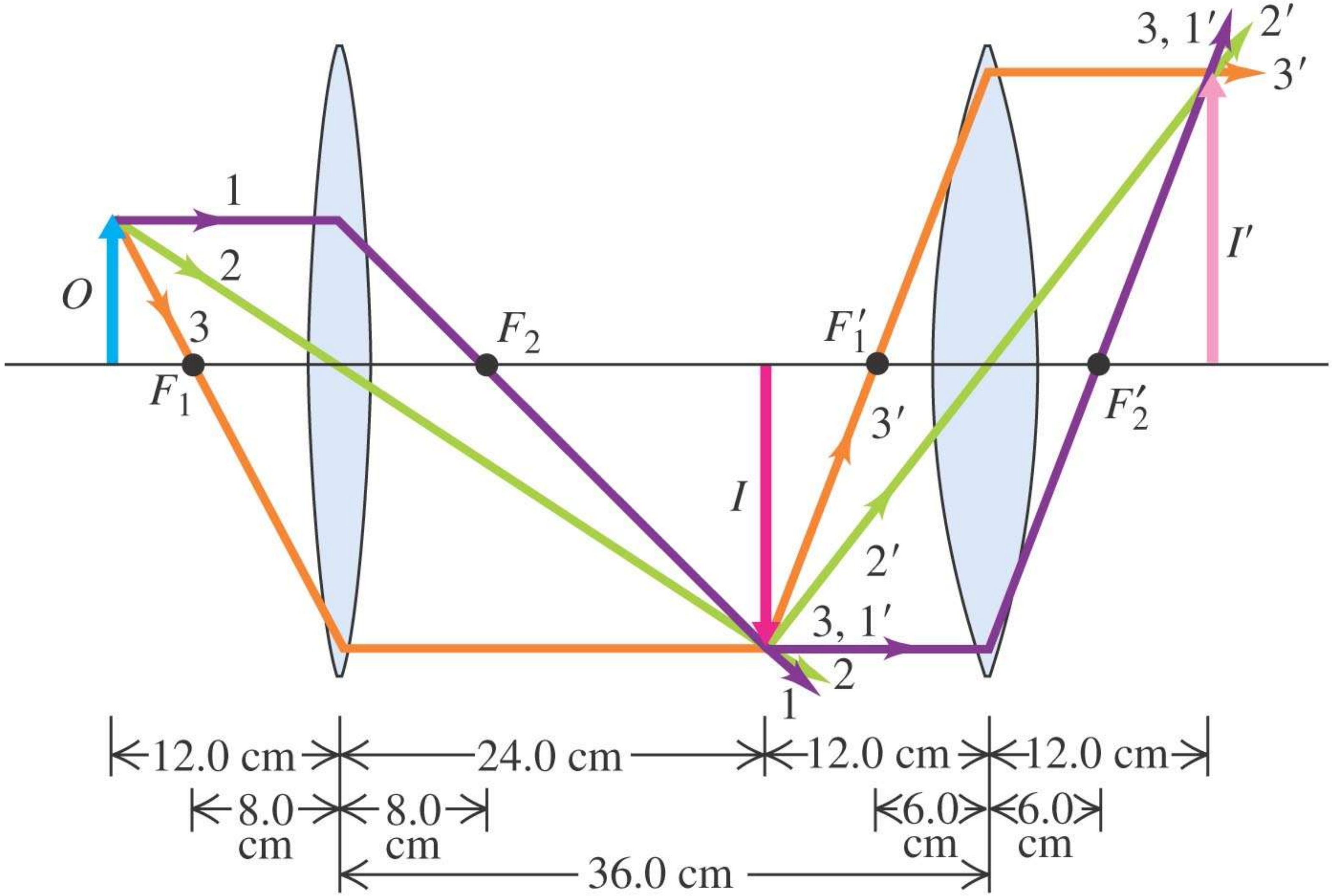
Example: Compound Lenses



Example: Component Lens

For the 1st Lens: $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1} \rightarrow \frac{1}{12}+\frac{1}{s_1'}=\frac{1}{8} \rightarrow \frac{1}{s_1'}=\frac{12-8}{96}=\frac{1}{24}$

$s_1'=24cm$

This image from the 1st lens is on the light incoming side of lens #2, so that:

$$s_2=36cm-s_1'=+12cm$$

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2} \rightarrow \frac{1}{12}+\frac{1}{s_2'}=\frac{1}{6} \rightarrow \frac{1}{s_2'}=\frac{2-1}{12}=\frac{1}{12}$$

$s_2'=12cm$ (final image is 12 cm on the outgoing side of lens #2)

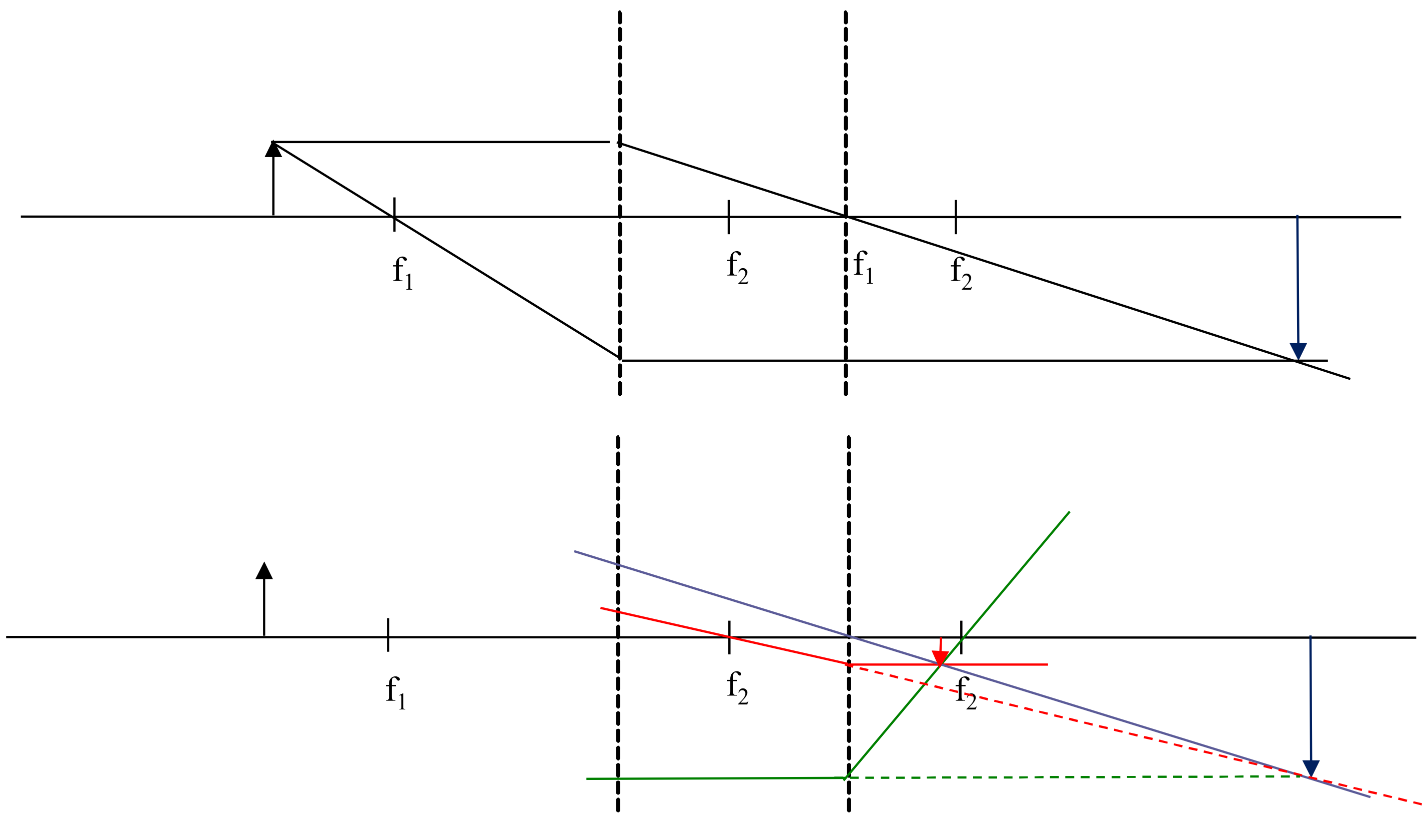
The *combined* lateral magnification is the product from both lenses,

$$m_{tot}=m_1m_2=\left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right)=\frac{24cm}{12cm}\frac{12cm}{12cm}=+2$$ (upright and real)

Another Example of Component Lens

Two converging lens with $f_1 = 20cm$ and $f_2 = 10cm$ and the lens are 20 cm apart.

Object is located 30cm to the left of lens #1, find the location of final image.



Another Example of Component Lens

For the 1st Lens: $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1} \rightarrow \frac{1}{30}+\frac{1}{s_1'}=\frac{1}{20} \rightarrow \frac{1}{s_1'}=\frac{3-2}{60}=\frac{1}{60}$

$s_1'=60cm$

This image from the 1st lens is on the **outgoing** side of lens #2, so that:

$$s_2=-40cm$$

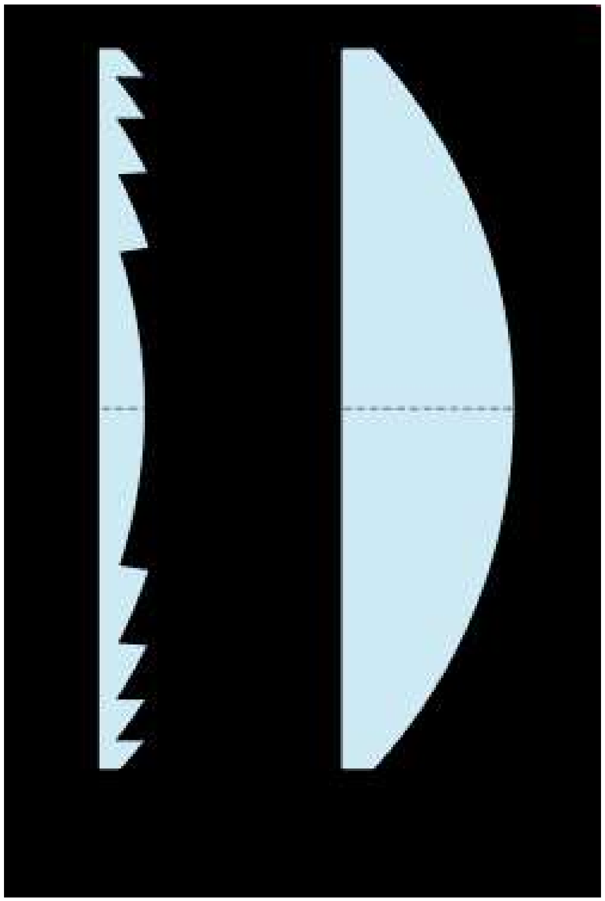
$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2} \rightarrow -\frac{1}{40}+\frac{1}{s_2'}=\frac{1}{10} \rightarrow \frac{1}{s_2'}=\frac{4+1}{40}=\frac{1}{8}$$

$s_2'=8cm$ (final image is 8 cm on the outgoing side of lens #2)

The *combined* lateral magnification is the product from both lenses,

$$m_{tot}=m_1m_2=\left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right)=-\frac{60cm}{30cm}\left(-\frac{8cm}{-40cm}\right)=-0.4$$ (inverted and real)

Fresnel Lens

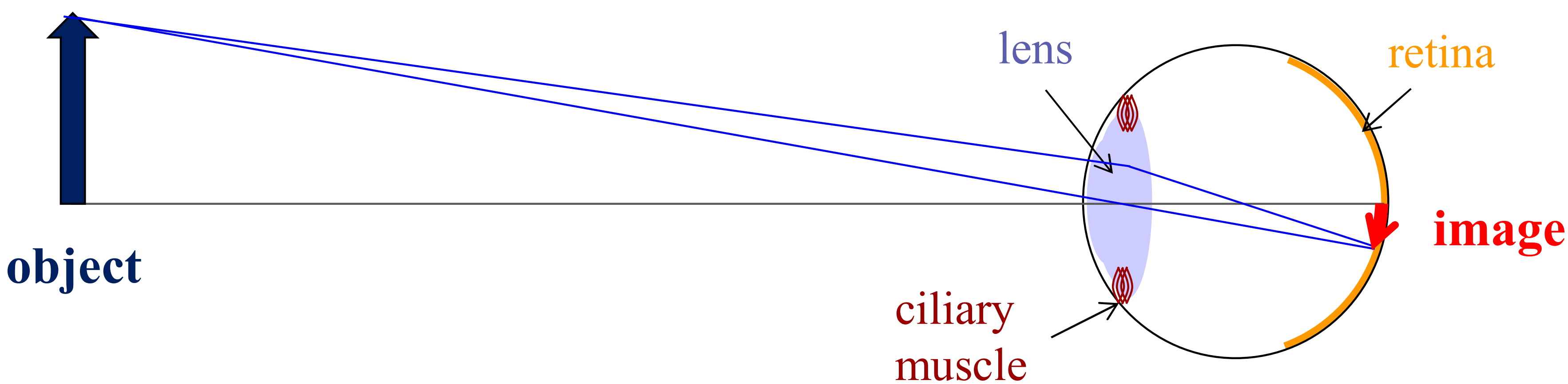


Use in headlights and lighthouse lamps to save weight.

Fresnel Lens for the lighthouse at Jones Point in Old Town Alexandria

D$_{\text{fresnel lens}}$

The Eye

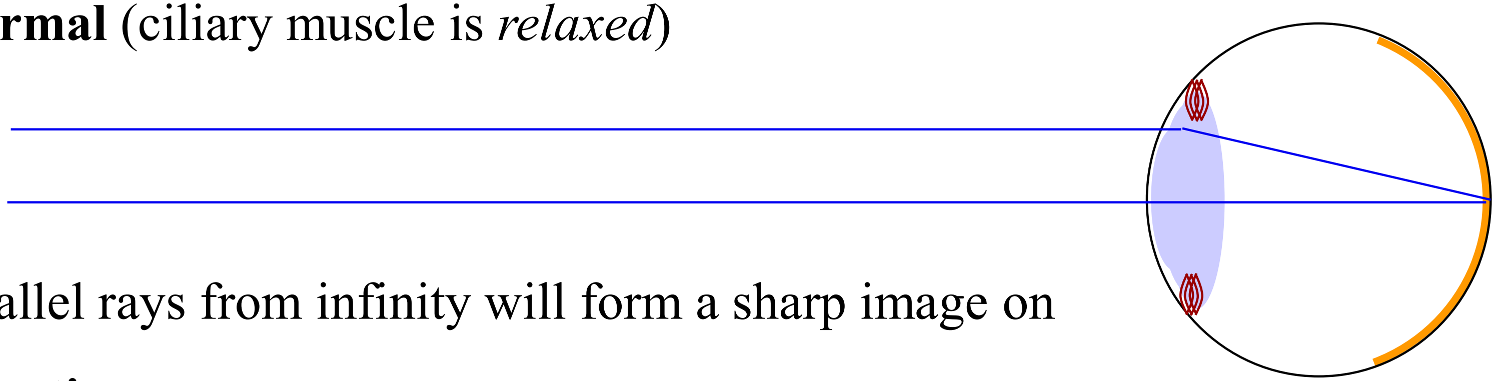


For an object to be seen clearly…

The **lens** must adjust its radius of curvature using its **ciliary muscle** to change the curvature of the **lens** so as to form the image sharply on the **retina**.

