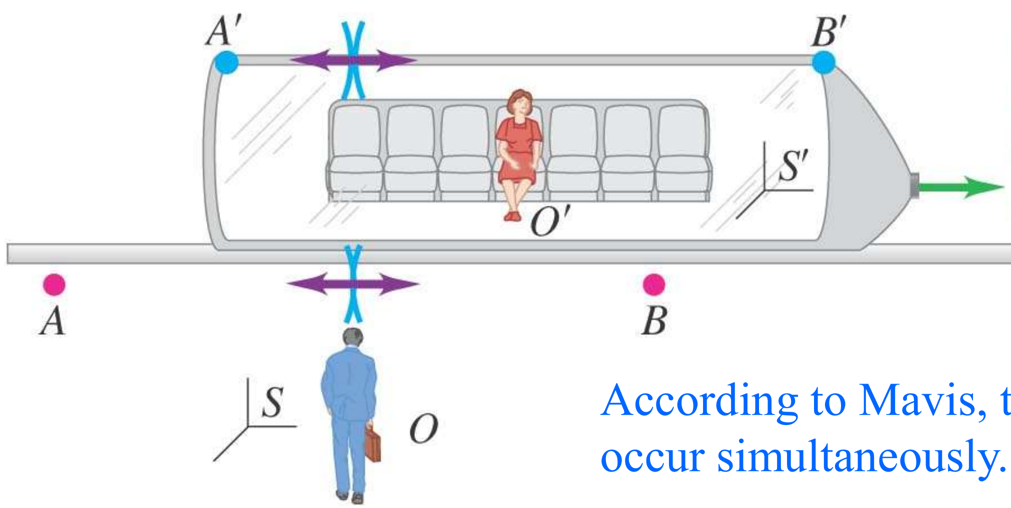


Relativity of Simultaneity

On the ground (*S*-frame) after some more time *t*,



Because Mavis sees the light at the front of the train first, she believes that the lightning at the front of the train hit first.



Then, a bit later, when Stanley see the lights from *A* and *B* reach him *simultaneously*, Mavis in *S*'- frame still waiting for the light from the lighting strike at the rear of the train to reach her.

According to Mavis, the lighting strikes at *A*' and *B*' did not occur simultaneously. Stanley agrees with Mavis' observation.

Unreality Check

In the (incorrect) Galilean view, *according to Stanley*, $c_{A'}$ and $c_{B'}$ inside the train will be modified according to their relative speed u.

So that light signals from *A*' will speed up and signal from *B*' will slow down in Stanley's viewpoint.

$$\left.\begin{aligned} c_{A'} &= c + u \\ c_{B'} &= c - u \end{aligned}\right\}$$

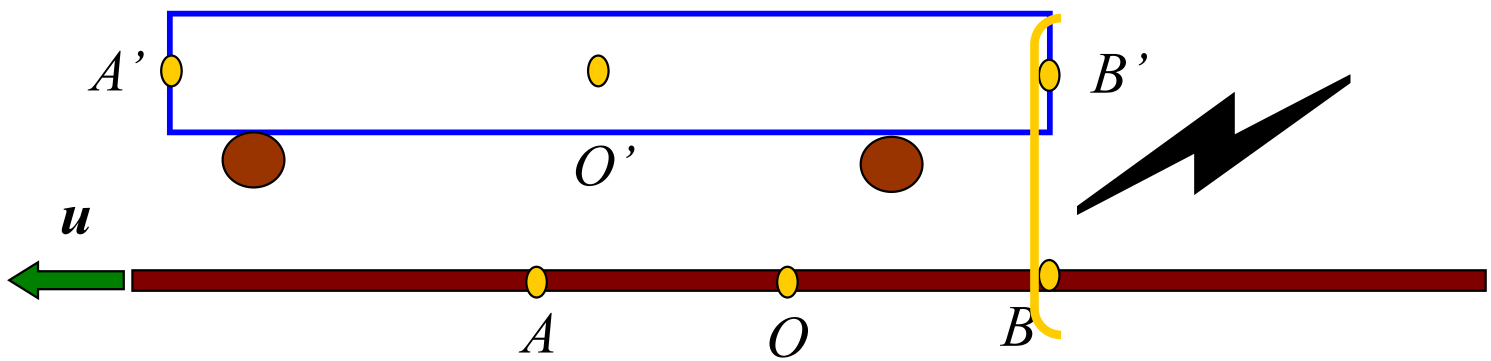
Speed of light from *A*' and *B*' will be modified by the box car's speed u.

