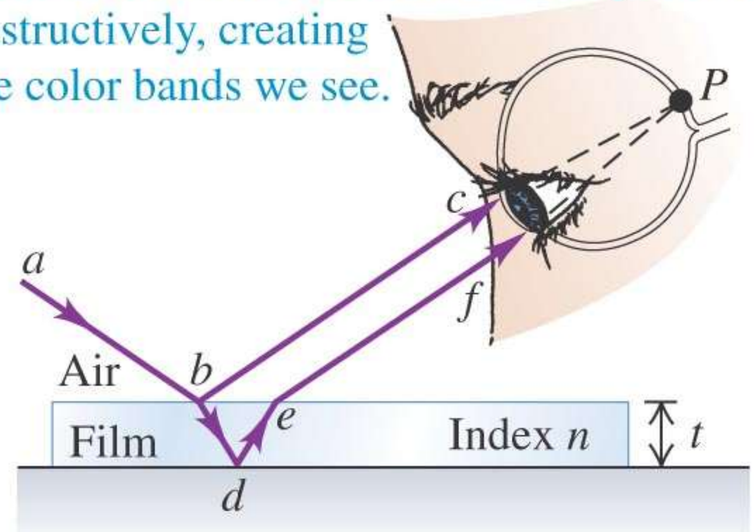
# Interference in Thin Films

Color fringes observed from an oil slick on water or on a soap bubble are the white-light *interference* patterns produced by the *reflected* light off a *thin film* of oil or soap.



Light reflected from the upper and lower surfaces of the film comes together in the eye at $P$ and undergoes interference.

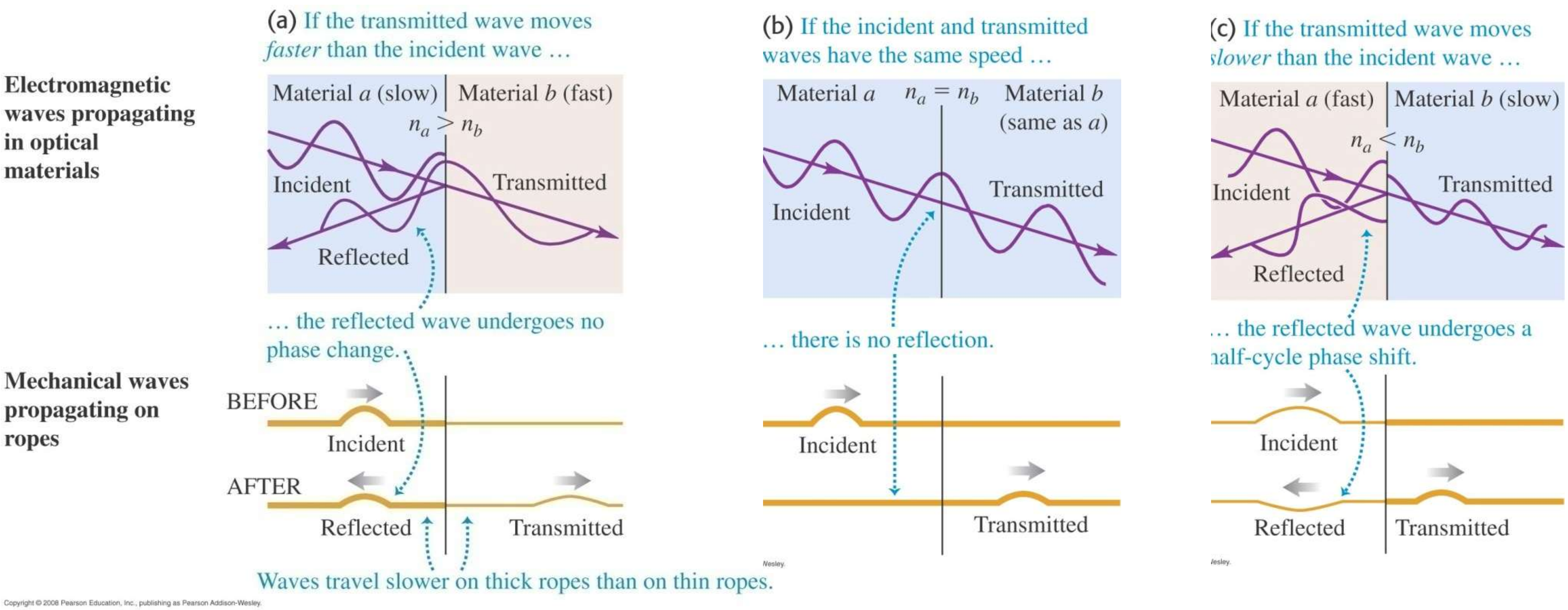
Some colors interfere constructively and others destructively, creating the color bands we see.



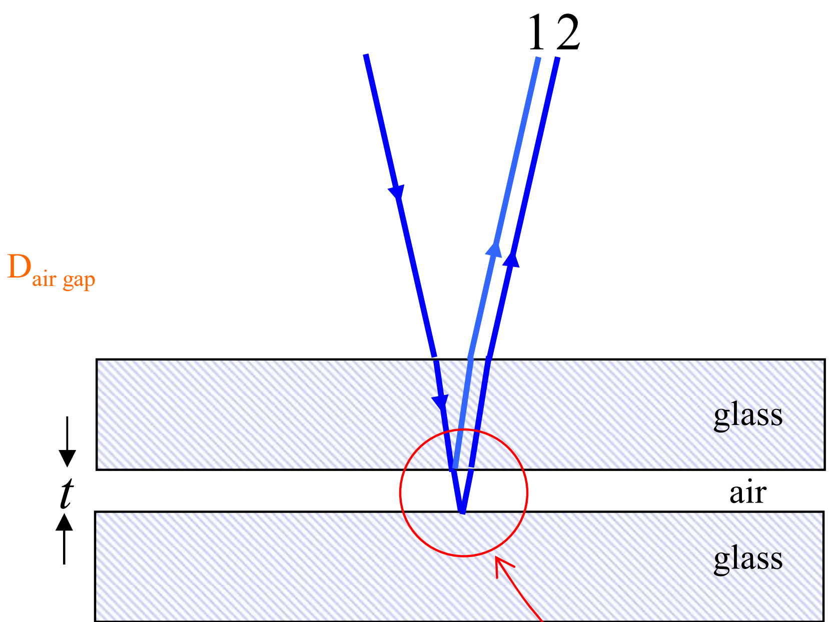
# Phase Shifts During Reflection

From Maxwell's Equations, one can show that the reflected wave will suffer a 180° or $\lambda/2$ phase shift if it is reflected off from a medium with a *larger n*.

$$E_r = \frac{n_a - n_b}{n_a + n_b} E_i \qquad \text{(for normal incidence)}$$



# Interference from a Thin Air Gap




**Assumptions**:
- Thickness of air gap $t$ is small
- Thickness of glass is large
- Incident light is nearly *normal* at the upper plate.

Interference due to this small gap

wave #1: reflected from top interface of air gap:

$$n_{glass} > n_{air} \quad \Rightarrow \quad \text{no phase shift}$$

wave #2: reflected from bottom interface of air gap:

$$n_{air} < n_{glass} \quad \Rightarrow \quad 180^{\circ} \ (\text{or } \pi) \text{ phase shift}$$

# Interference from a Thin Air Gap

Now, consider the conditions for interference:

Constructive: wave #1 and wave #2 upon reflection must have a *net* phase difference of multiples of $2\pi$, i.e.,

$$m(2\pi), \quad m = 0, 1, 2, \cdots$$

- wave #1: suffers no phase shift during reflection
- wave #2: acquires a $\pi$ ($180^o$) phase shift during reflection and it also gains additional phase shift due to path difference = $2t$ in the air gap.

So, the *net* phase diff accumulated between wave #1 and #2 = $\pi + \frac{2t}{\lambda}(2\pi)$

For const interf, $\Longrightarrow$

$$\frac{2t}{\lambda}(2\pi) + \pi = m(2\pi), \quad m = 1, 2, 3, \cdots$$

$$= 0, 2\pi, 4\pi, \cdots$$

$$\frac{2t}{\lambda}(2\pi) = \cancel{\pi}, \pi, 3\pi, \cdots$$

# Interference from a Thin Air Gap

Solving for only positive $t$'s, we have $2t = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \cdots$

Rewriting, $$2t = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \cdots$$ (condition for Con. Int. from thin film where one of the waves suffers a phase shift)

Destructive: wave #1 and wave #2 upon reflection must have a *net* phase difference of

$$\left(m + 1/2\right)(2\pi), \quad m = 0, 1, 2, \cdots$$

Again, the net phase diff accumulated between wave #1 and #2 = $\pi + \frac{2t}{\lambda}(2\pi)$

$$\frac{2t}{\lambda}(2\pi) + \pi = \left(m + \frac{1}{2}\right)(2\pi), \quad m = 0, 1, 2, \cdots$$
$$= \pi, 3\pi, 5\pi, \cdots$$