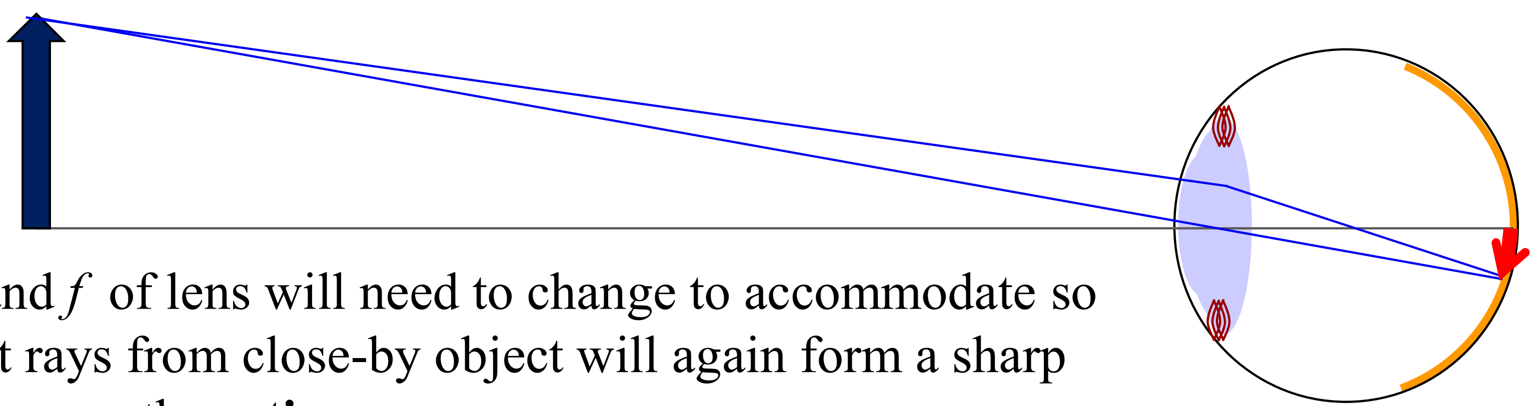
The Eye

Normal (ciliary muscle is *relaxed*)



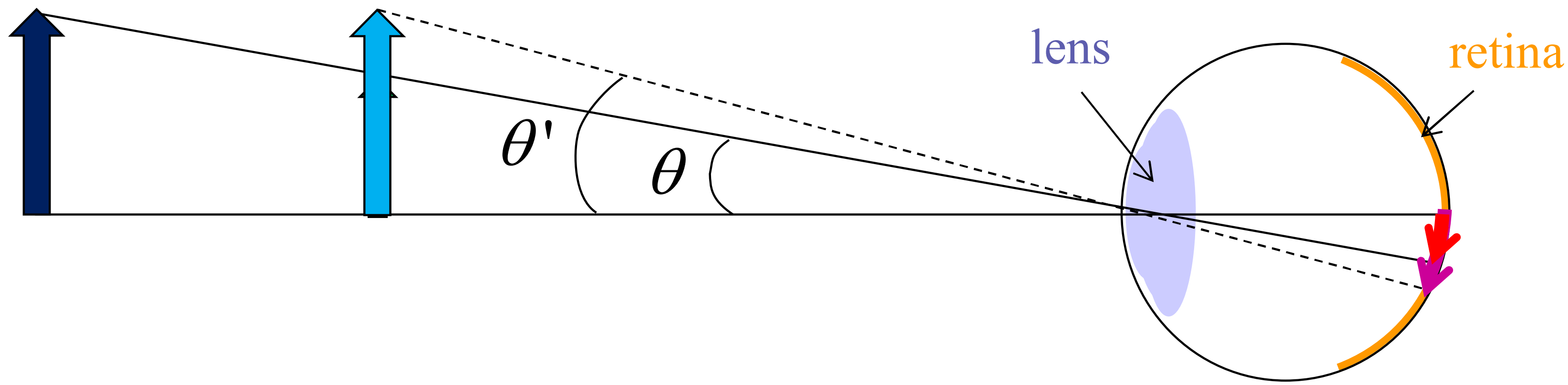
Parallel rays from infinity will form a sharp image on the **retina**.

Accommodation (ciliary muscle *contracts*): rays are not coming in parallel from infinity



R and f of lens will need to change to accommodate so that rays from close-by object will again form a sharp image on the **retina**.

Angular Size of an Object



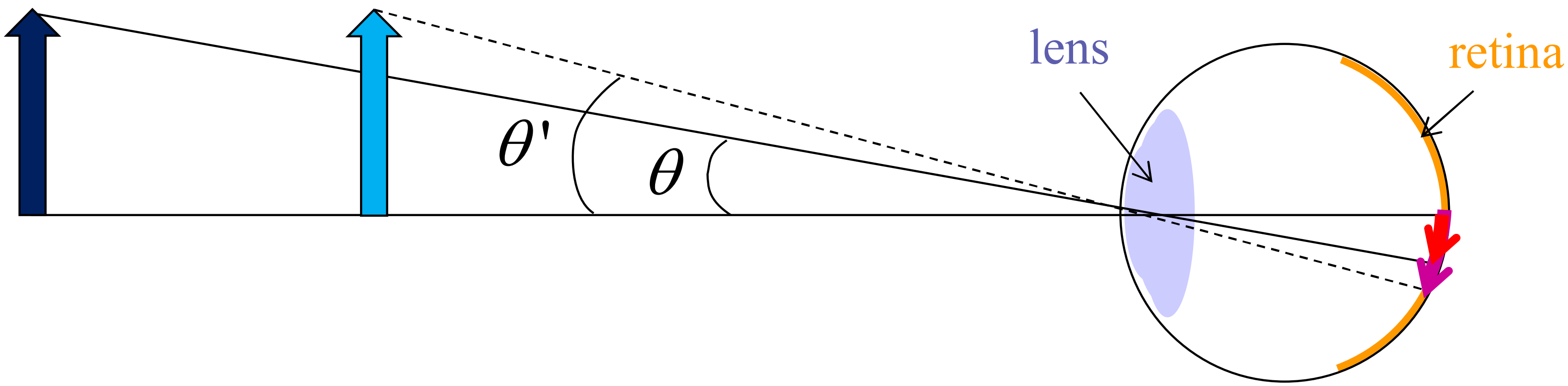
→ The *perceived* size of an object depends on its actual image size on the retina:

- Both the blue (bigger) the green (smaller) arrows will be perceived to have the same angular size

- Moving the blue arrow *closer* to the eye will result in a bigger image on the retina and thus a bigger *perceived* angular size

⇨ The perceived **angular size** of an object is determined by the angle θ subtended by the image on the retina

Angular Magnification



Thus, an object can appear to be magnified if it moves closer to the eye and the **Angular Magnification** M is defined as the ratio:

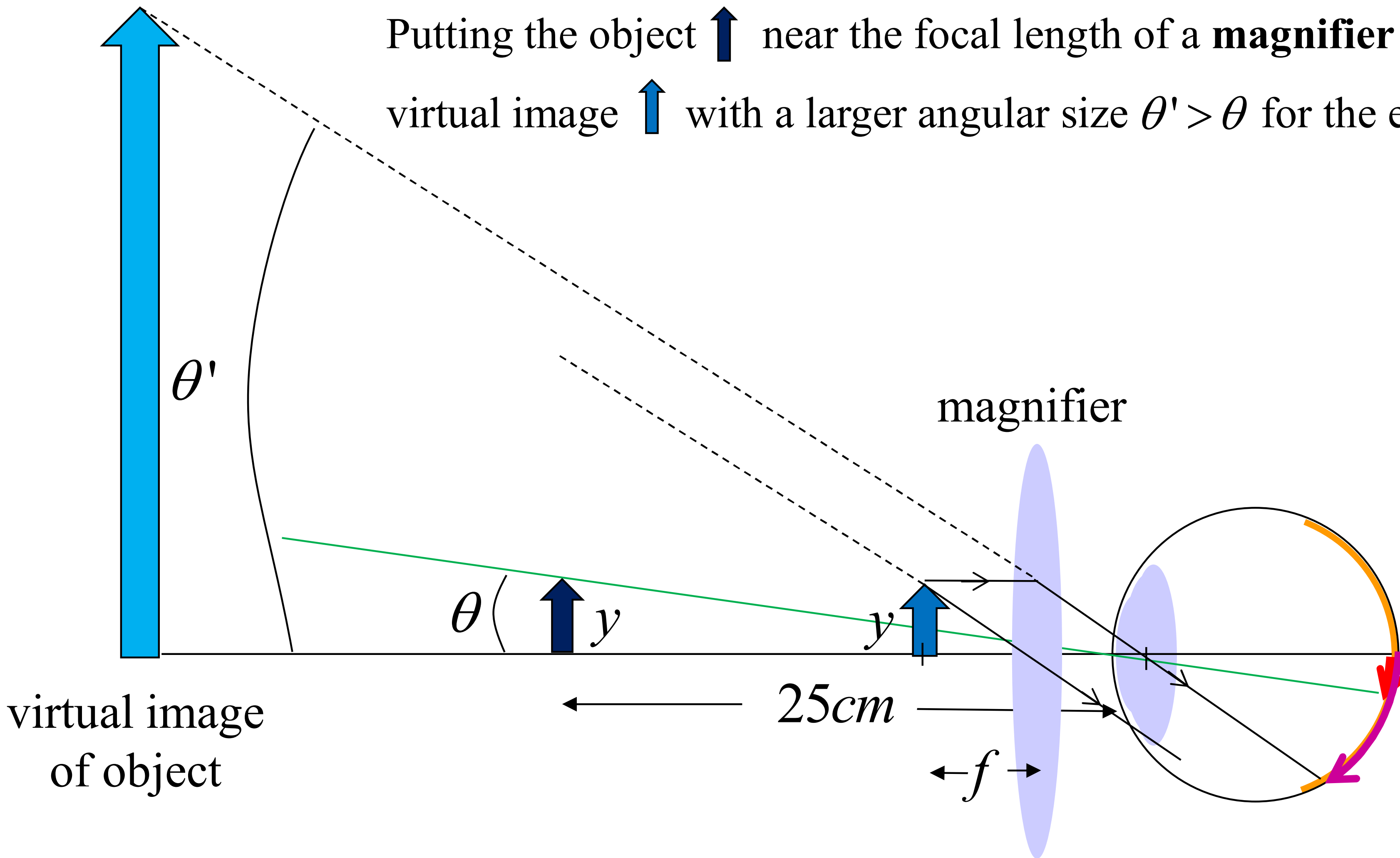
$$M = \frac{\theta'}{\theta}$$

Note: The *largest* angular magnification is achieved at the *closest* distance from the eye (the **near point**) where the lens can still form a sharp image at the retina.

~ 25 cm (for healthy young adult)

Magnifier

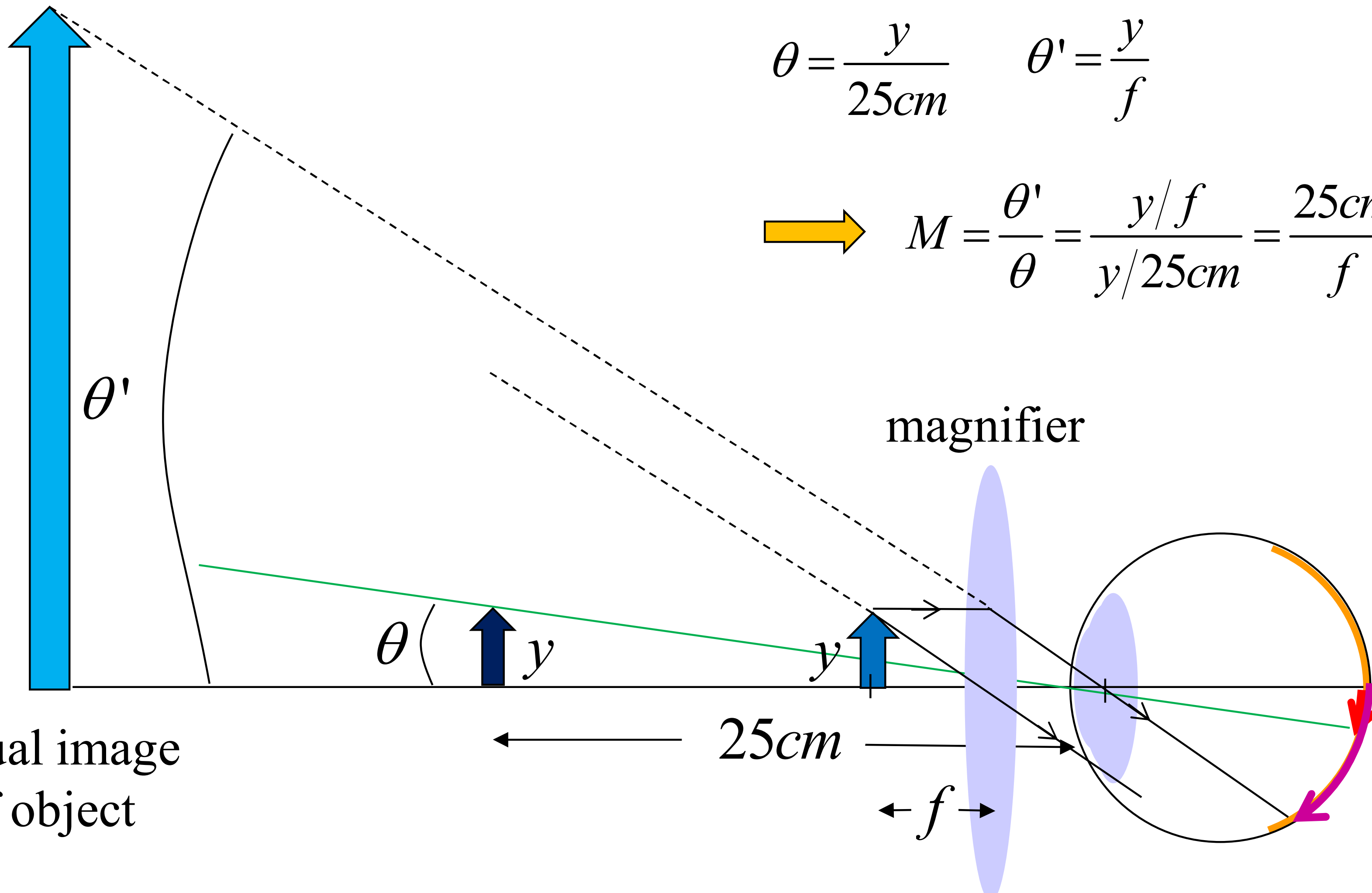
Putting the object near the focal length of a **magnifier** forms a large virtual image with a larger angular size $\theta' > \theta$ for the eye.