This changes in speed for c will result in signals from *A*' and *B*' to arrive at *O*' at the same time so that Stanley will say that *both* Marvis and himself agree that the lighting strikes simultaneously.

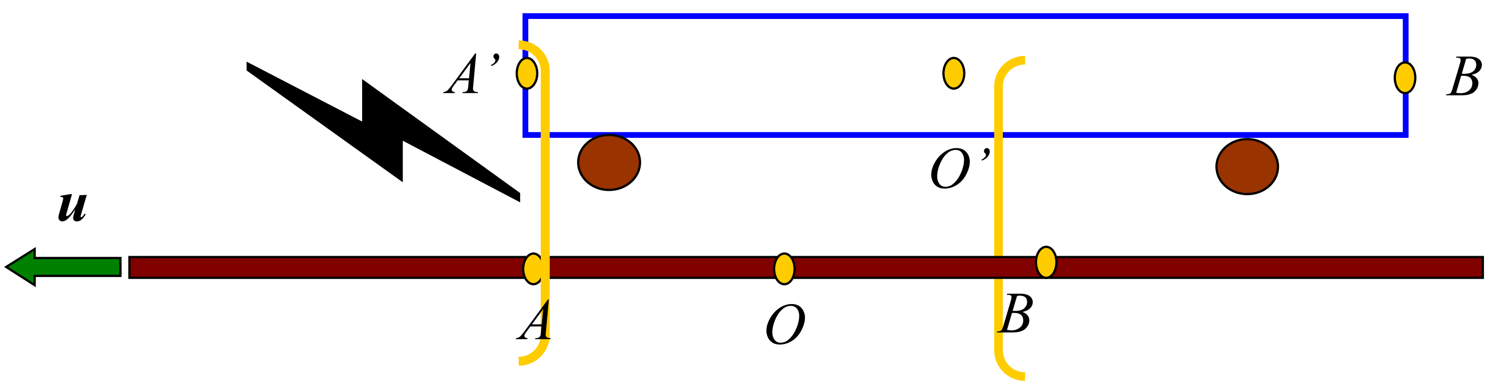
Sequence of Events according to Mavis (S'- frame) [box car *stationary*]

At $t' = 0$, lightning strikes on B and B':

Note:
1. platform is length contracted and the box car's rest length is longer.
2. Speed of light is the same in both frame according to O'.