# Interference from a Thin Air Gap

Rearranging the equation, we have,

$$\frac{2t}{\lambda}(2\pi)+\pi=\pi,3\pi,5\pi\cdots$$

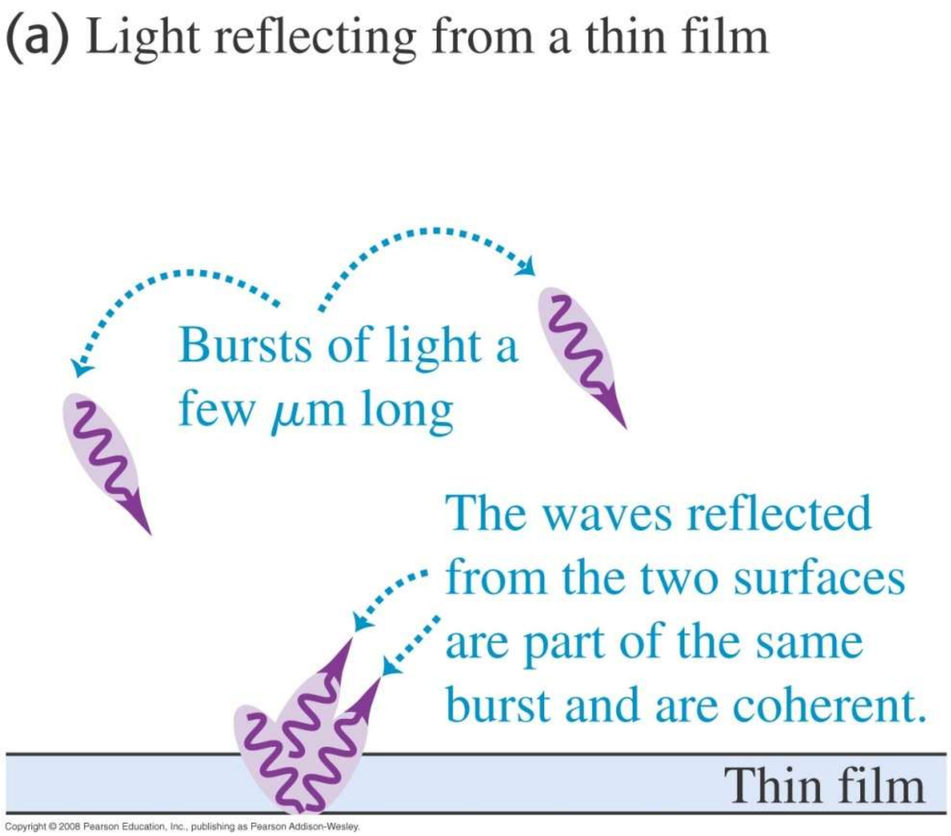
$$\frac{2t}{\lambda}(2\pi)=0,2\pi,4\pi$$

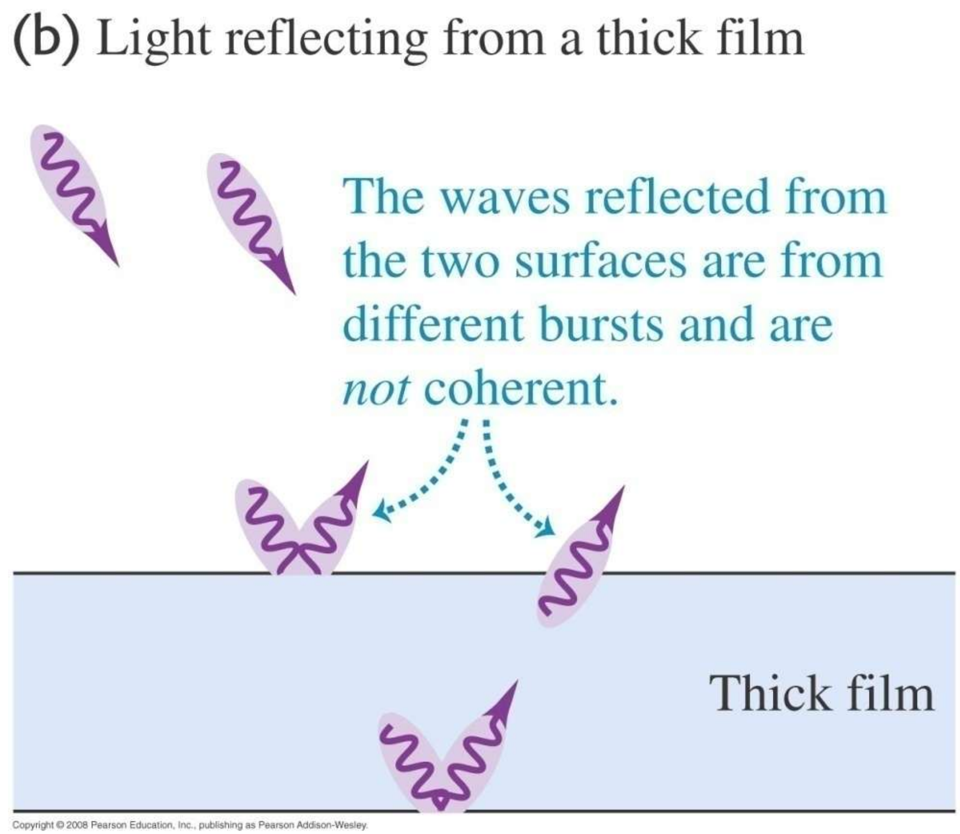
Finally, we have $2t=m\lambda,\quad m=0,1,2,\cdots$ (condition for Des. Int. from thin film where one of the waves suffers a phase shift)

# Thin File Interference Demo

https://youtu.be/s8vLq2HsrHM

# Thin and Thick Films

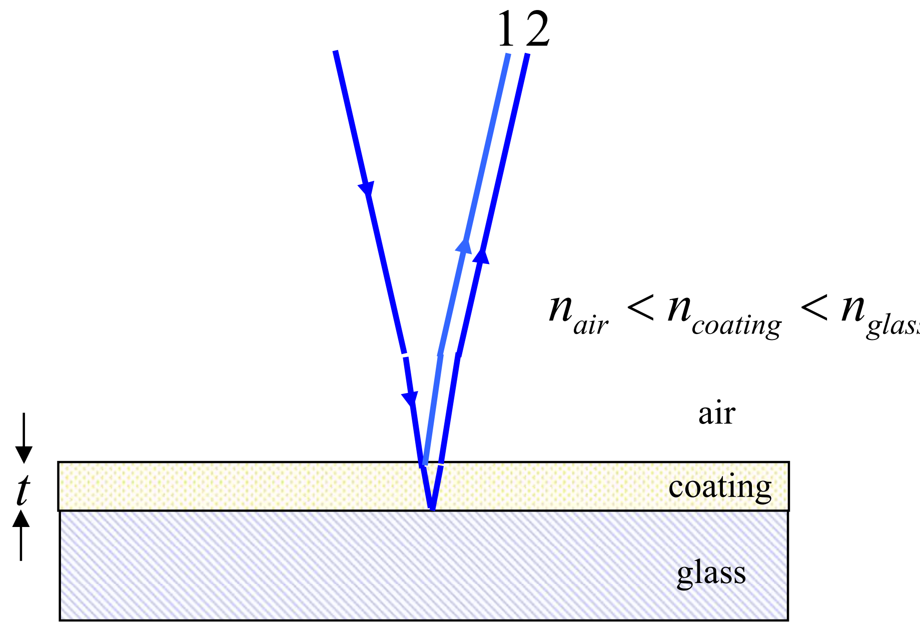



Interference effects can be observed

Interference effects are difficult to observe

# Another Thin Film Example (non-refractive coating on lens)


wave #1: reflected from top interface of the coating:

$$n_{air} < n_{coating} \quad \Rightarrow \quad 180^{o} \text{ (or } \pi\text{) phase shift}$$

wave #2: reflected from bottom interface of the coating:

$$n_{coating} < n_{glass} \quad \Rightarrow \quad 180^{o} \text{ (or } \pi\text{) phase shift}$$

# Interference from a Thin Film

Since both wave #1 and #2 suffers the same phase shift upon reflection, the *net* phase difference will be from the path difference ($2t$) only. So, we have the standard condition (net phase diff. due to *path diff.* only),

There is one more consideration: the path difference is accumulated in a medium with $n_{coating}$ so that the relevant wavelength should be $\lambda_n = \lambda / n_{coating}$.

Constructive:

$$2t = m\lambda_n, \quad m = 0, 1, 2, \cdots$$

$$2n_{coating}t = m\lambda, \quad m = 0, 1, 2, \cdots$$

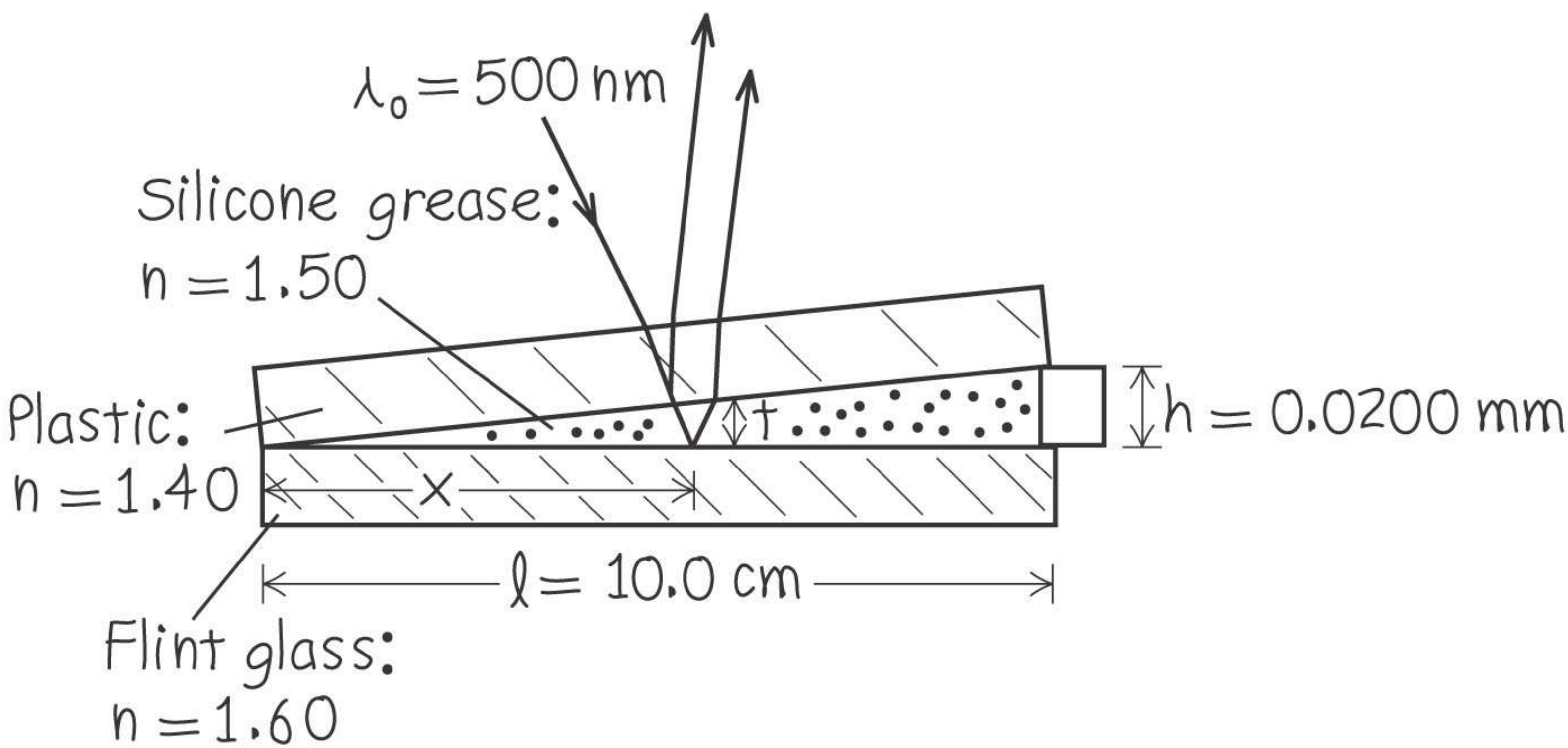
Destructive:

$$2t = \left(m + \frac{1}{2}\right)\lambda_n, \quad m = 0, 1, 2, \cdots$$

$$2n_{coating}t = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \cdots$$

Note: In addition to wavelength modification, the RHS dependence are switched with respect to the air gap case.