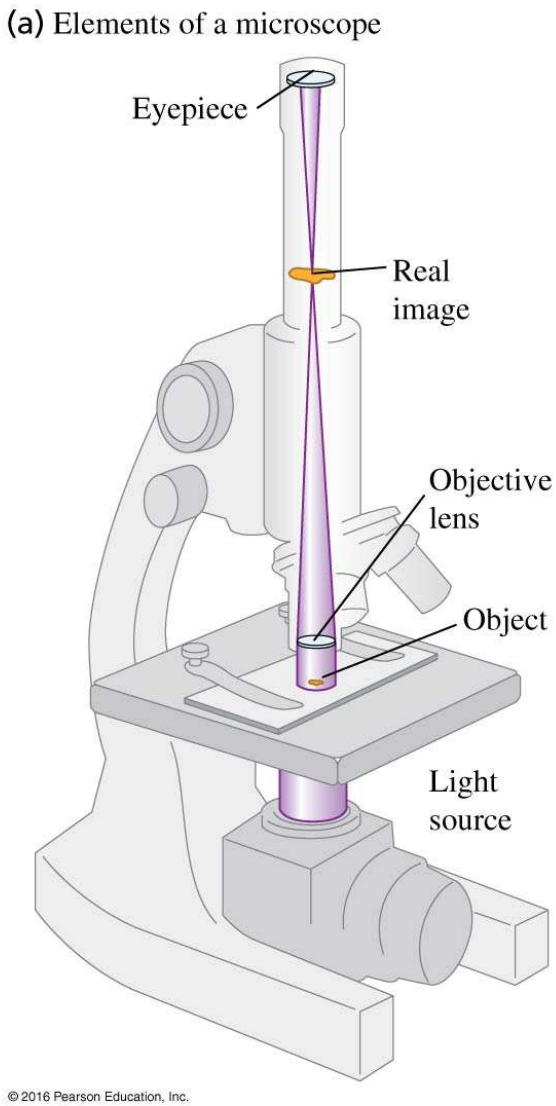
Angular Magnification of a Magnifier

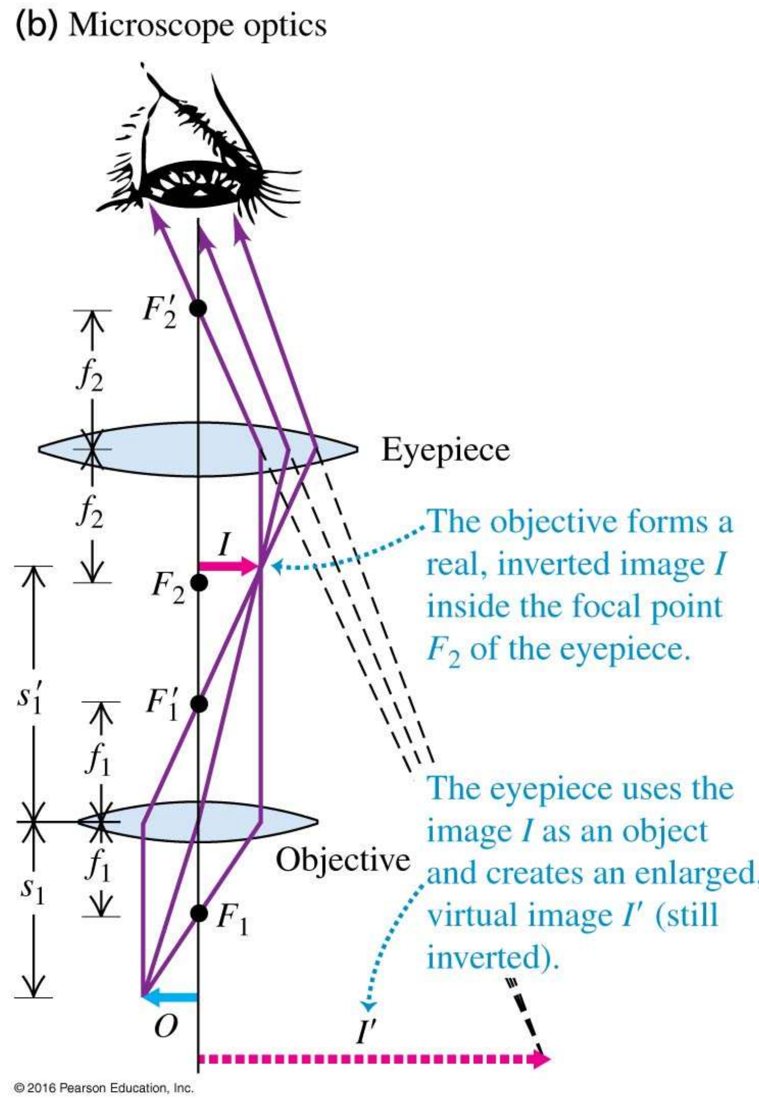
$$\theta = \frac{y}{25cm} \qquad \theta' = \frac{y}{f}$$

$$\Rightarrow \quad M = \frac{\theta'}{\theta} = \frac{y/f}{y/25cm} = \frac{25cm}{f}$$

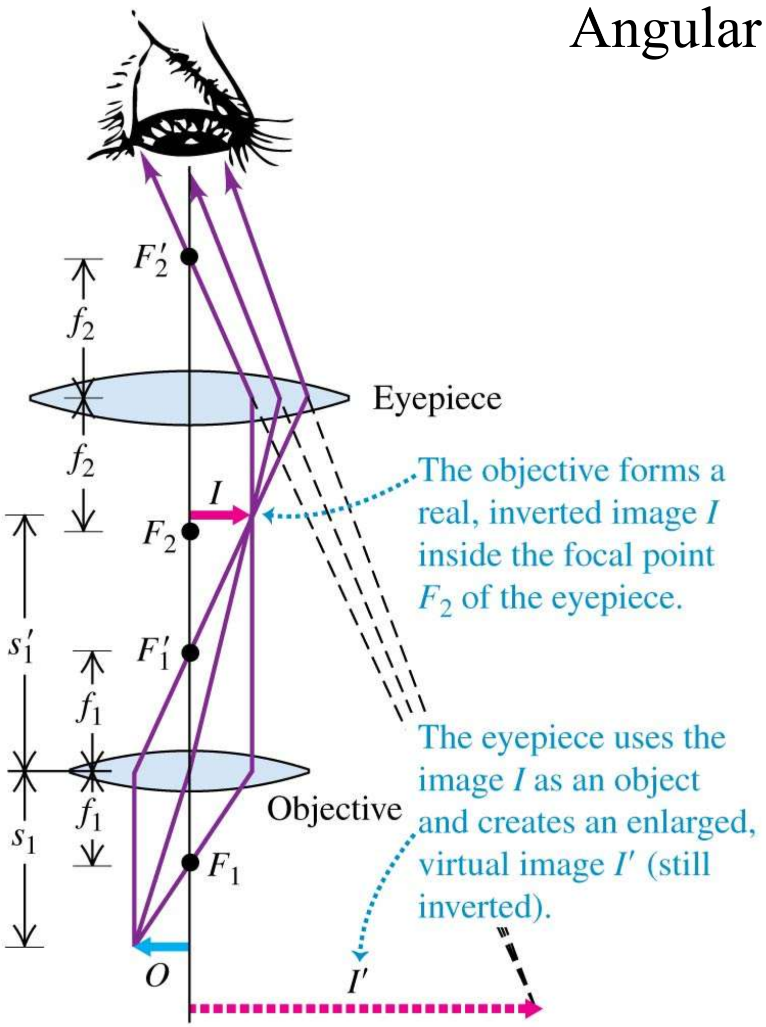


Compound Microscope





Magnification of a Compound Microscope



Angular Magnification of a Compound Microscope:

Lateral Magnification from Objective × Agular Magnification from Eyepiece

$$M_{total} = m_{objective} M_{eyepiece}$$

$$m_{objective} = \frac{s_1'}{s_1} \xrightarrow{s_1 \approx f_1} = \frac{s_1'}{f_1}$$

$$M_{eyepice} = \frac{25cm}{f_2}$$

$$\Rightarrow \quad M_{total} = \frac{(25cm)\, s_1'}{f_1 f_2}$$

Fermat's Principle
Pierre de Fermat (1601-1665)

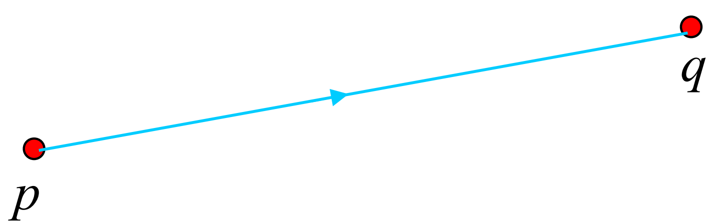
A general mathematical principle that can be used to analyze light path:

"When a light ray travels between two points, its path is the one that requires the *least* time."

Application #1: uniform material [*n* (or *v*) is the same everywhere!]

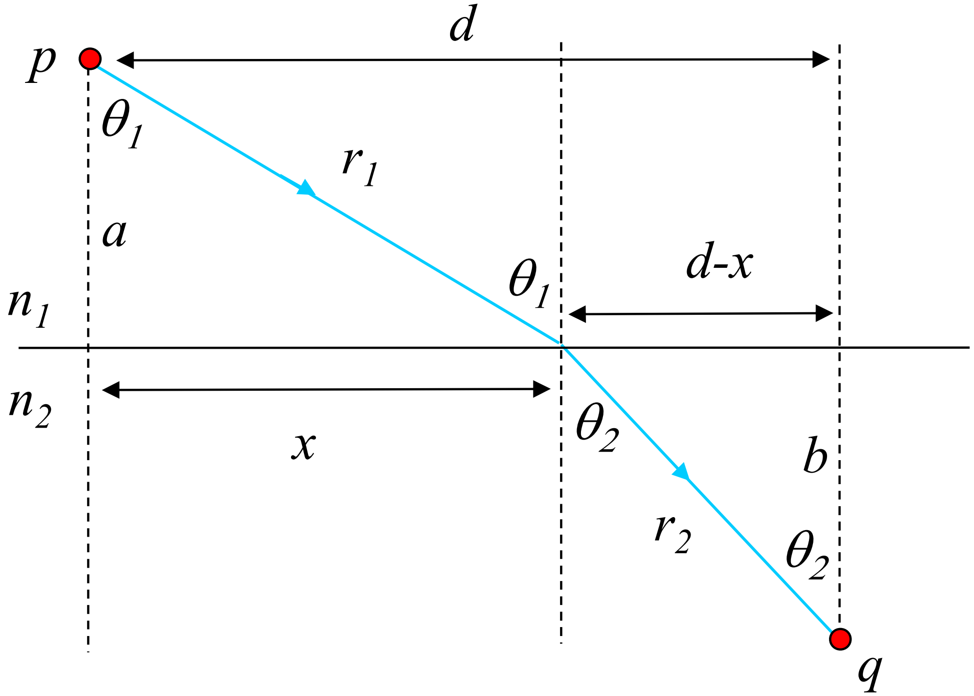
$t = \frac{d}{v} \quad \rightarrow$ Between any two points, the least time requires the *shortest* distance in an uniform medium.

Light will travel in a straight line in an uniform medium.



Fermat's Principle
Pierre de Fermat (1601-1665)

Application #2: Snell's Law



Within n_1 and n_2, light travels in straight lines and total time of travel from p to q is,

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2}$$

$$= \frac{\sqrt{a^2 + x^2}}{c/n_1} + \frac{\sqrt{b^2 + (d-x)^2}}{c/n_2}$$

Note: With two different speeds, the fastest way to get from p to q is *not* necessary a straight line !