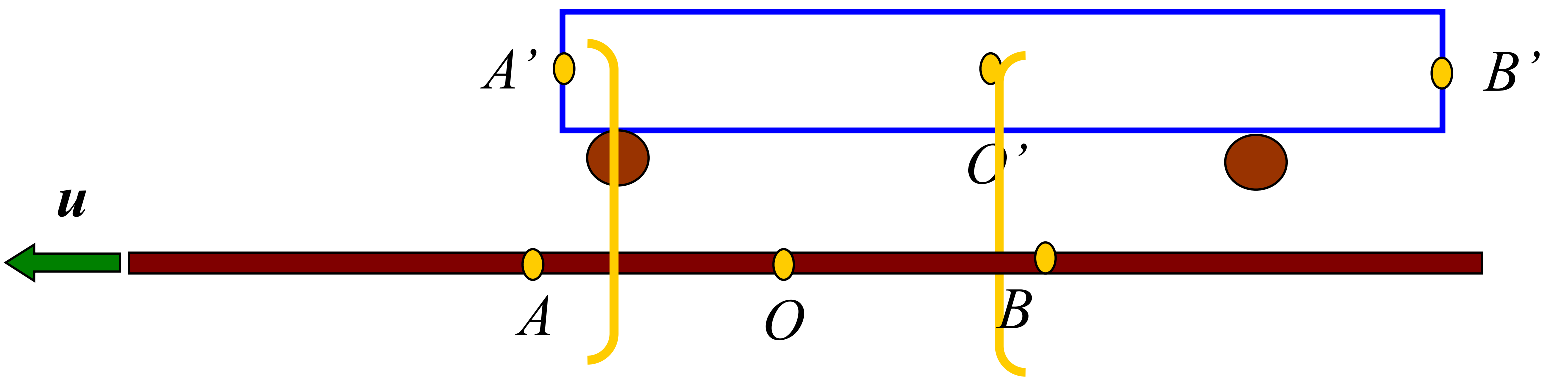


Some time later, lightning strikes on A and A':



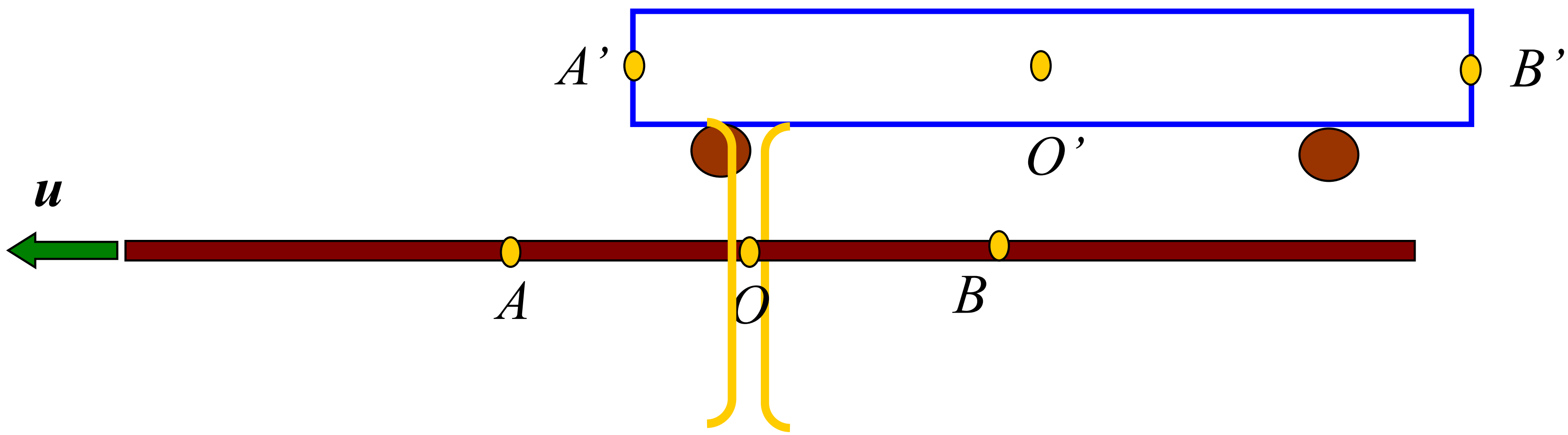
Sequence of Events according to Mavis (*S*'- frame)

Some time later, the light pulses reach *O'*



Sequence of Events according to Mavis (*S*'- frame)

Some more time later, the light pulses reach *O* at the same time but light pulse from A' still has not reach O' yet.