# Example 35.4: Thin Film





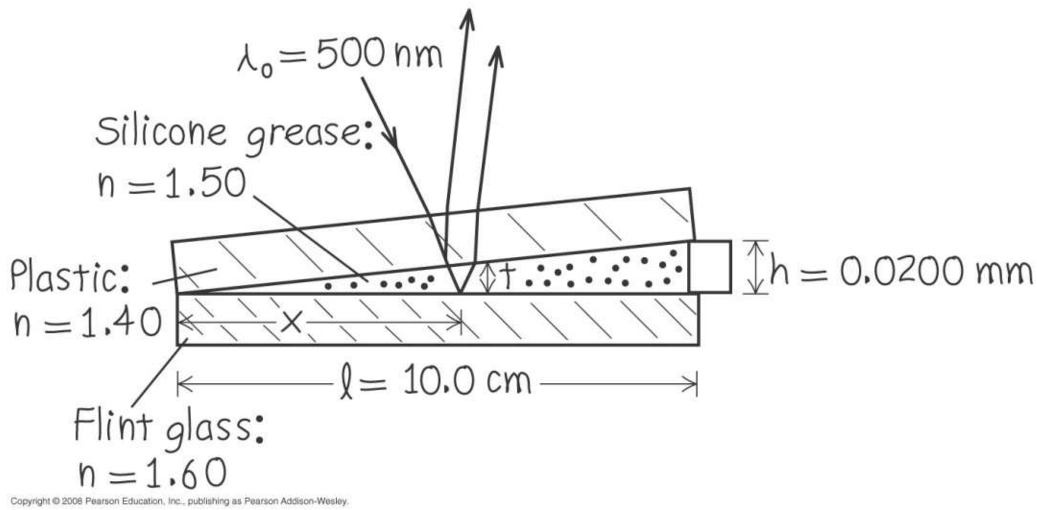

Question:

a. Will there be a bright or dark fringe close to the point of contact?

b. What is the distance *x* to the next bright fringe?

# Example 35.4

Since $n_{plastic} < n_{silicone}$ and $n_{silicone} < n_{glass}$




*Both* wave #1 and #2 suffer a phase shift.

So, close to the point of contact ($t \simeq 0$), the reflected wave #1 and #2 will arrive at the eyes *in phase* (bright fringe).

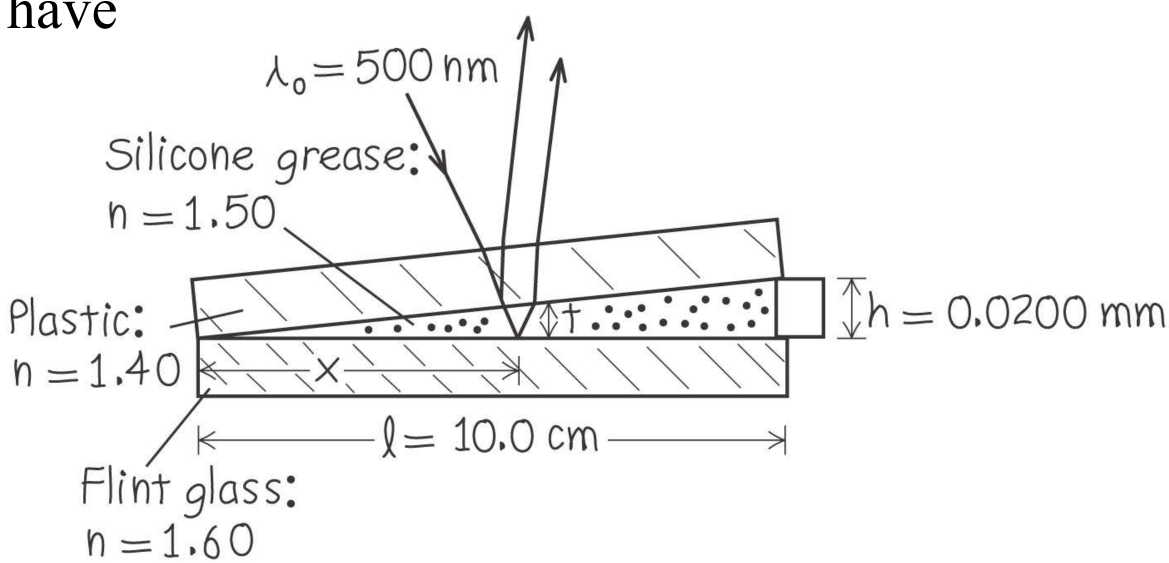
To find the location of the next constructive interference, we use,

$$2n_{silicone}t = \lambda_0 \ (m = 1)$$

# Example 35.4

From the two similar triangles, we have

$$\frac{h}{l} = \frac{t}{x} \quad \rightarrow \quad t = \frac{hx}{l}$$





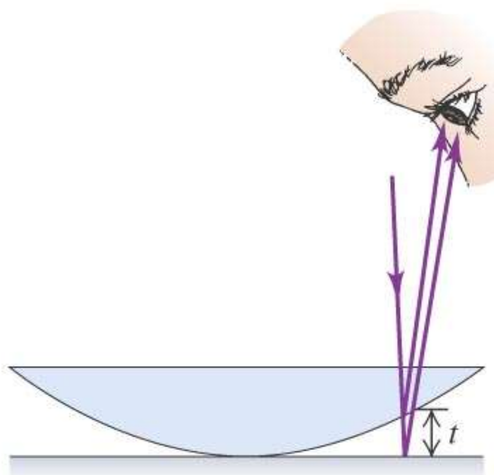
Substitute $t$ into the previous eq,

$$2n_{silicone}t = 2n_{silicone}\frac{hx}{l} = \lambda_0$$

$$x = \frac{l\lambda_0}{2n_{silicone}h} = \frac{(100mm)(500\times10^{-6}\,mm)}{2(1.50)(0.0200mm)} = 0.833mm$$