Fermat's Principle (Application to Snell's Law)

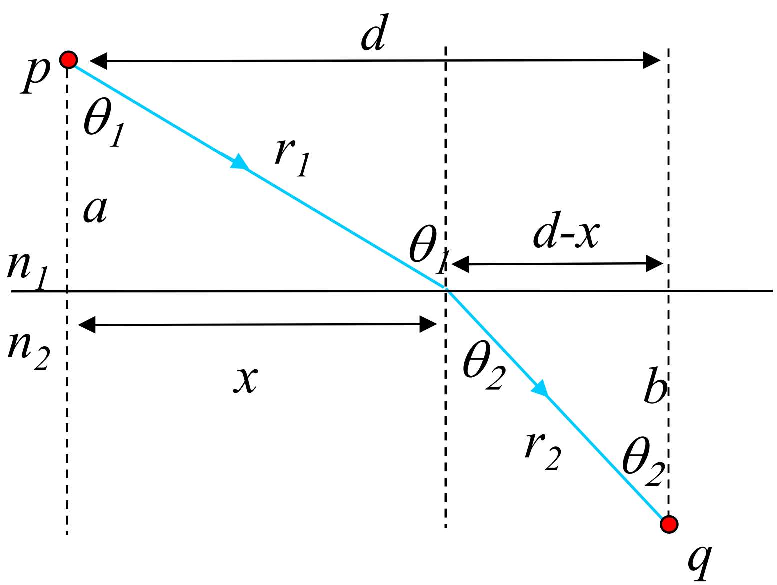
$$t(x) = \frac{\sqrt{a^2 + x^2}}{c/n_1} + \frac{\sqrt{b^2 + (d-x)^2}}{c/n_2}$$

Find the value of x (the crossing point) such that the total travel time is minimized.

$$\frac{dt}{dx} = 0 \quad \rightarrow \quad \frac{n_1}{c}\left(\frac{1}{2}\right)\frac{2x}{\sqrt{a^2 + x^2}} + \frac{n_2}{c}\left(\frac{1}{2}\right)\frac{2(d-x)(-1)}{\sqrt{b^2 + (d-x)^2}} = 0$$

Fermat's Principle (Application to Snell's Law)

$$\Rightarrow \quad \frac{n_1 x}{\sqrt{a^2 + x^2}} - \frac{n_2 (d-x)}{\sqrt{b^2 + (d-x)^2}} = 0$$



$$r_1^2 = a^2 + x^2$$
$$r_2^2 = b^2 + (d-x)^2$$

$$\sin\theta_1 = x/r_1$$
$$\sin\theta_2 = (d-x)/r_2$$

$$\Rightarrow \quad n_1 \sin\theta_1 - n_2 \sin\theta_2 = 0$$
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

(Snell's Law)

Physics 262

George Mason University

Prof. Paul So

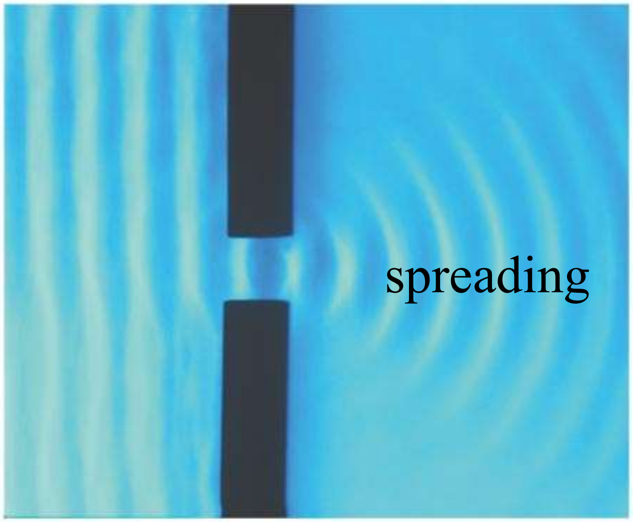
Chapter 35: Interference

- Interference and Coherent Sources
- Two-Source Interference of Light
- Intensity of Interference Patterns
- Interference in Thin Films
- The Michelson Interferometer

Wave Nature of Light

- Previous Chapters (Geometric Optics) $\lambda \ll L$
  - Rays Model is an approximation of EM waves with rays pointing in the direction of propagation
- Next Couple of Chapters (Wave/Physical Optics) $\lambda \sim L$
  - Like water waves, light *spreads* and *interferes* with each other.
  - Observed phenomena *cannot* be accounted for by rays:

Diffraction

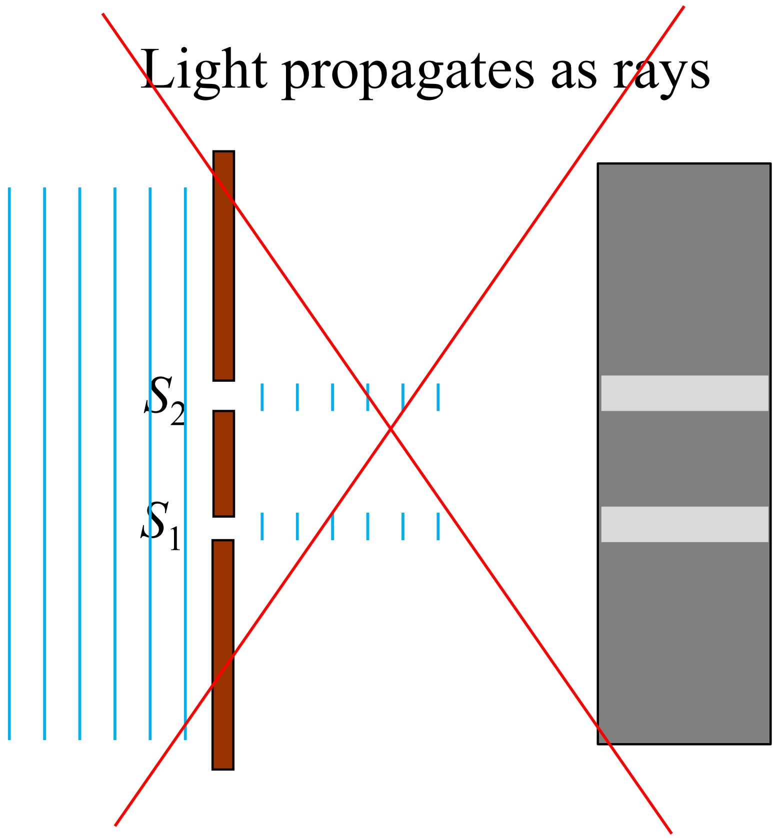


Interference

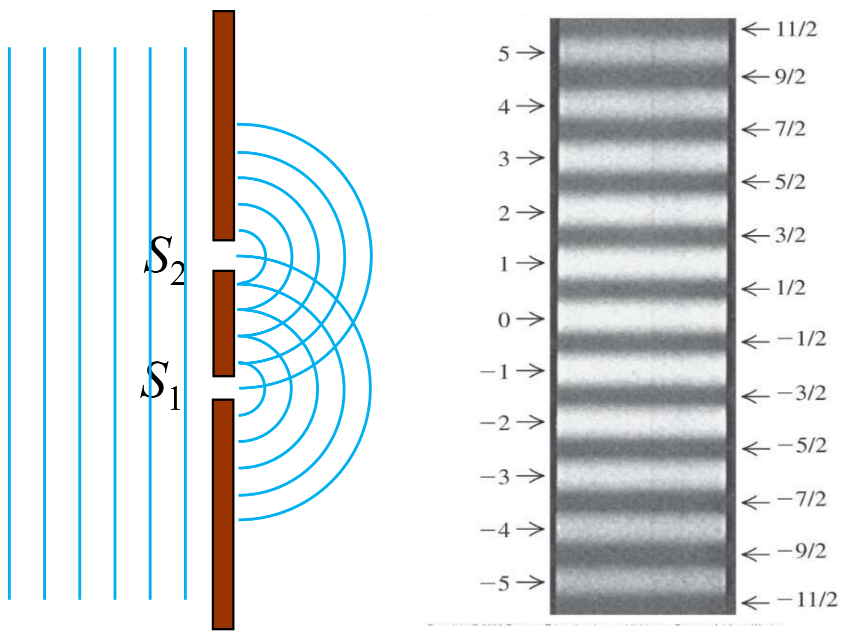
constructive/destructive interference patterns

Wave Nature of Light: Diffraction & Interference

Light propagates as rays

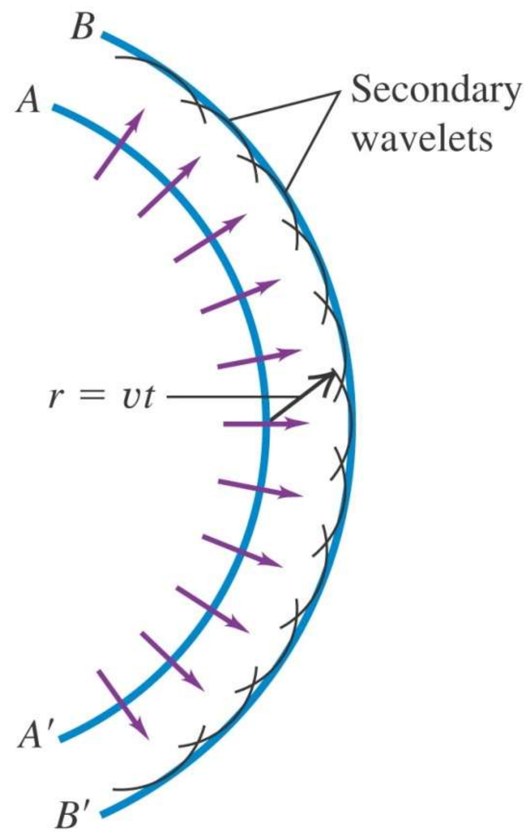


Light propagates as waves



Huygens’ Principle

Christiaan Huygens (1629-195): The Huygens’ Principle can be used to predict the spreading of light wave. It is a geometrical construction using every point on a wave front as the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.



Interference and Superposition

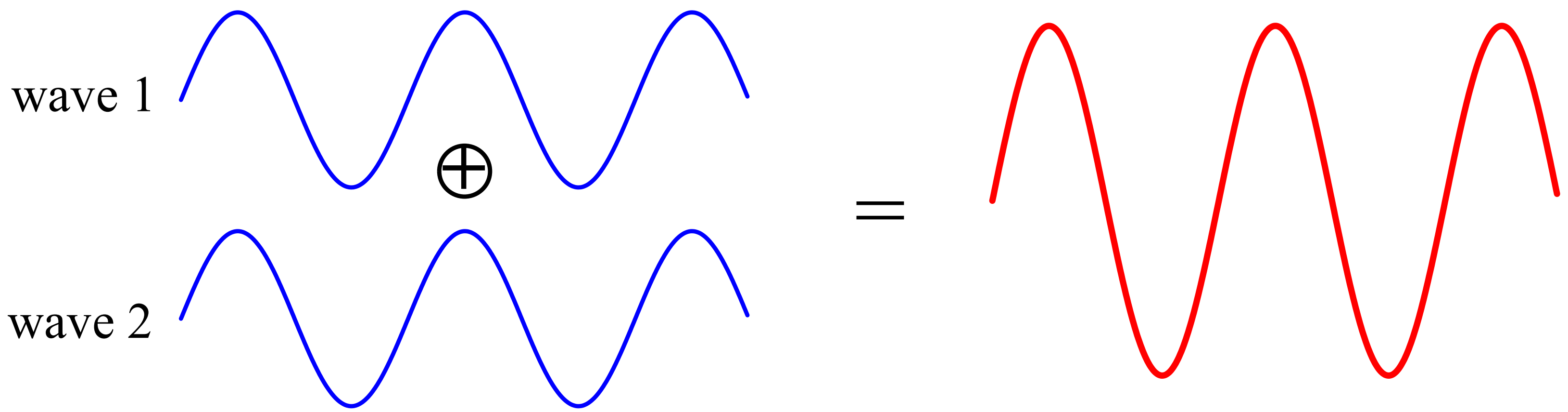
- **Interference** refers to a situation in which two or more waves overlap in space.

  → The resultant displacement at any point is governed by the **principle of superposition**.

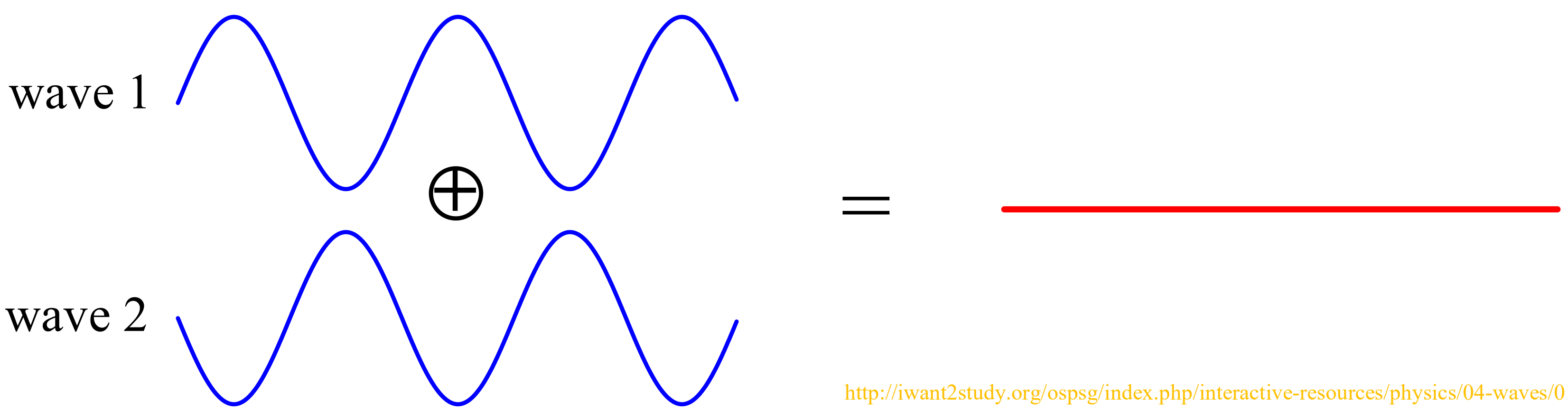
  “the *resultant* disturbance at any point and at any instant is found by *adding* the instantaneous disturbance that would be produced at the point by the *individual* waves as if each waves was present *alone*.”

Superposition and Interference

Constructive Interference (+ peaks aligns w/ + peaks)



Destructive Interference (+ peaks aligns w/ - peaks: $\lambda/2$ apart)



http://iwant2study.org/ospsg/index.php/interactive-resources/physics/04-waves/01-superposition/384-wave1d01

Conditions for Observable Sustained Interference

1. The sources have to be **coherent**

Coherent means that…

→ The individual waves must maintain a *constant phase relationship* (oscillate in unison) with each other.

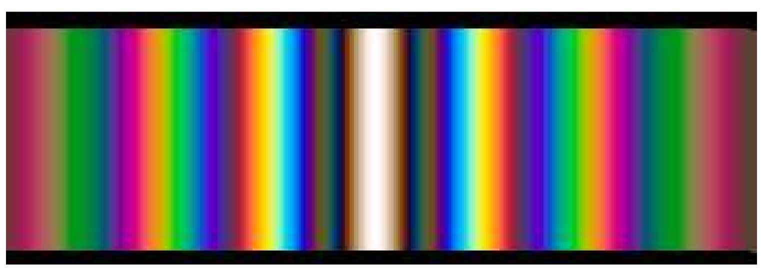
- e.g. two speakers driven by the same amplifier

- two regular light bulbs *don't* interfere since they are not coherent. (Emission from a light bulb is from a *thermal* process of *random* motions of charged particles in the filament.)