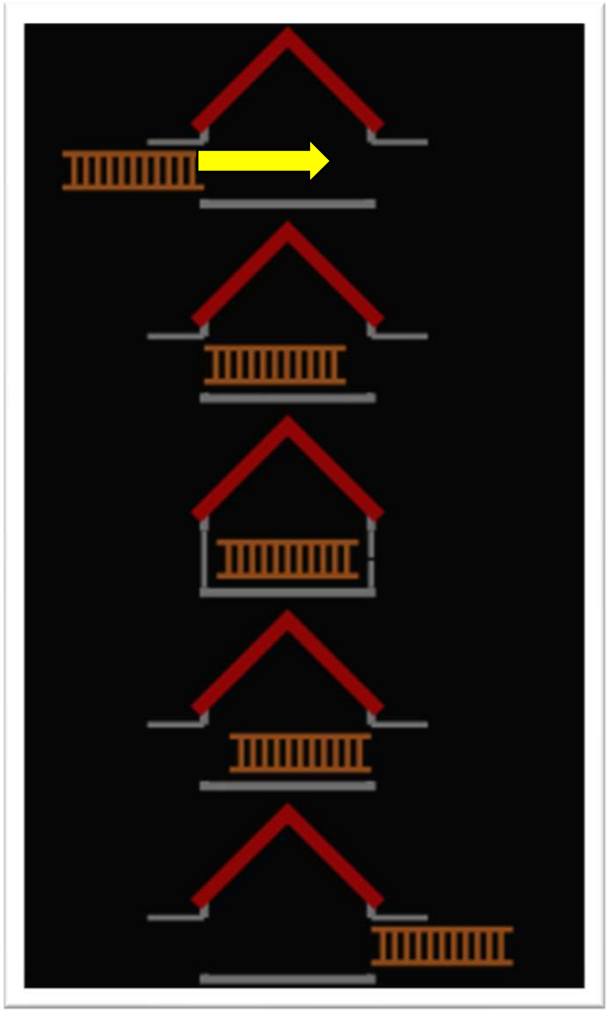


Mavis see the flashes at *different* time but agrees with Stanley that the light signals in Stanley's frame arrive at him at the same time. Both see the same sequence of events but the notion of simultaneity is relative.