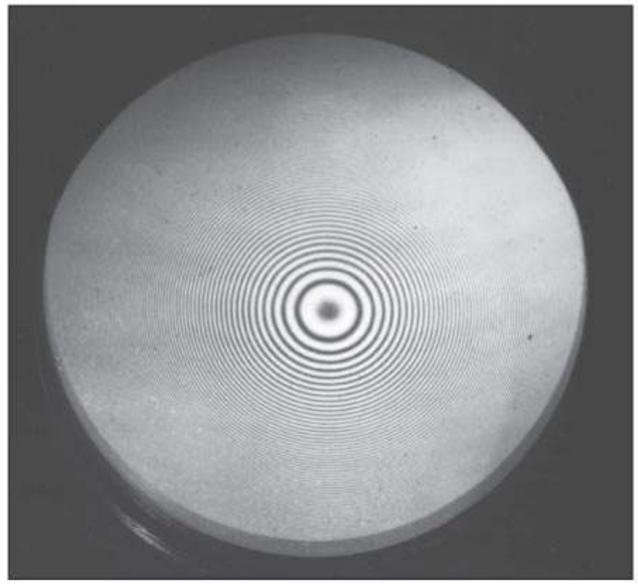
# Newton’s Rings

(a) A convex lens in contact with a glass plane

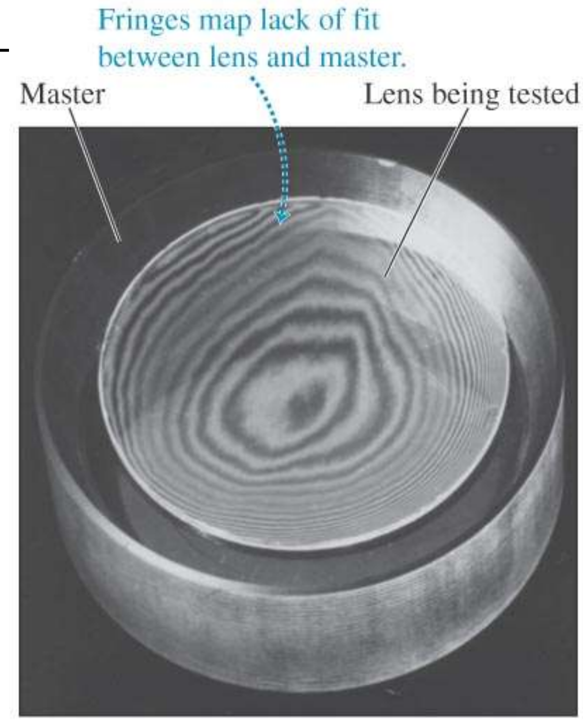





(b) Newton’s rings: circular interference fringes





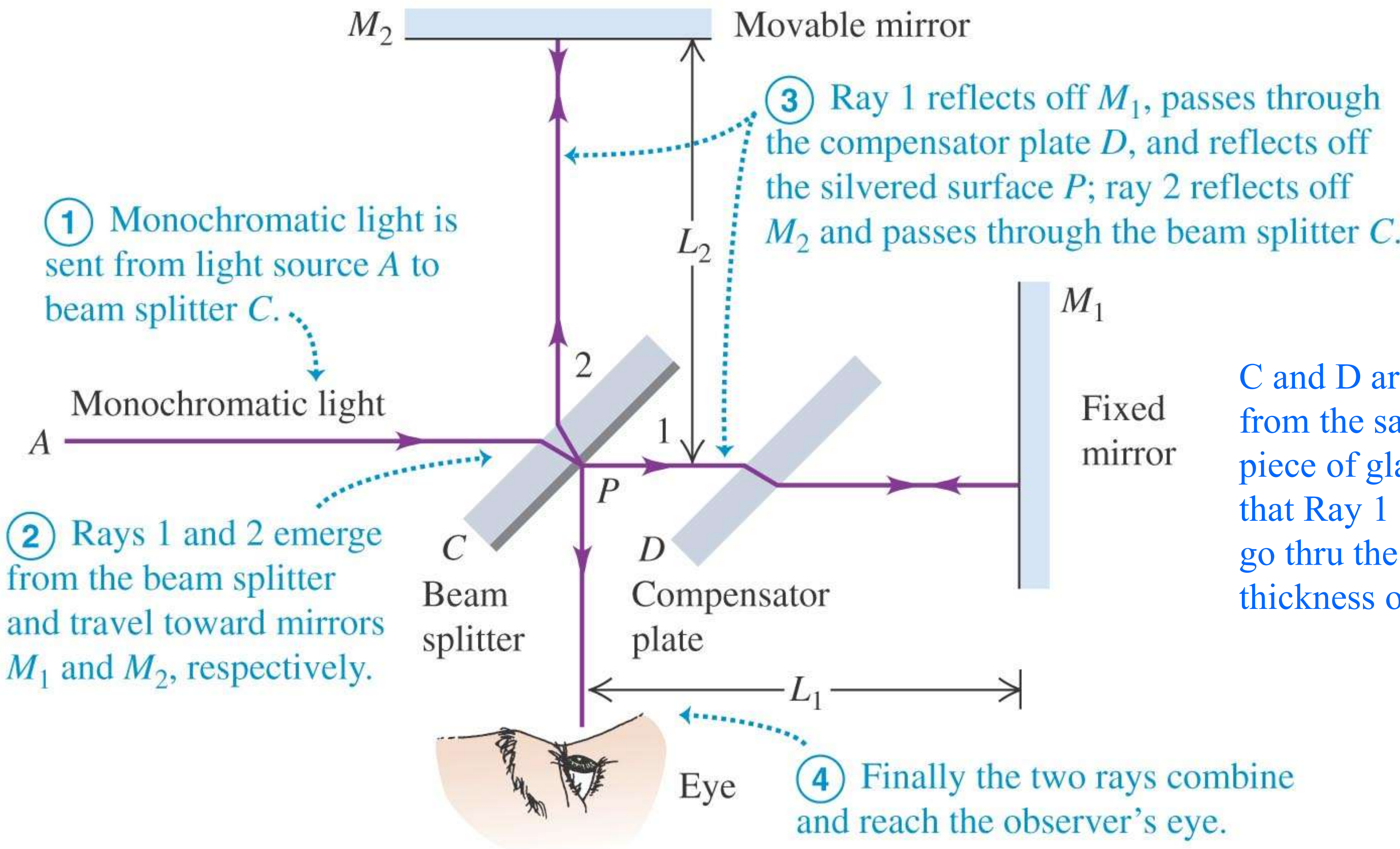





When viewed in monochromic light, the interference pattern is a set of concentric rings called the Newton’s rings.

Since each fringe corresponds to a path difference ~ $\lambda$, the lack of symmetry of these rings can be used to check for precision in lens making extremely accurately.

# Michelson Interferometer




C and D are cut from the same piece of glass so that Ray 1 & 2 will go thru the same thickness of glass.

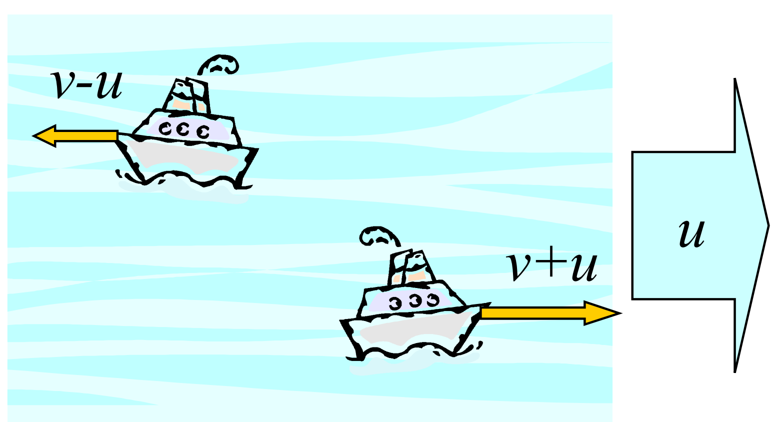
Distances comparable to $\lambda$ can be measured with ease using this device by counting fringes.

# Michelson-Morley Experiment

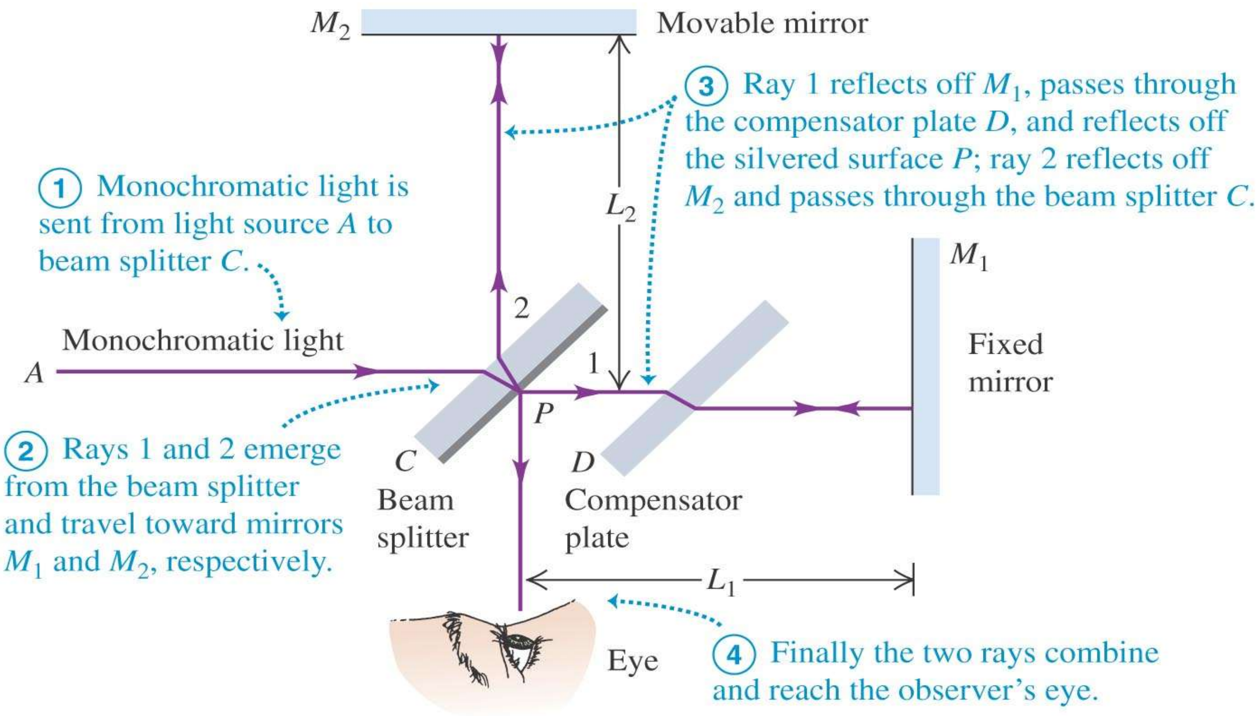
In the 1880s (before our full understanding of electromagnetic theory and special relativity), scientists believe that light travels in a medium called **ether** (similar to sound waves travel in air and water waves travel in water).

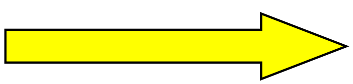
Albert Michelson and Edward Morley set out to measure the property of the either and ended up showing that there is no ether.

Similar to a boat (light) traveling in a flowing river ("ether"), the speed of light was expected to depends on its relative motion with respect to the ether.



# Michelson-Morley Experiment





ether wind direction

Expectation:

- The ether wind will affect the horizontal and vertical branch of the device *differently*
- The resulting interference fringes depend on the path difference between the two branches
- Different pattern will result if device is *rotated* 90 degrees

Result: No observable difference implies there is *no* ether!