Conditions for Observable Sustained Interference

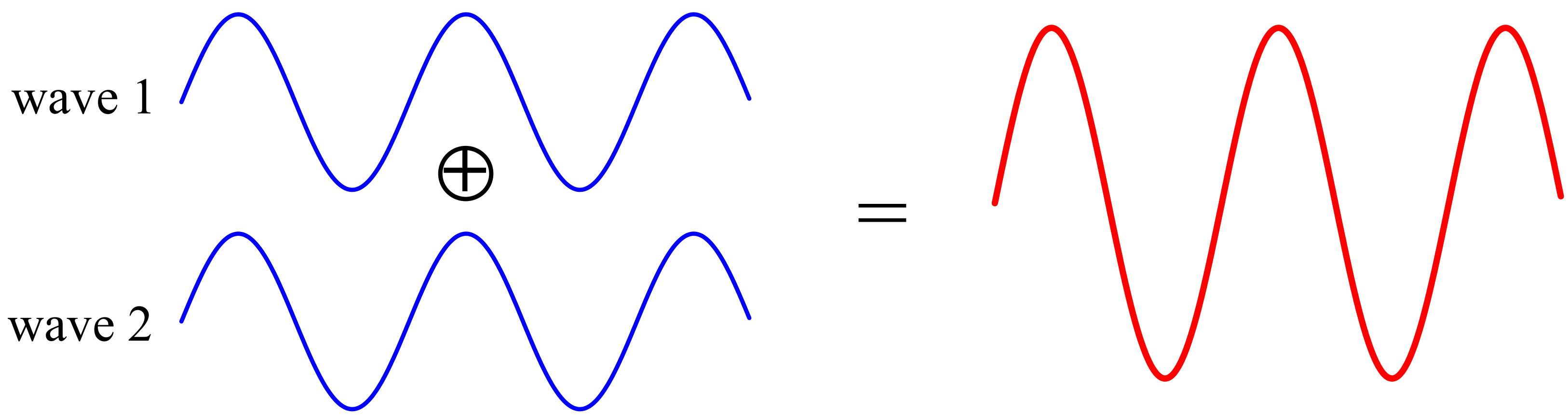
2. The waves need to have the same polarization.
3. Two or more interfering waves must have the same wavelength (monochromatic)

→ You can have white light interference pattern (if the source is coherent) but the effect will appear for different colors corresponding to the diff. interference patterns for diff. λ in the white light.

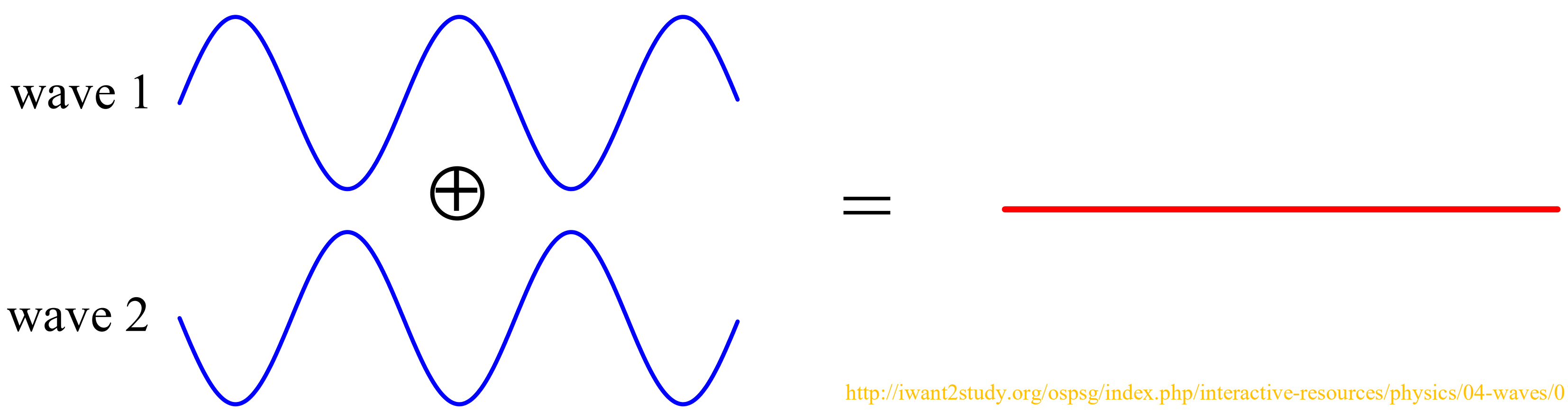


Superposition and Interference

Constructive Interference (+ peaks aligns w/ + peaks)



Destructive Interference (+ peaks aligns w/ - peaks: $\lambda/2$ apart)

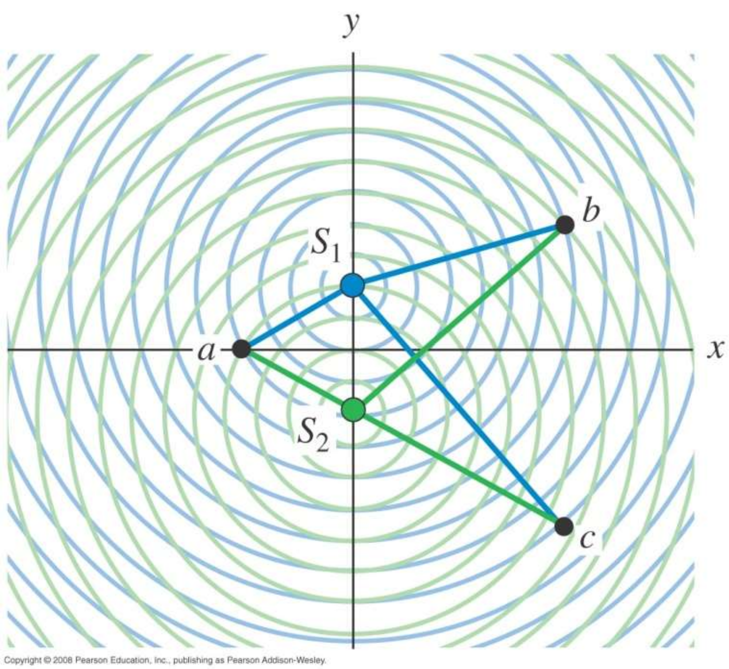


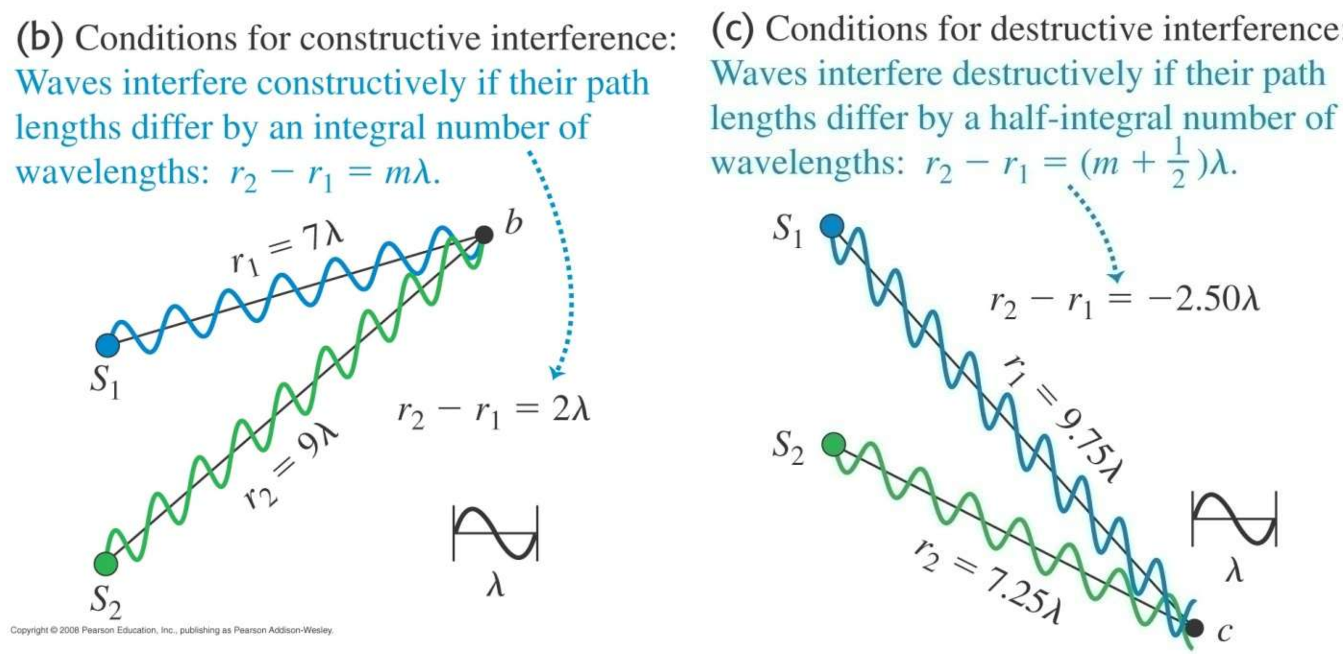
http://iwant2study.org/ospsg/index.php/interactive-resources/physics/04-waves/01-superposition/384-wave1d01

Interference and Path Difference

(a) Point *a* is symmetric with respect to the two coherent sources. Waves will arrive in phase *constructively*: $r_2 - r_1 = 0$.

$D_{\text{two sheets}}$





r_1 : distance to S_1

r_2 : distance to S_2

$r_2 - r_1 =$ **path difference**

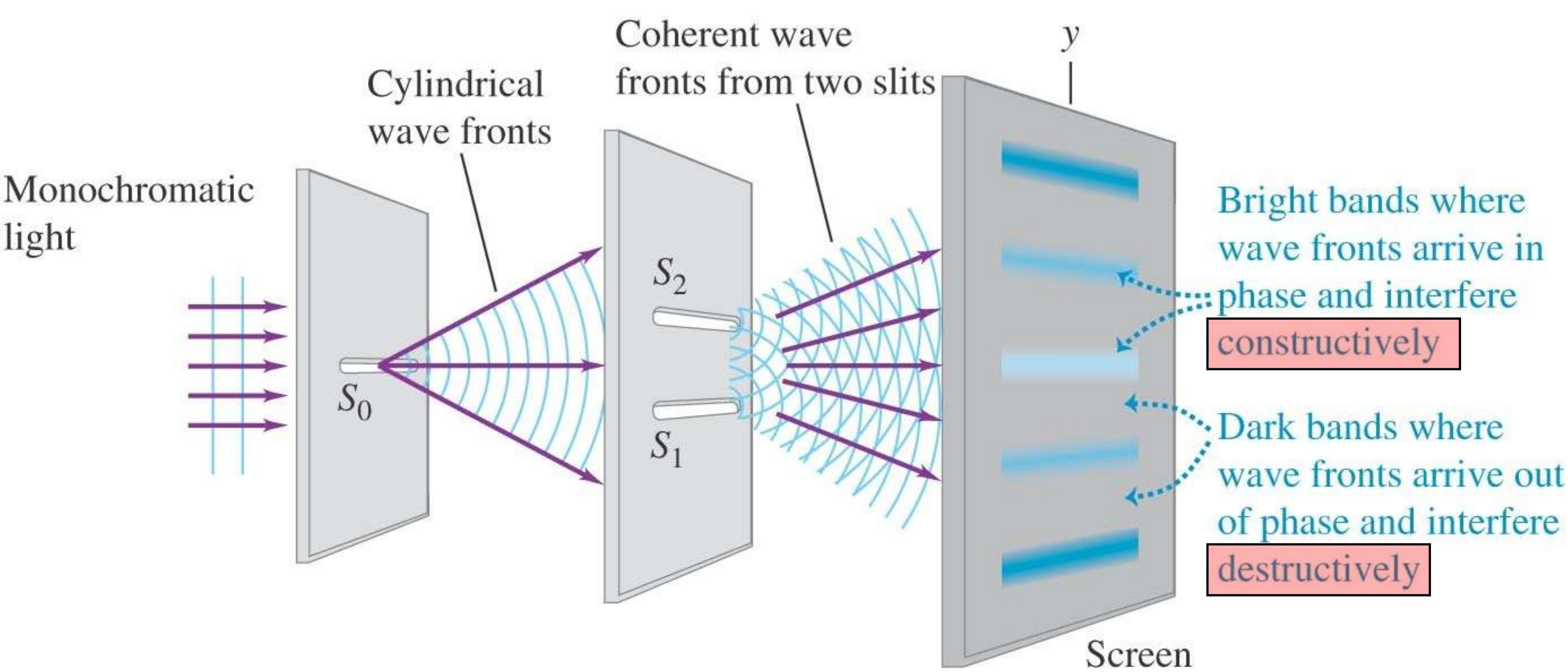
Constructive Inter.