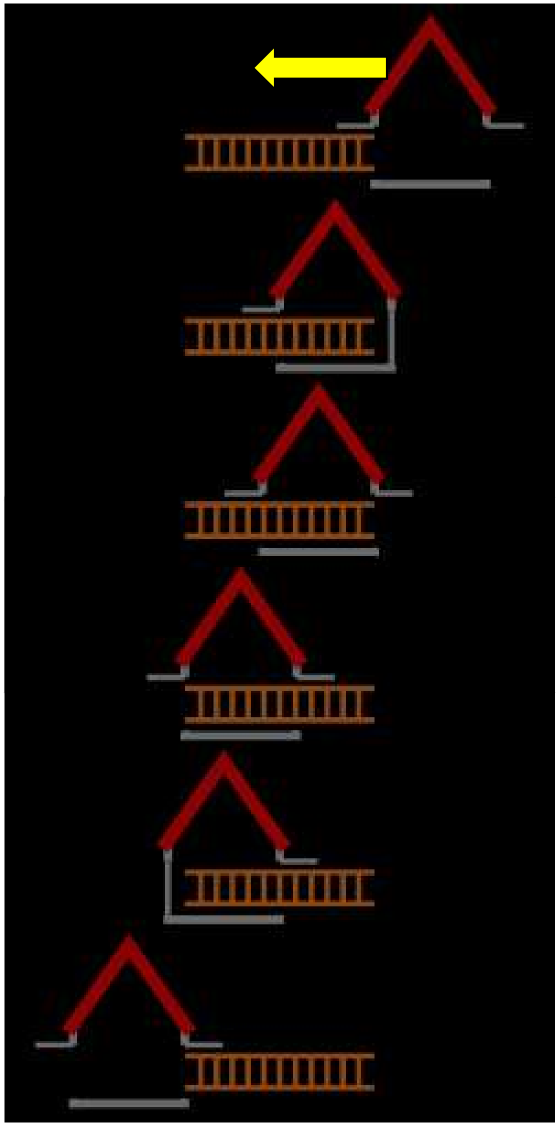
Ladder | Barn Paradox

Pictures from wikipedia

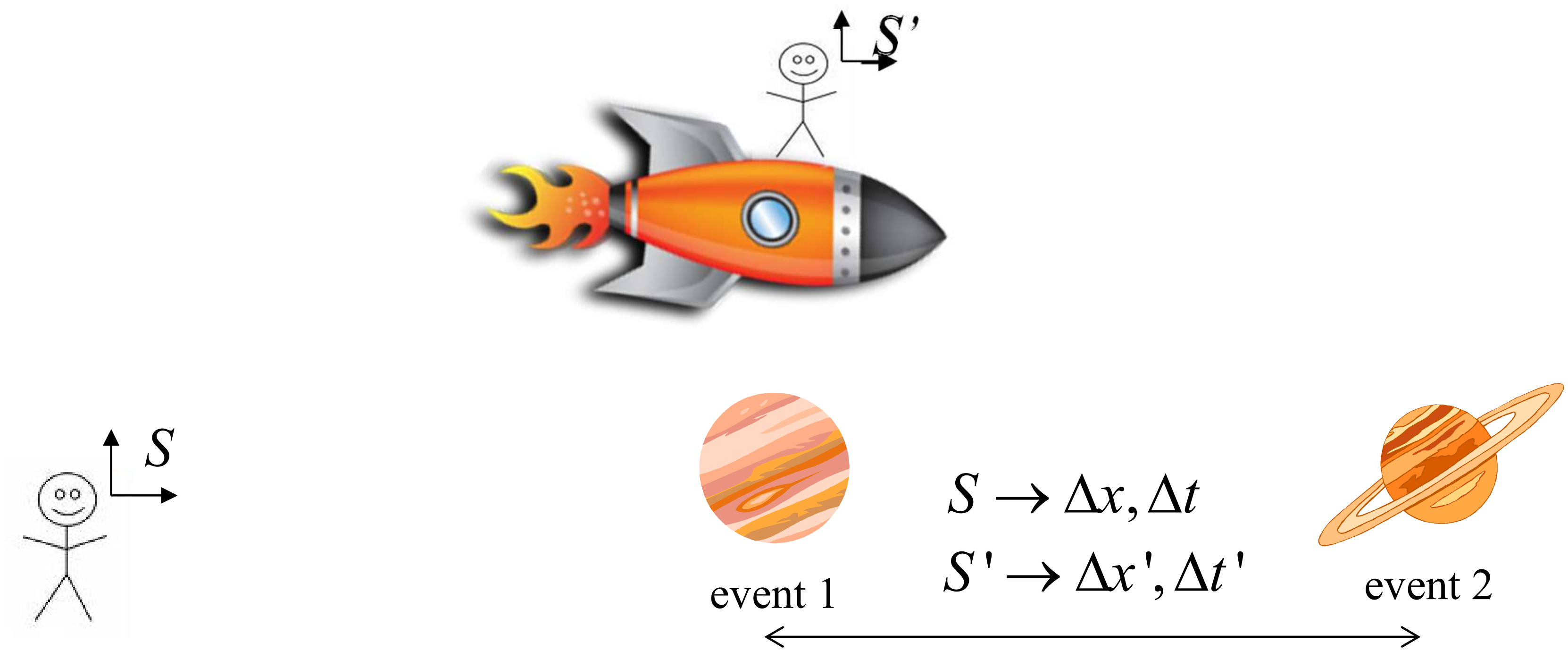
Barn's Frame



Ladder's Frame



Relative Speed of a Spaceship



$\Delta x \rightarrow proper$

$\Delta x' \rightarrow$ contracted

$\Delta x' = \Delta x / \gamma$

$\Delta t \rightarrow dilated$

$\Delta t' \rightarrow proper$

$\Delta t = \gamma \Delta t'$

$$\Rightarrow \quad u' = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x / \gamma}{\Delta t / \gamma} = \frac{\Delta x}{\Delta t} = u$$