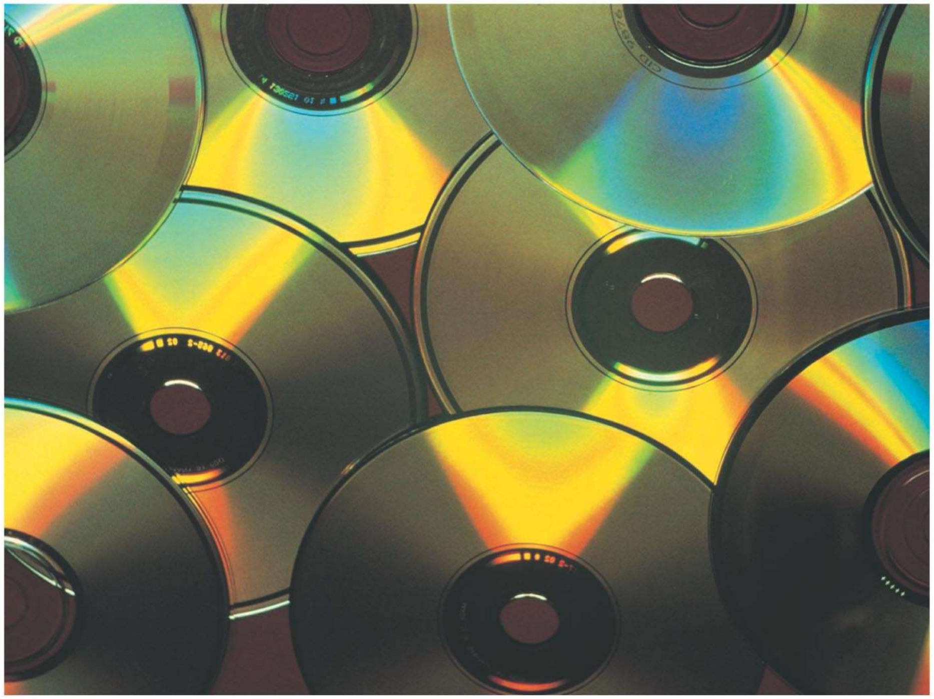
# PHYS 262

George Mason University

Prof. Paul So

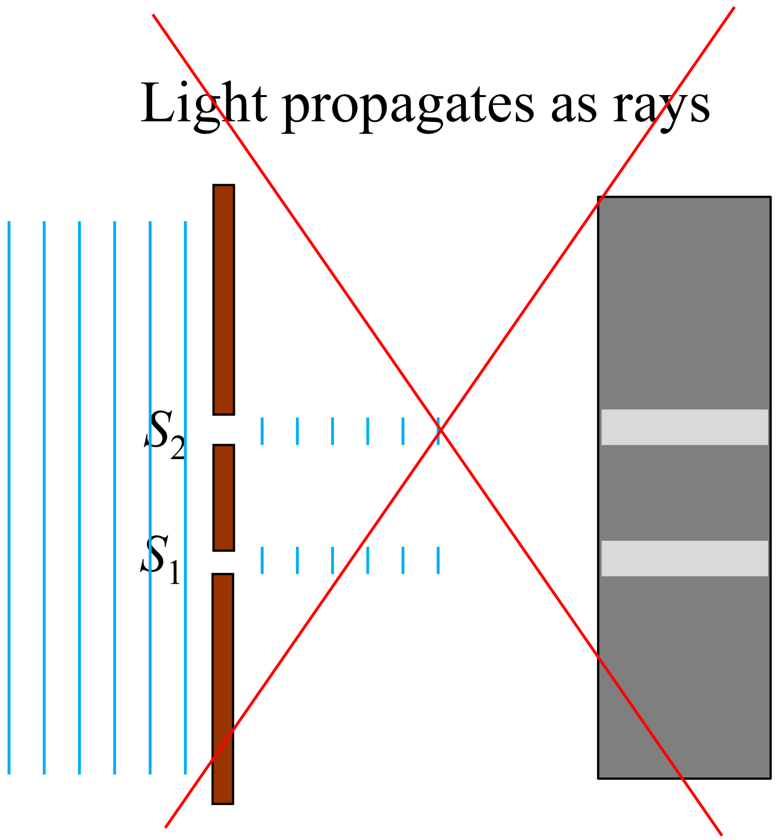
# Chapter 36: Diffraction

- Diffraction and Huygens' Principle
- Diffraction from a Single Slit
- Intensity in the Single-Slit Pattern
- Double-Slit Diffraction
- Diffraction Grating
- x-Ray Diffraction
- Resolving Power