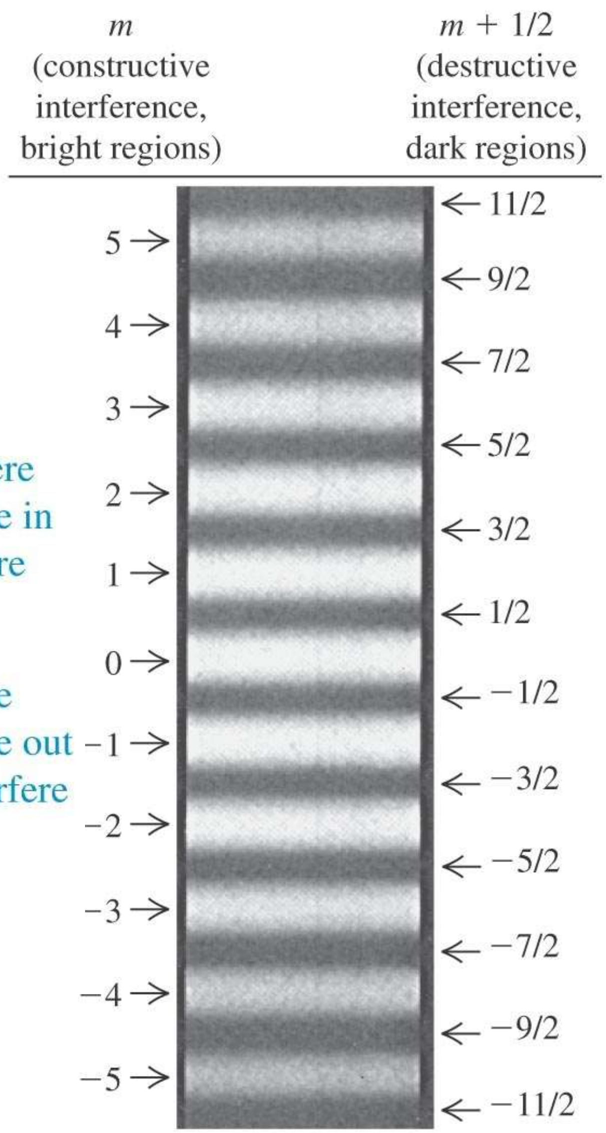
$$r_2 - r_1 = m\lambda$$
$$(m = 0, \pm 1, \pm 2, \cdots)$$

Destructive Inter.

$$r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda$$
$$(m = 0, \pm 1, \pm 2, \cdots)$$

Young's Double Slit Experiment





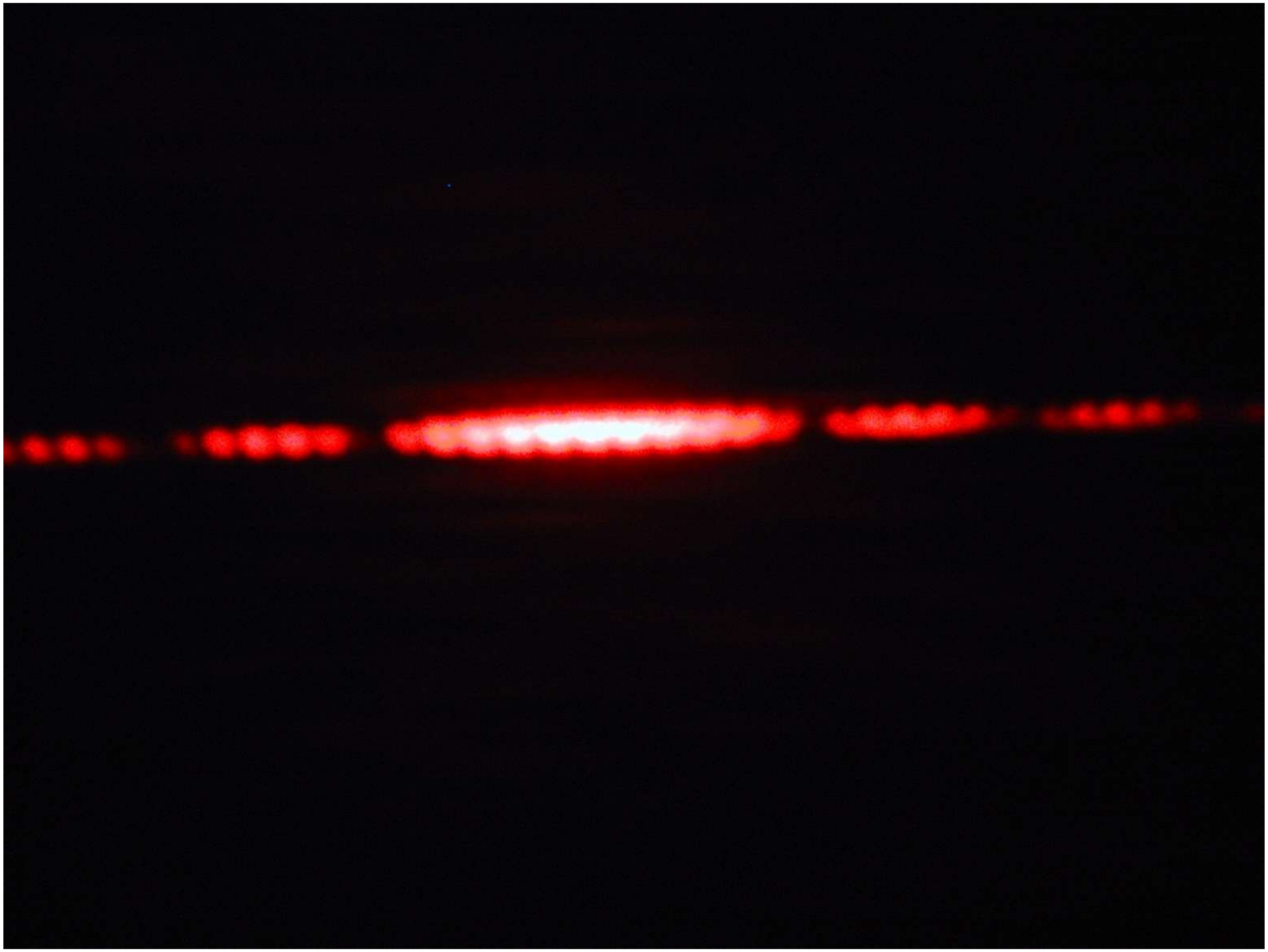
Spreading of light behind slits
Recall Huygen's Principle

$D_{2\ slits}$

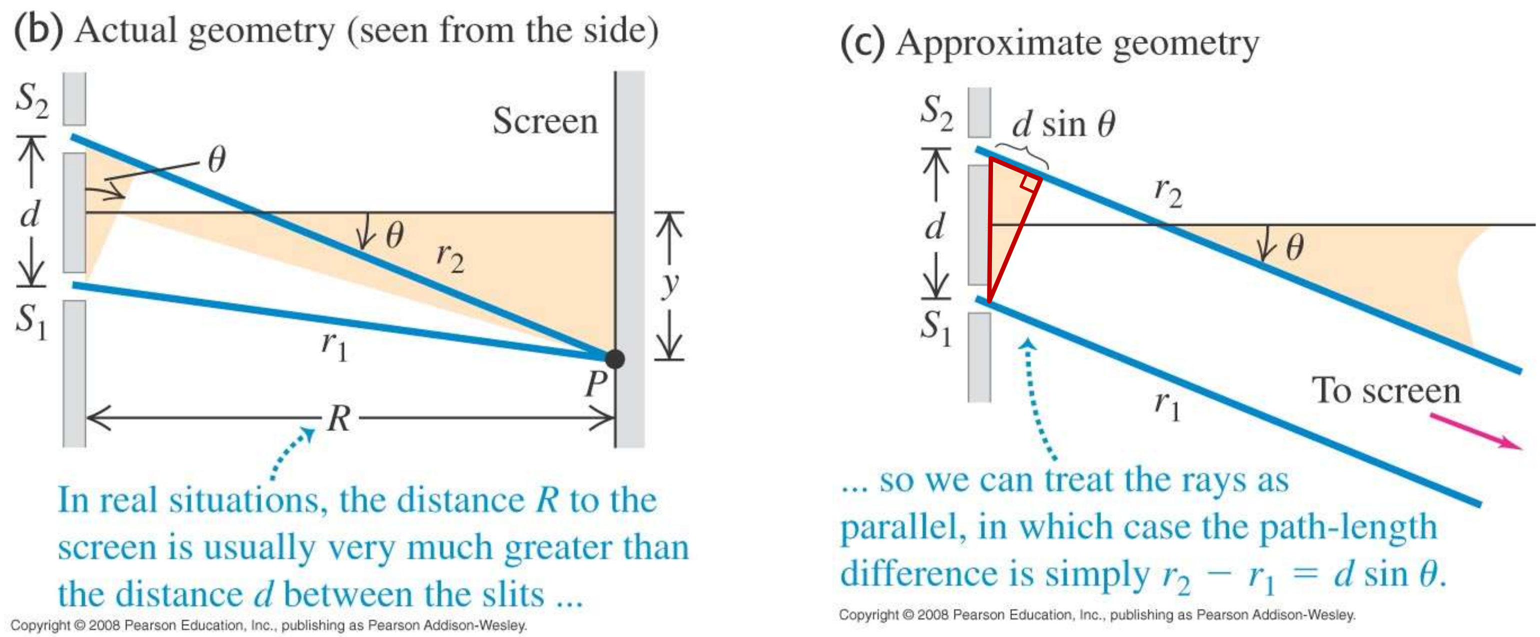
Double Slit Interference Demo

https://youtu.be/J-v7F4LWDvU

Young's Double Slit Experiment



Young's Double Slit Experiment



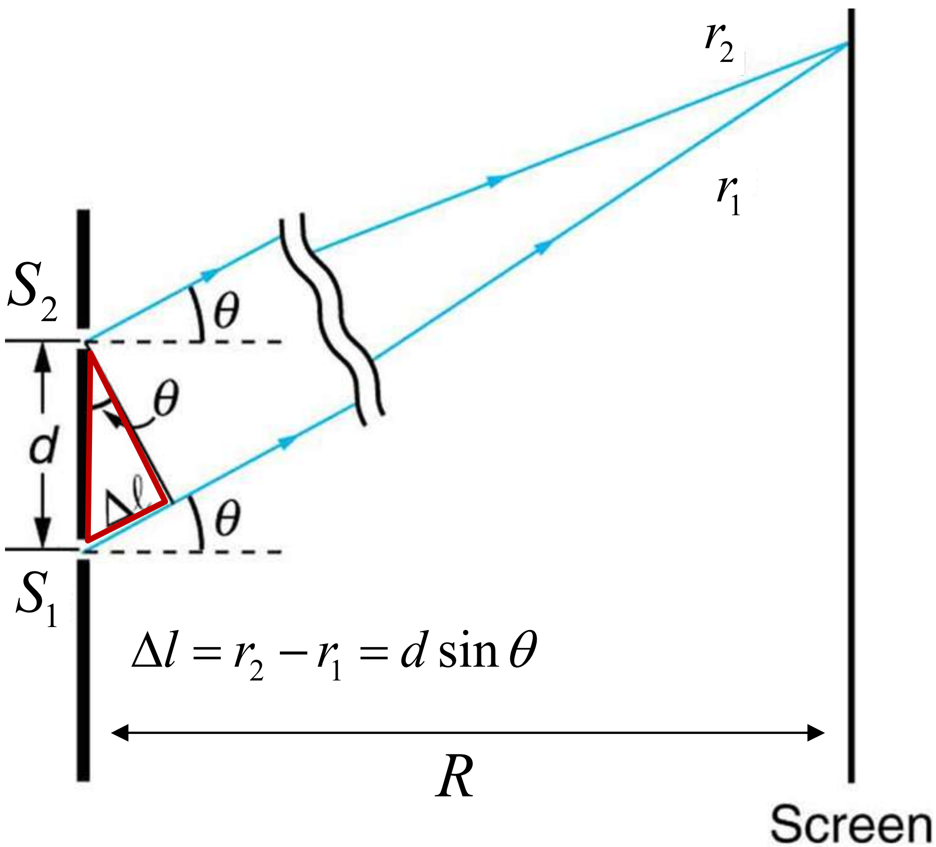
If screen is far away so that $R >> d$, we can assume rays from S_1 and S_2 to be approximately parallel and the red triangle becomes a right-triangle.

Then, from the simplified geometry (right panel), we have an explicit expression for the **path difference**:

$$r_2 - r_1 = d \sin \theta$$

(θ is the angular location of observation point P on the screen.)

Young's Double Slit Experiment



If screen is far away so that $R >> d$, we can assume rays from S_1 and S_2 to be approximately parallel and the small red triangle becomes a right-triangle.

Then, from the simplified geometry (right panel), we have an explicit expression for the **path difference**:

$$r_2 - r_1 = d \sin\theta$$

(θ is the angular location of observation point P on the screen.)

Constructive/Destructive Two-Slit Interference

Applying the conditions for constructive/ destructive interference, we have the following conditions:

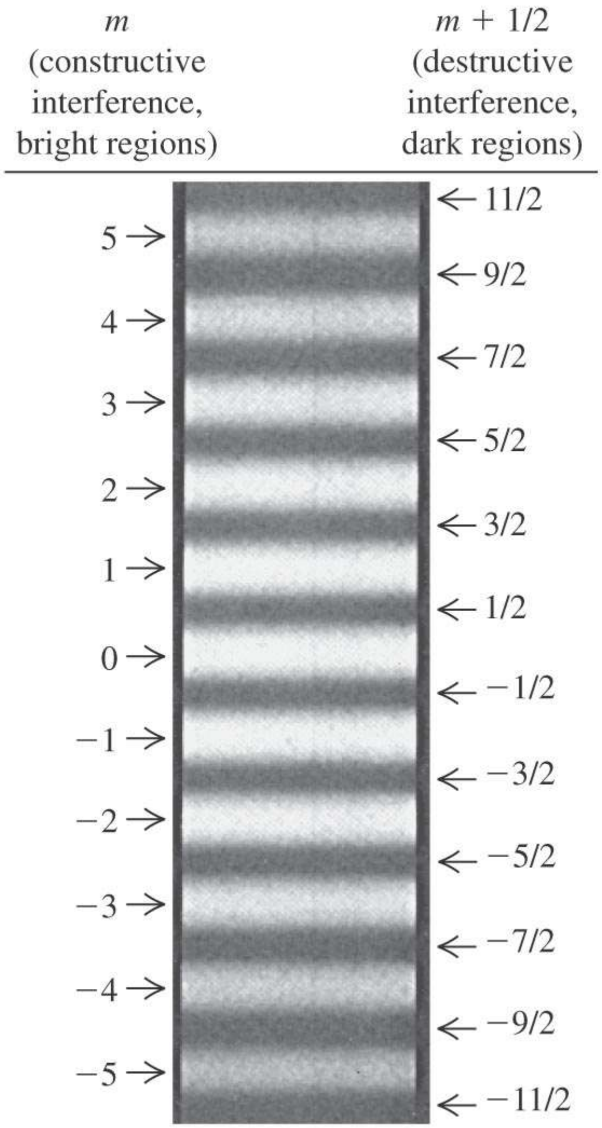
Constructive Interference: Two Slit Interference

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \cdots)$$

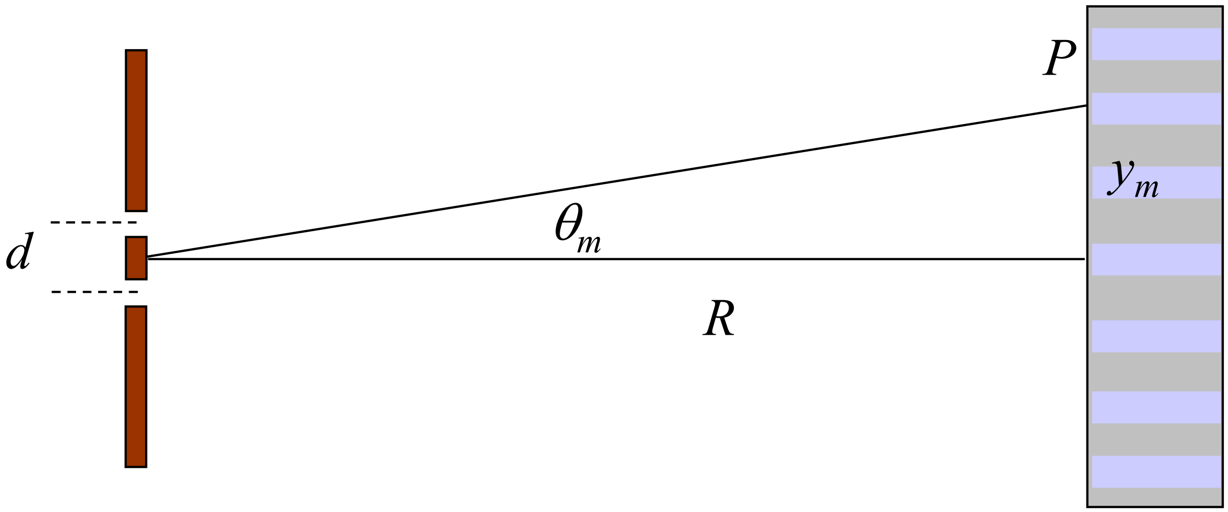
Destructive Interference: Two Slit Interference

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \cdots)$$

- The bright/dark bands in the pattern are called fringes
- m is the *order* of the fringes



Locating Fringes



In normal situations,

$$\begin{cases} R \sim 1m \\ d \leq \text{tenths of mm} \\ \lambda \leq \text{tenths of } \mu\text{m} \end{cases}$$

(400-700 nm)

So, typically, we have the condition that $d << R$ so that θ is *small*.

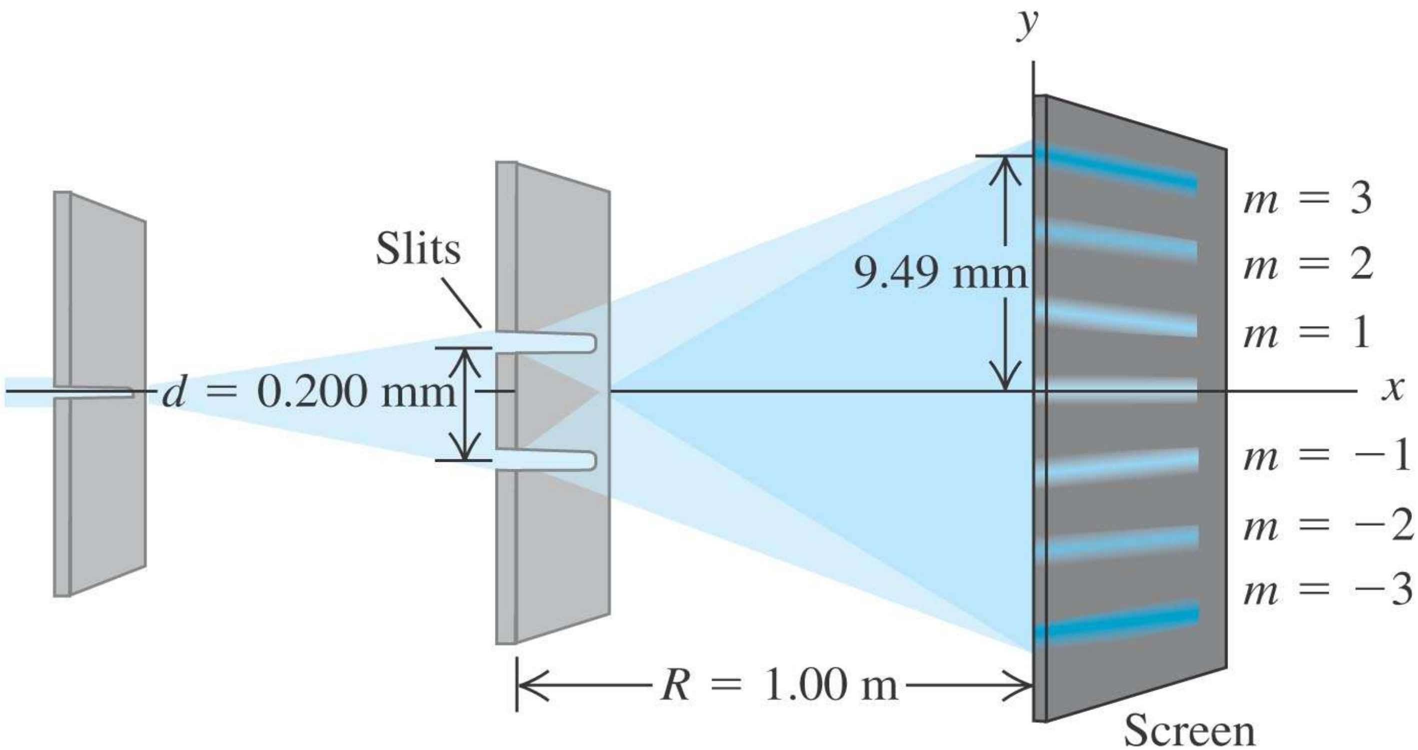
Thus, we can approximate $\sin\theta \cong \tan\theta \cong \theta$ (θ needs to be in radian !)

The linear distance to a particular ordered fringe (y_m) is given by: $y_m = R\tan\theta_m$

With the small angles approximation, we have:

$$y_m = R\tan\theta_m \cong R\sin\theta_m = R\frac{m\lambda}{d}$$

Example 35.1



Determine the wavelength of the light from location of y_3.

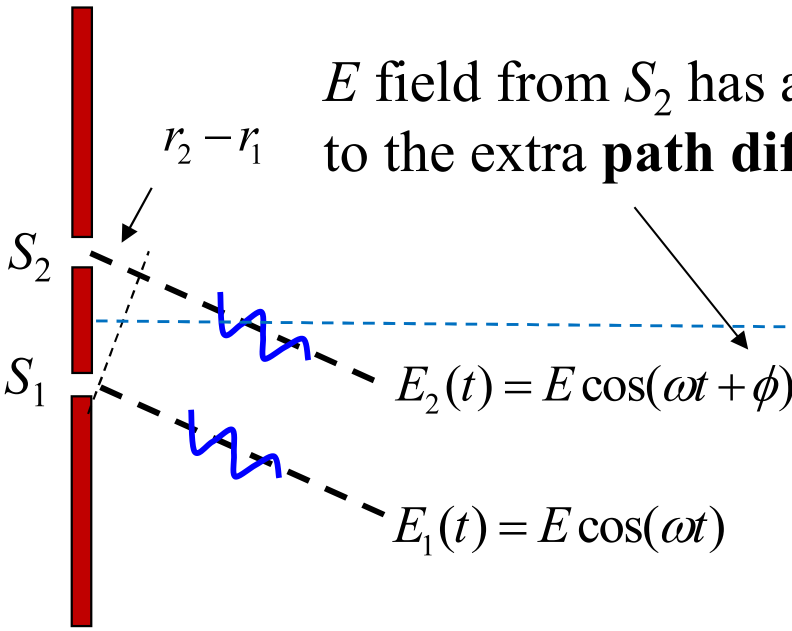
$$y_3 = R\frac{3\lambda}{d} \quad \rightarrow \quad \lambda = d\frac{y_3}{3R} = \left(0.2\times10^{-3}m\right)\frac{9.49\times10^{-3}m}{3\left(1.00m\right)} = 6.33\times10^{-7}m = 633nm$$

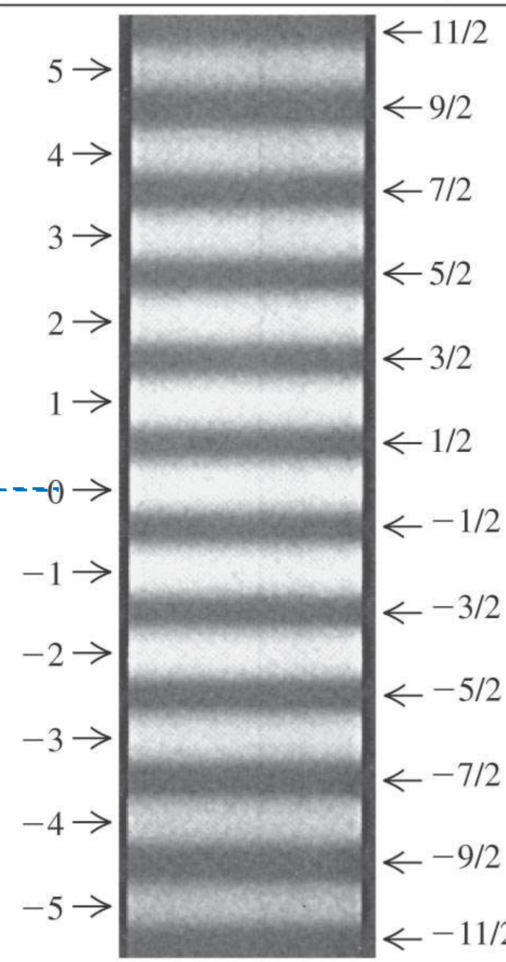
Intensity of Interference Patterns

Wish to find $I(\theta)$ on a screen far away …

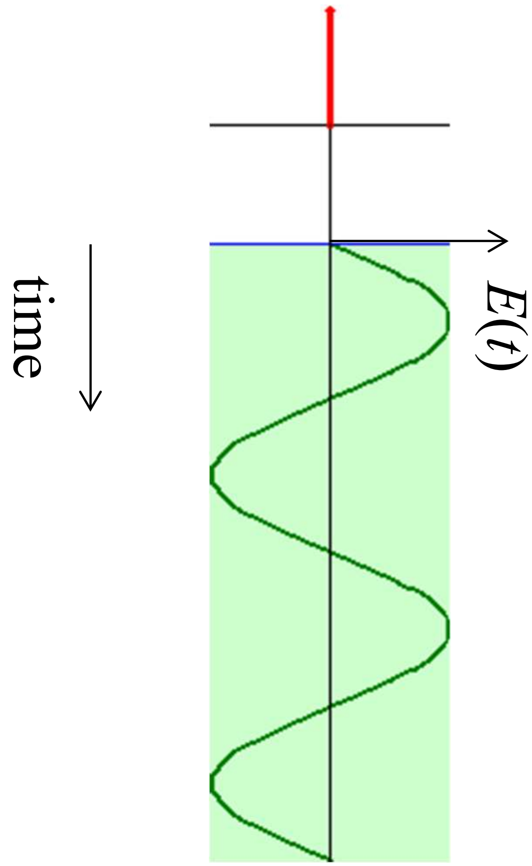
Let consider the E fields coming from the double slits:

E field from S_2 has a **phase lag** ϕ due to the extra **path difference**, r_2 - r_1.





Phasor in Action

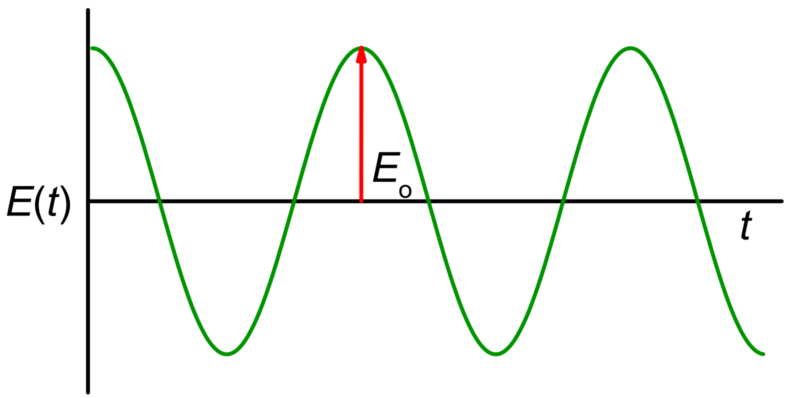


$$E(t) = E_o \cos(\omega t)$$

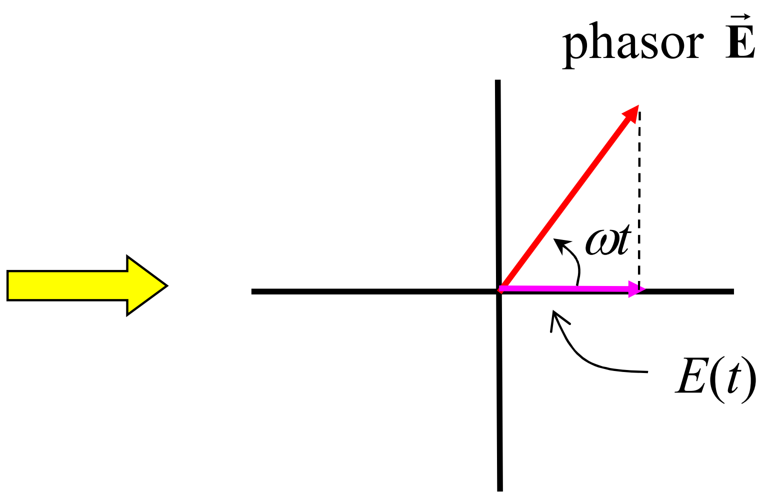
- E field as a vector (**phasor**) rotating in the x-y plane with an angular frequency ω.
- The time variation of this E field, $E(t)$ is given as the horizontal projection (light red) of the phasor (dark red).

Intensity of Interference Patterns

Phasor Representation of an *E* Field:



$$E(t) = E_o \cos(\omega t)$$

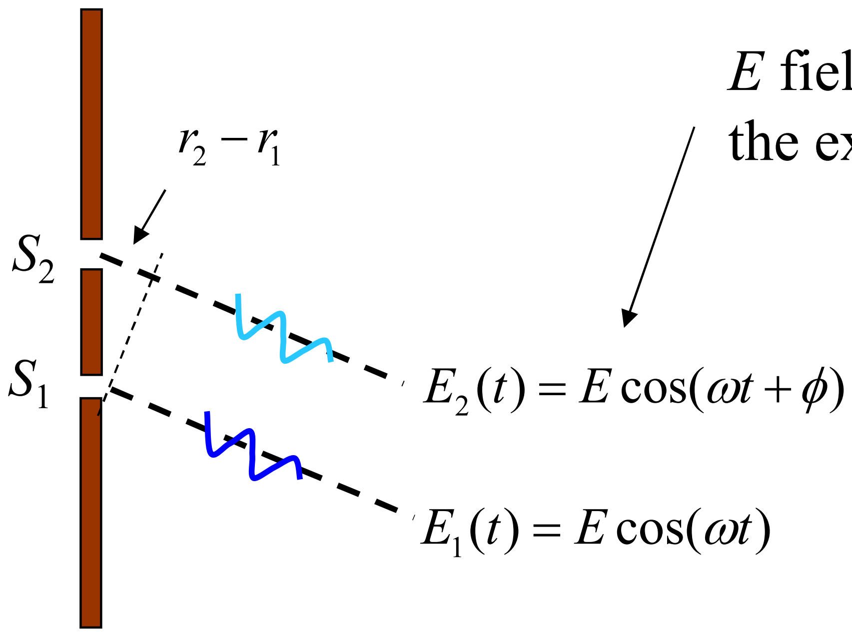


- *E* field as a vector (**phasor**) $\vec{\mathbf{E}}$ rotating in the x-y plane with an angular frequency ω.
- The time variation of this *E* field, $E(t)$ is given as the horizontal projection (light red) of the phasor $\vec{\mathbf{E}}$ (dark red).