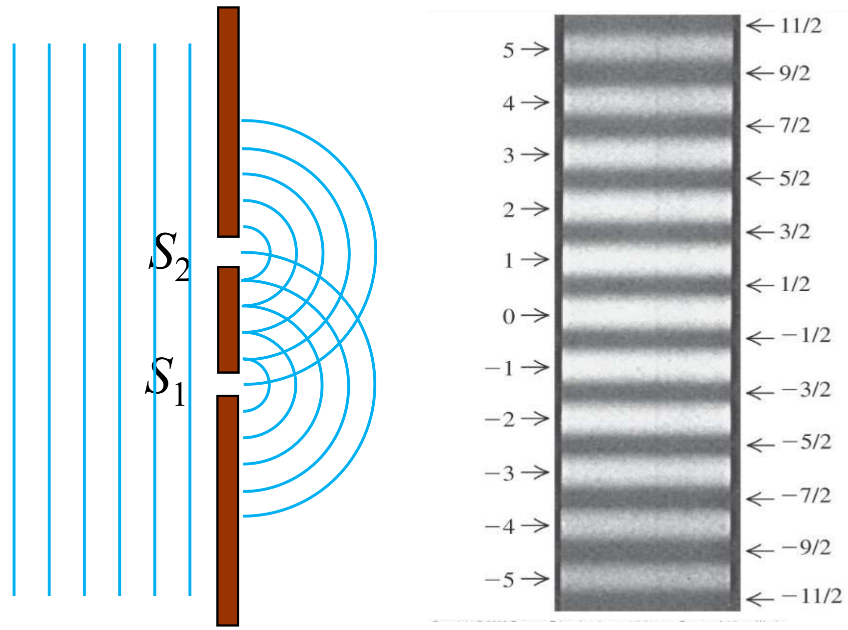


# Wave Nature of Light: Diffraction & Interference from Two Slits

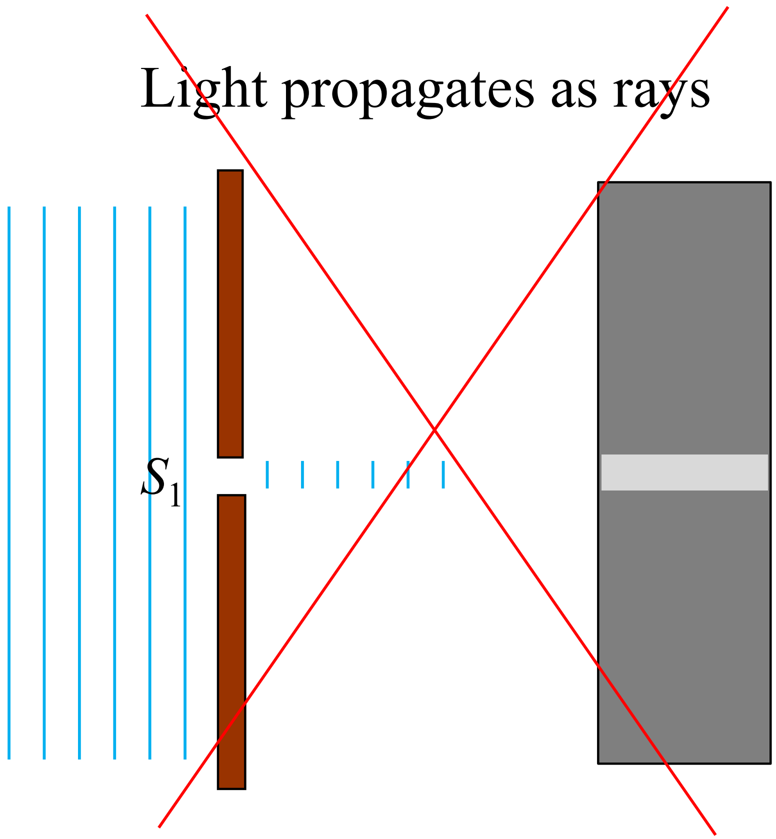
Light propagates as rays

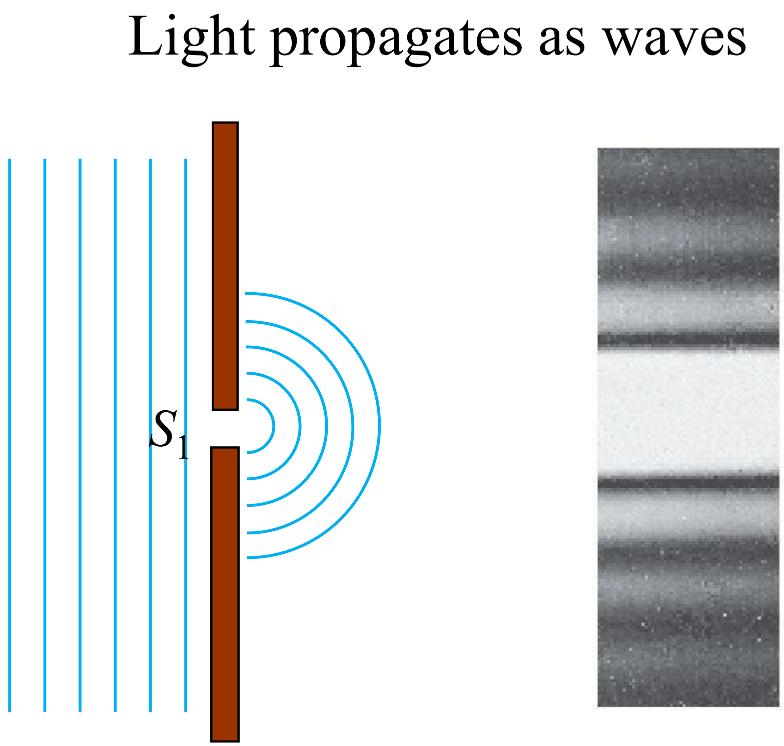



Light propagates as waves

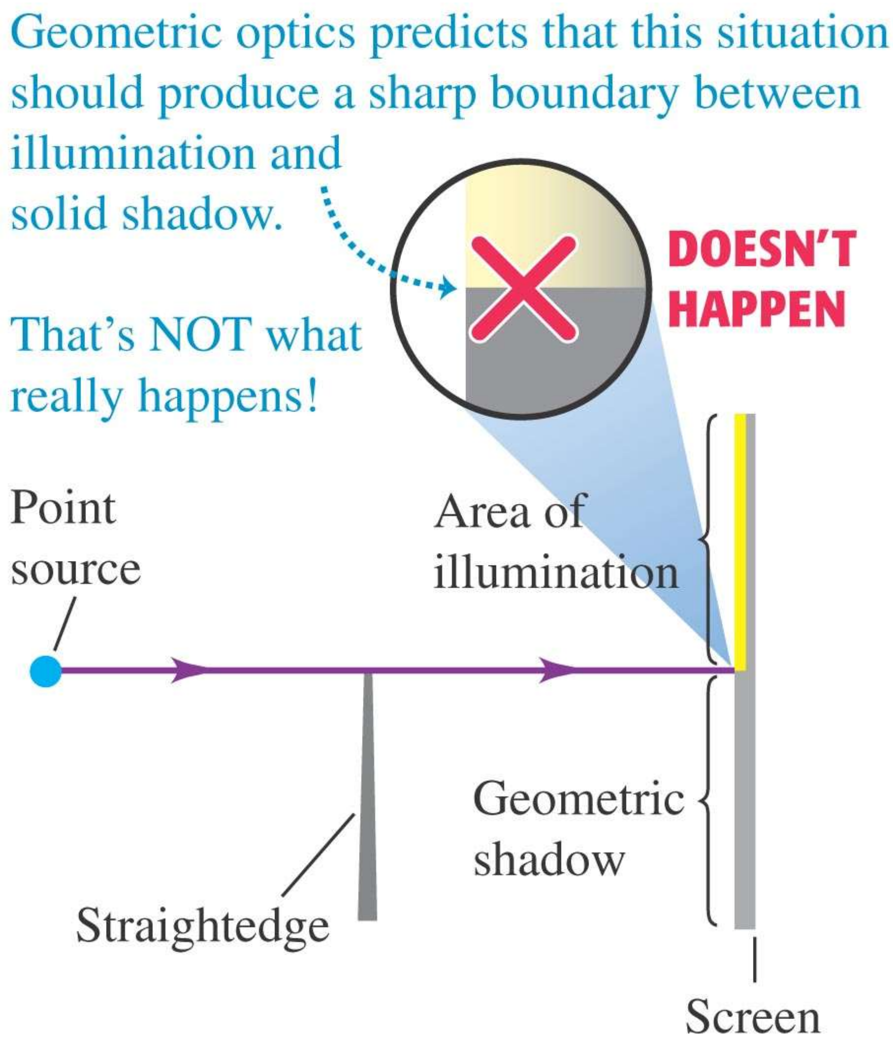


# Wave Nature of Light: Diffraction of a Single Slit



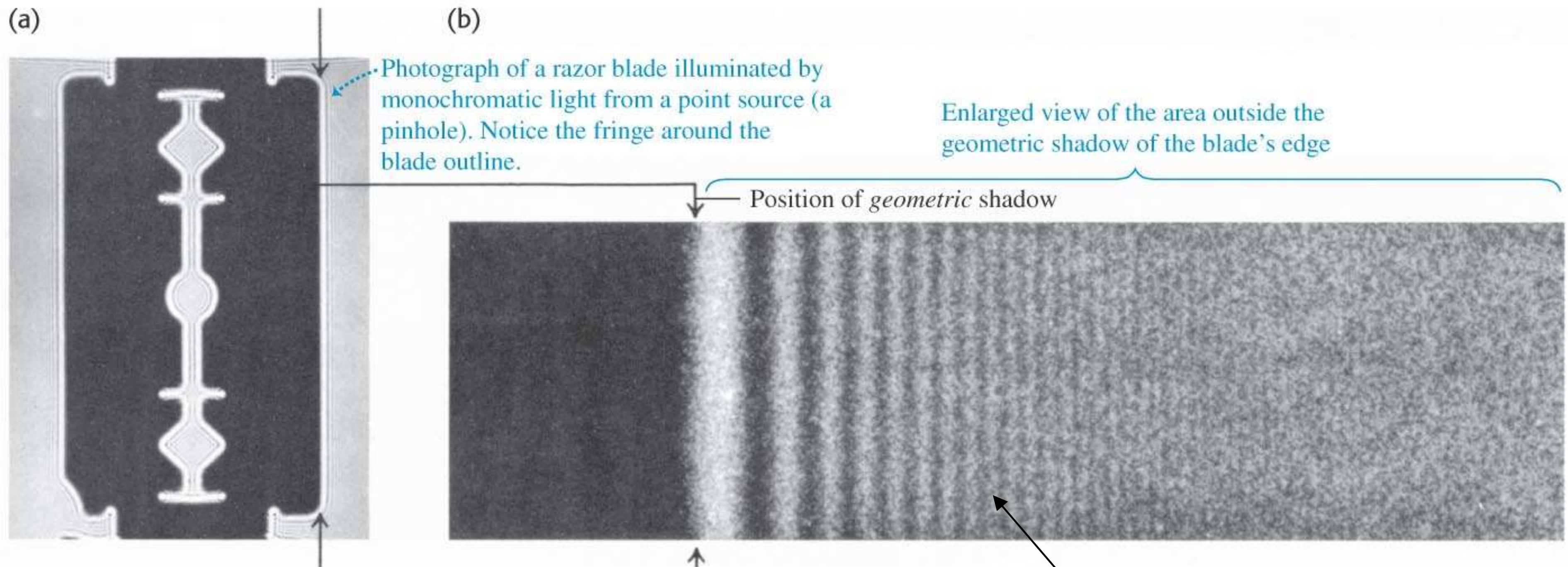



# Diffraction from Sharp Edges



# Diffraction from Sharp Edges

This is what actually happens in real experiments.

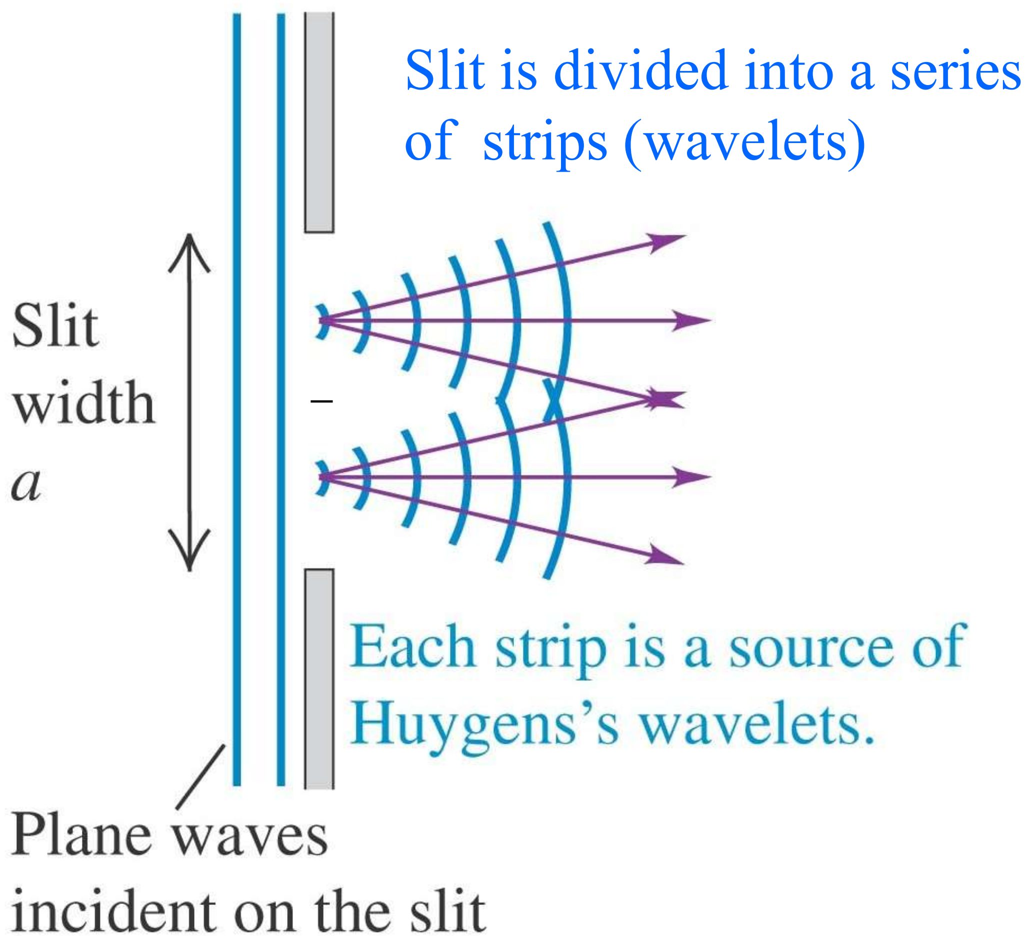





Interference fringes seen beyond geometric shadow

# Diffraction and Huygen's Principle

Consider a simpler case: a single slit






The spreading out of waves thru small apertures or by sharp edges is called **diffraction**.

Waves spread out from each points along the slit as wavelets creating interference patterns beyond and around sharp edges.

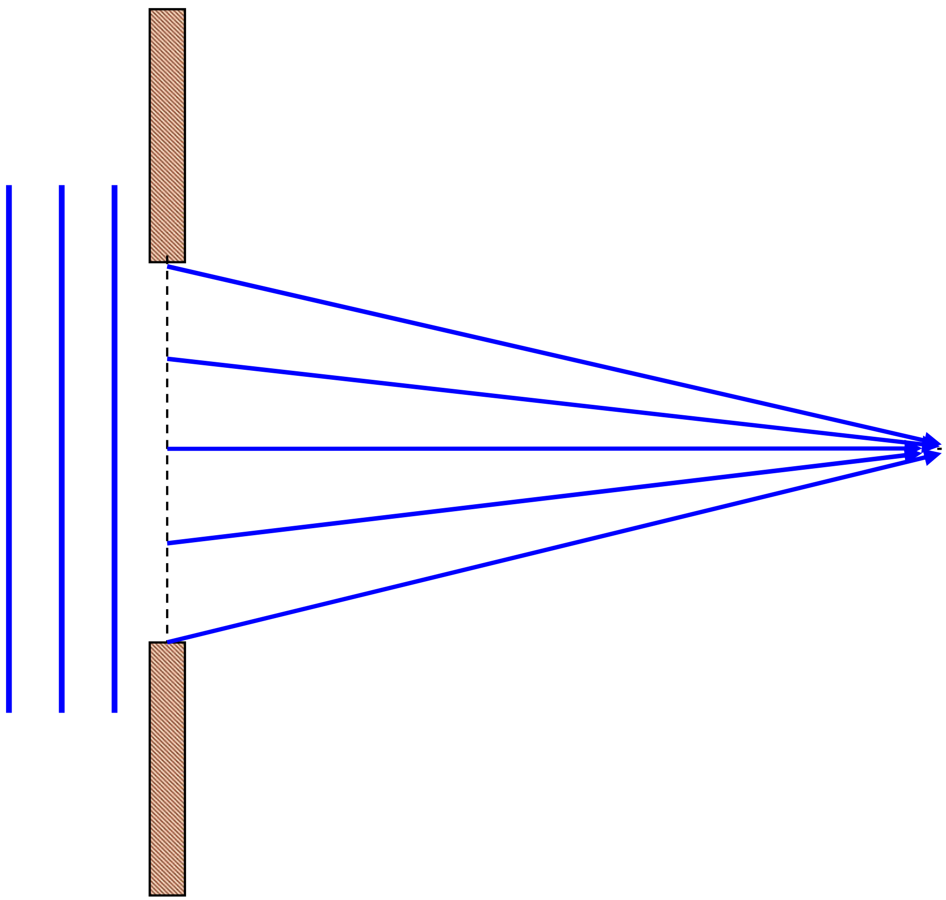
Similar to the two-source interference pattern, these wavelets interfere as they spread out and create the diffraction pattern.

# Diffraction from Narrow Slit

https://youtu.be/JcjDO5VMTiI

# Single-Slit Diffraction

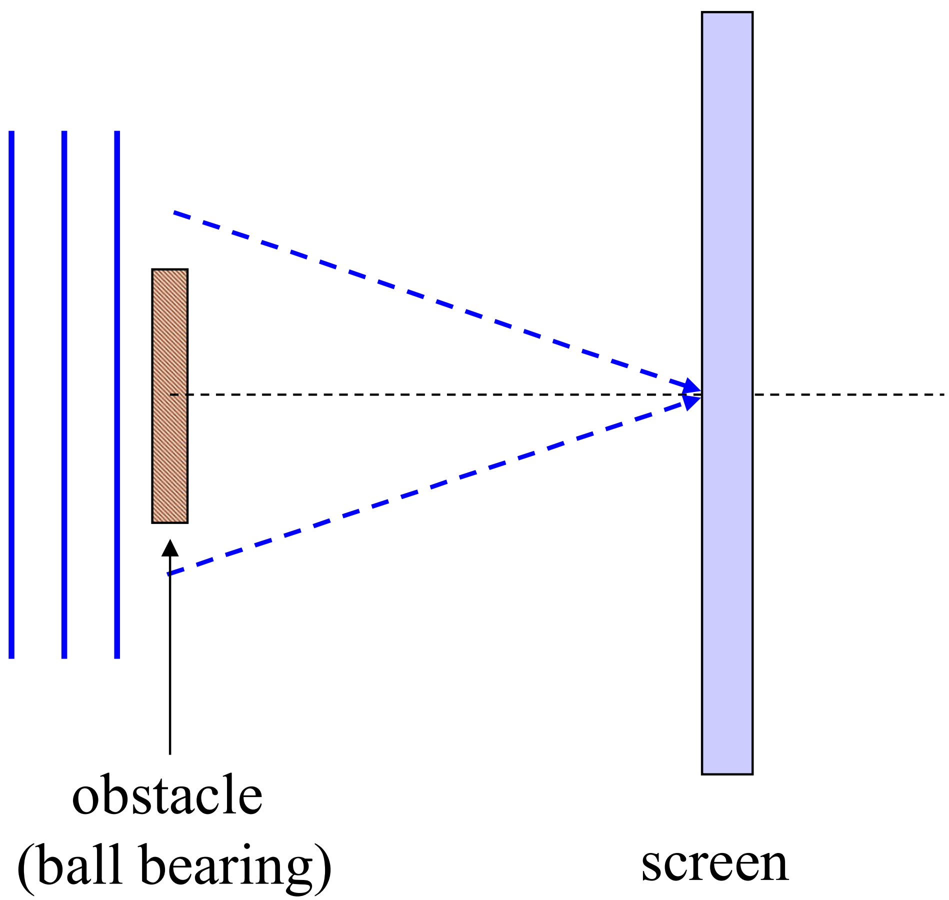
Central Maximum ($\theta = 0$, straight ahead)



All waves from top half of slit travel the same distance to the screen as waves from bottom half. They arrive *in phase* at the central mid-point → constructive interference.

There will be a **bright fringe** in the *middle* at $\theta = 0$.

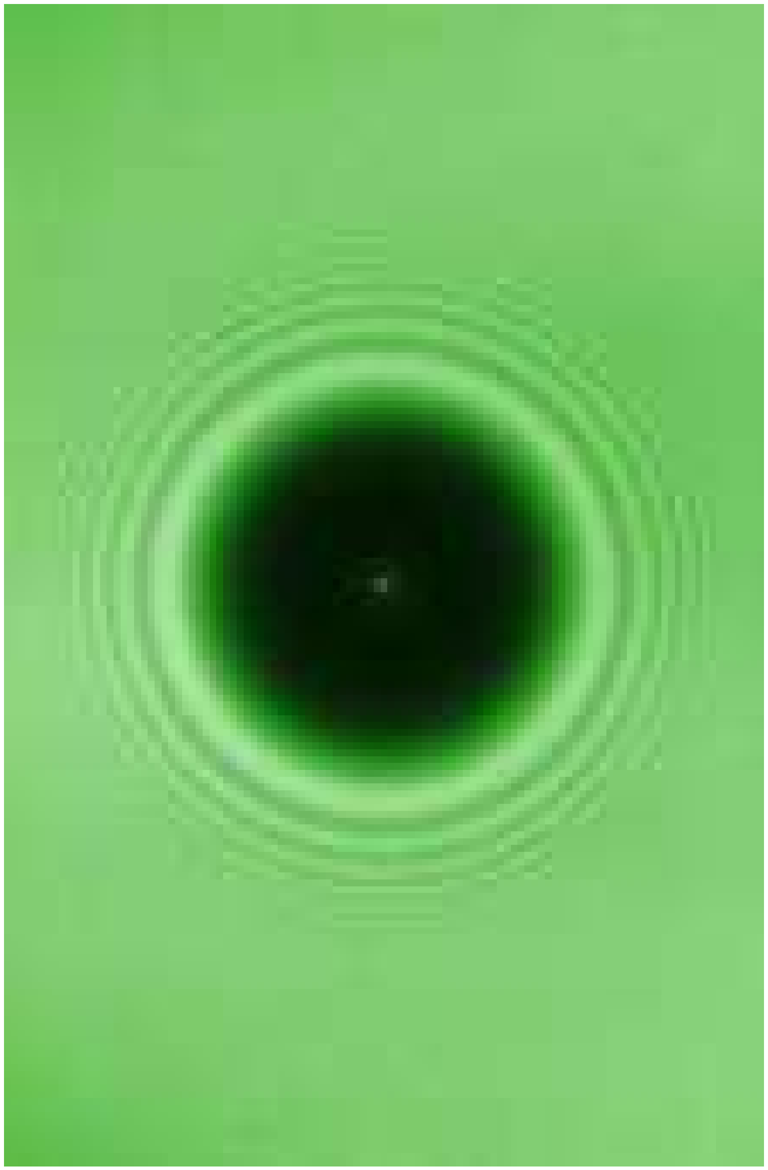
# Central Maximum: Poisson's Spot

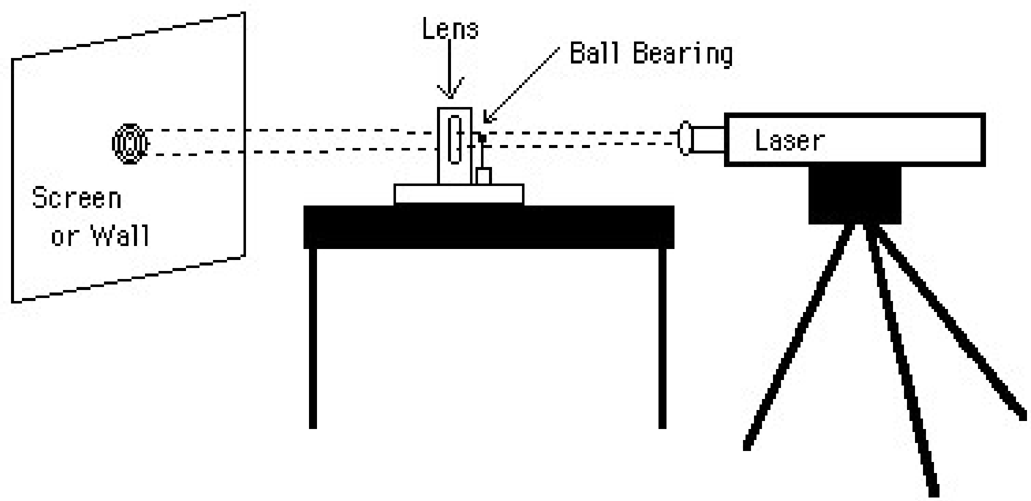



Wave spreading around from the top will travel the *same* distance as the wave spreading around from the bottom.

At the mid-point ($\theta = 0$), these waves will interfere *constructively* and create a *bright* spot although it is in the *shadow region*.