Intensity of Interference Patterns

Recall that there are two coherent E fields with a slight *phase difference* coming from the double slits:

E field from S_2 has a **phase lag** ϕ due to the extra **path difference**, r_2 - r_1.

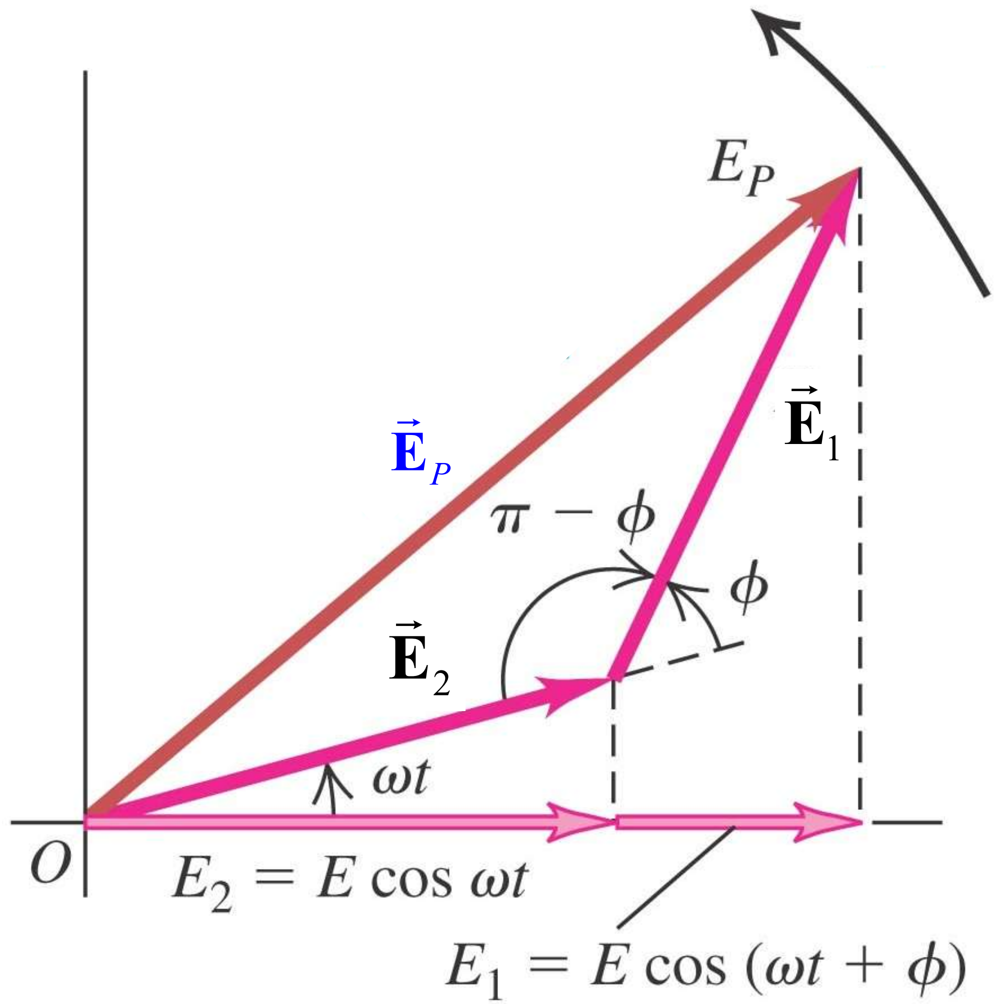


Intensity of Interference Patterns

At a given point on the screen far away from the two slits, the total E field at P, E_P, is given by the vector-sum of the two phasors $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$.

To find the magnitude of the resultant phasor $\vec{\mathbf{E}}_P$, E_P, we use the law of cosines.

$$E_P^2 = E^2 + E^2 - 2E^2 \cos(\pi - \phi)$$



Intensity of Interference Patterns

Using the symmetry of the cosine function, $\cos(\pi-\phi)=-\cos\phi$

we have, $E_P^2 = E^2 + E^2 - 2E^2\cos(\pi-\phi)$

$$E_P^2 = 2E^2 + 2E^2\cos\phi$$

$$E_P^2 = 2E^2(1+\cos\phi)$$

Using another trig identity, $1+\cos\phi = 2\cos^2(\phi/2)$

we have, $E_P^2 = 4E^2\cos^2\left(\frac{\phi}{2}\right)$. This gives, $\boxed{E_P = 2E\left|\cos\left(\frac{\phi}{2}\right)\right|}$.

Intensity of Interference Patterns

The intensity of an electromagnetic wave is given by the time average magnitude of the Poynting vector, S_{av}.

In general, the Poynting vector is proportional to the *square* of the magnitude of the electric field so for the intensity at P (eq 32.29),

We can write the expression as,

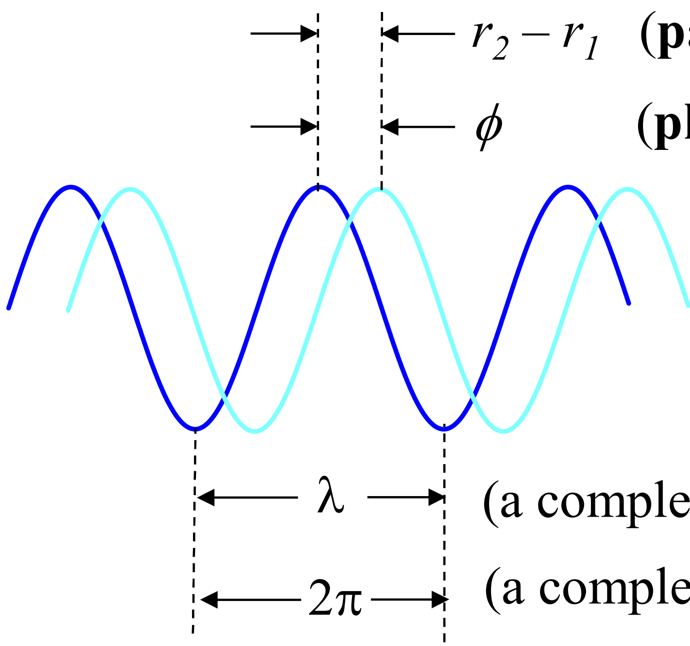
$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

where I_0 is the maximum intensity when $\phi = 0$.

Note: when the two waves are *in phase* ($\phi = 0$, straight ahead), the resultant intensity is at maximum ($I = I_0$) and when the two waves are exactly half-cycle *out of phase* ($\phi = \pi$), the resultant intensity is identically zero.

Phase Difference relates to Path Difference

Here, we have the lighter cyan wave slightly ahead of the blue wave.



$r_2 - r_1$ (**path difference** measured in *length*)

ϕ (**phase difference** measured in *radians*)

λ (a complete cycle measured in *wavelength*)

2π (a complete cycle measured in *phase*)

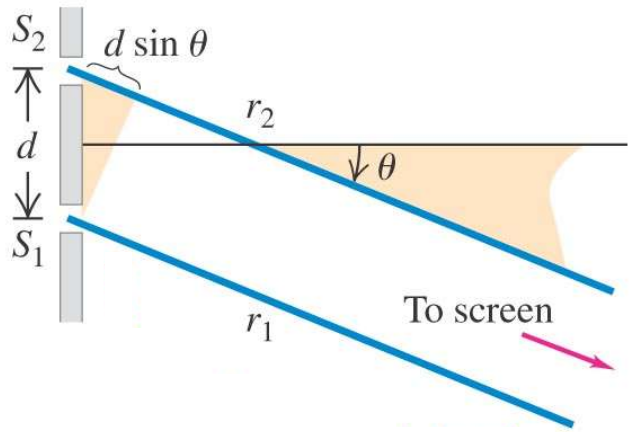
This gives the relation, $\dfrac{\phi}{2\pi} = \dfrac{r_2 - r_1}{\lambda}$ $\Longrightarrow$ $\boxed{\phi = \dfrac{2\pi}{\lambda}\left(r_2 - r_1\right) = k\left(r_2 - r_1\right)}$

where $k = 2\pi/\lambda$ is called the *wave number*.

Phase Difference depends on Path Difference

From our geometry, we have the following picture for the path difference:

$$r_2 - r_1 = d\sin\theta$$



θ is the *angular* position of the observation point P

Substituting this into to our previous equation, we have:

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = \frac{2\pi d}{\lambda}\sin\theta$$

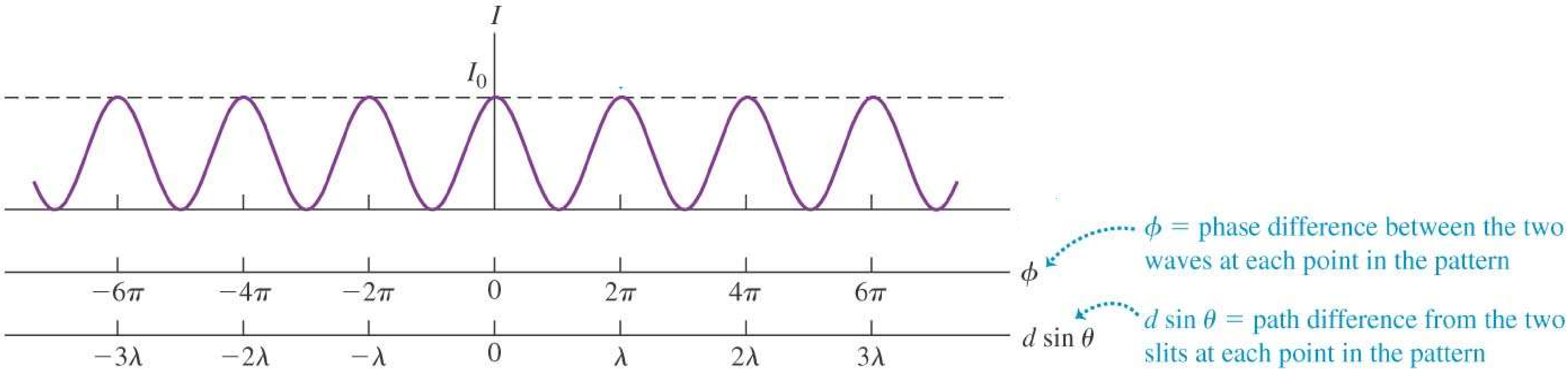
NOTE: We expressed one full cycle as 2π so that ϕ has to be in radian!

Intensity in Two-Slit Interference

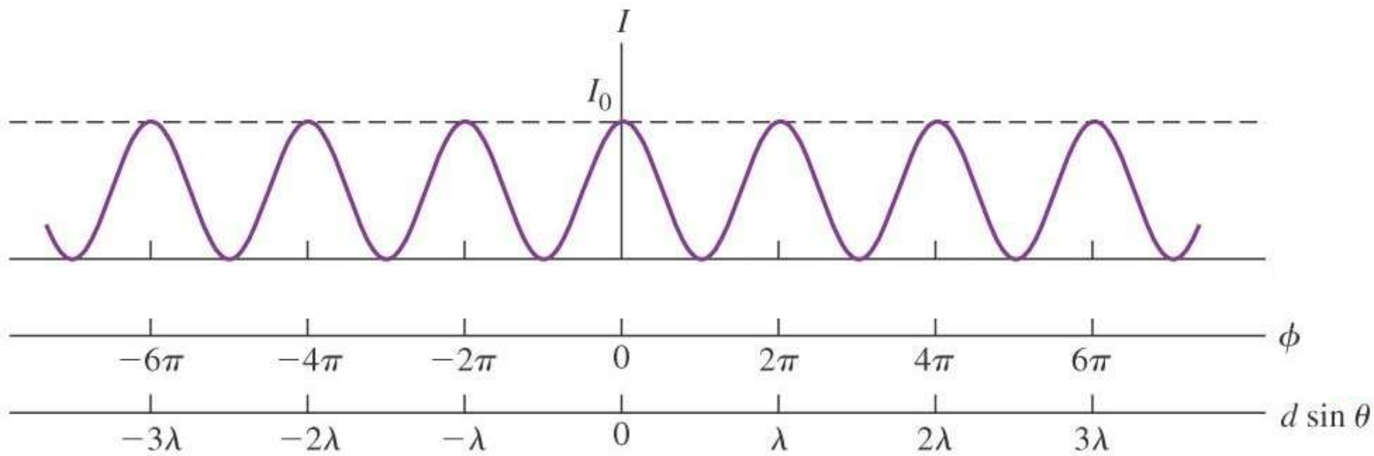
Putting the expression for the phase difference into our intensity equation for a two-slit interference pattern, we then have,

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right) = I_0 \cos^2\left(\frac{1}{2}\frac{2\pi d}{\lambda}\sin\theta\right)$$

$$\boxed{I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}$$



Intensity in Two-Slit Interference



$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

From the intensity equation, we can re-derive the conditions for the bright (*maximum*) and dark (*minimum*) fringes:

Maximum occurs when: $\frac{\pi d}{\lambda}\sin\theta = m\pi \quad \rightarrow \quad d\sin\theta = m\lambda \quad (m = 0, \pm 1, \cdots)$

Minimum occurs when: $\frac{\pi d}{\lambda}\sin\theta = (m + \frac{1}{2})\pi \quad \rightarrow \quad d\sin\theta = (m + \frac{1}{2})\lambda \quad (m = 0, \pm 1, \cdots)$

Interference in Thin Films

Color fringes observed from an oil slick on water or on a soap bubble are the white-light *interference* patterns produced by the *reflected* light off a *thin film* of oil or soap.