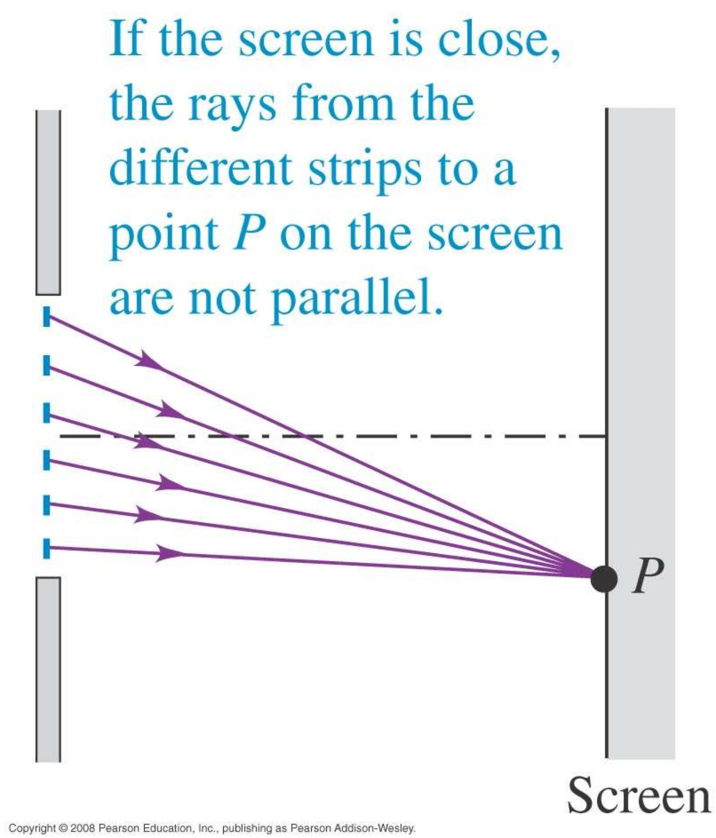
# The Poisson's Bright Spot



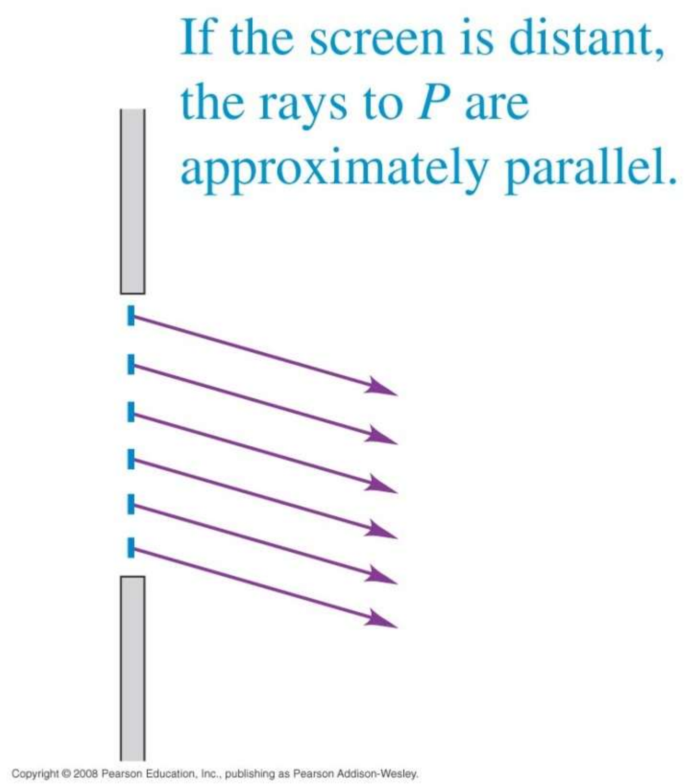


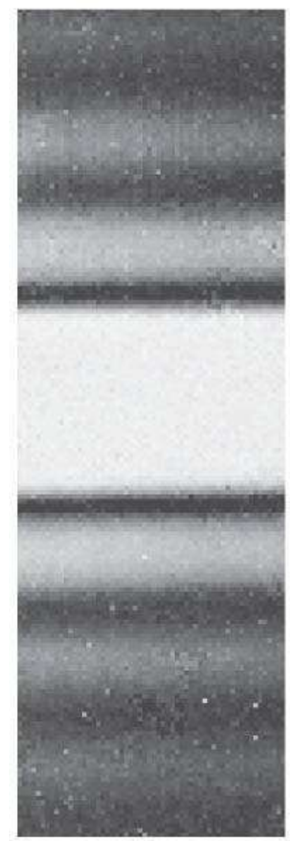
# Fresnel & Fraunhofer Diffraction

**(b)** Fresnel (near-field) diffraction




**(c)** Fraunhofer (far-field) diffraction

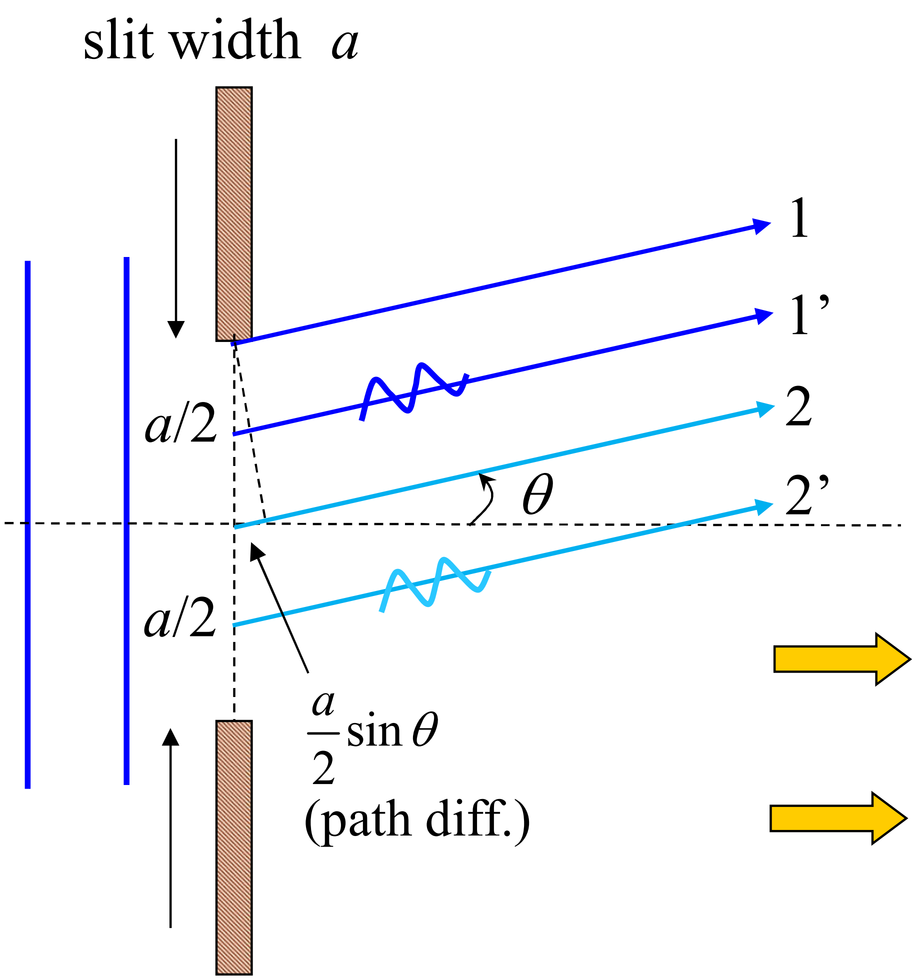


For simplicity, we will consider Fraunhofer Diffraction from now on.

# Single-Slit Diffraction: Dark Fringes

First Order Minimum: ($\theta > 0$ → slightly above (or below) the central max)




Divide the wavelets into 2 groups (top and bottom)

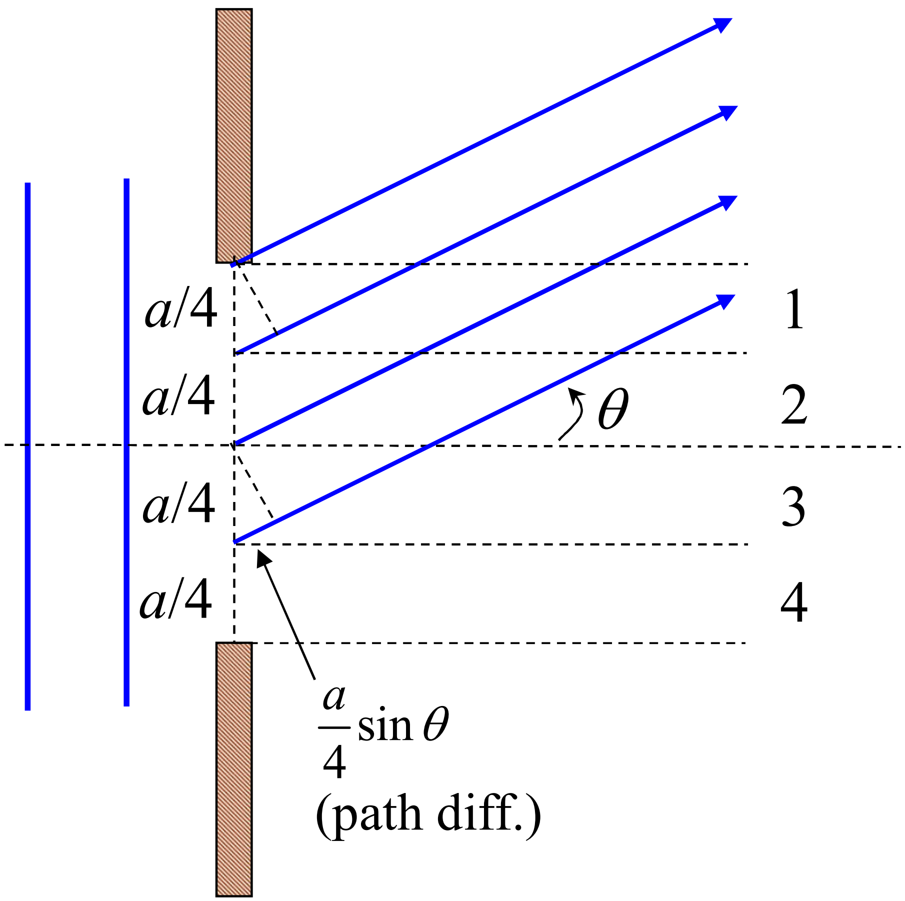
If wavelets from the top group *destructively interfere* with wavelets from the bottom group, we will have a dark spot on the screen at $\theta$.

$$r_2 - r_1 = r_{2'} - r_{1'} = \lambda/2$$

$$\frac{a}{2}\sin\theta = \lambda/2 \quad \text{or} \quad a\sin\theta = \lambda$$

# Single-Slit Diffraction: Dark Fringes

## Second Order Minimum




Consider the following arrangement with slightly **larger** $\theta$:

Divide the wavelets into 4 groups

If wavelets from each adjacent groups *destructively interfere*, we will have another dark spot on the screen at $\theta$.

$$r_2 - r_1 = r_3 - r_2 = r_4 - r_3 = \lambda/2$$

$$\frac{a}{4}\sin\theta = \lambda/2$$

or $a\sin\theta = 2\lambda$

# Single-Slit Diffraction: Dark Fringes

For higher order minimum with larger angular distance $\theta$, we can use the same argument by subdividing the slit into more groups (6, 8, 10, etc.).

This leads to the following general formula for the dark fringes:

$$a \sin\theta = m\lambda, \quad m = \pm 1, \pm 2, \cdots$$

Note:

1. $m = 0$ is *not* the first minimum !
   In fact, it is the location for the central max.
2. Secondary maximum occurs *near* $3\lambda/2$, $5\lambda/2$, etc. but not exactly.