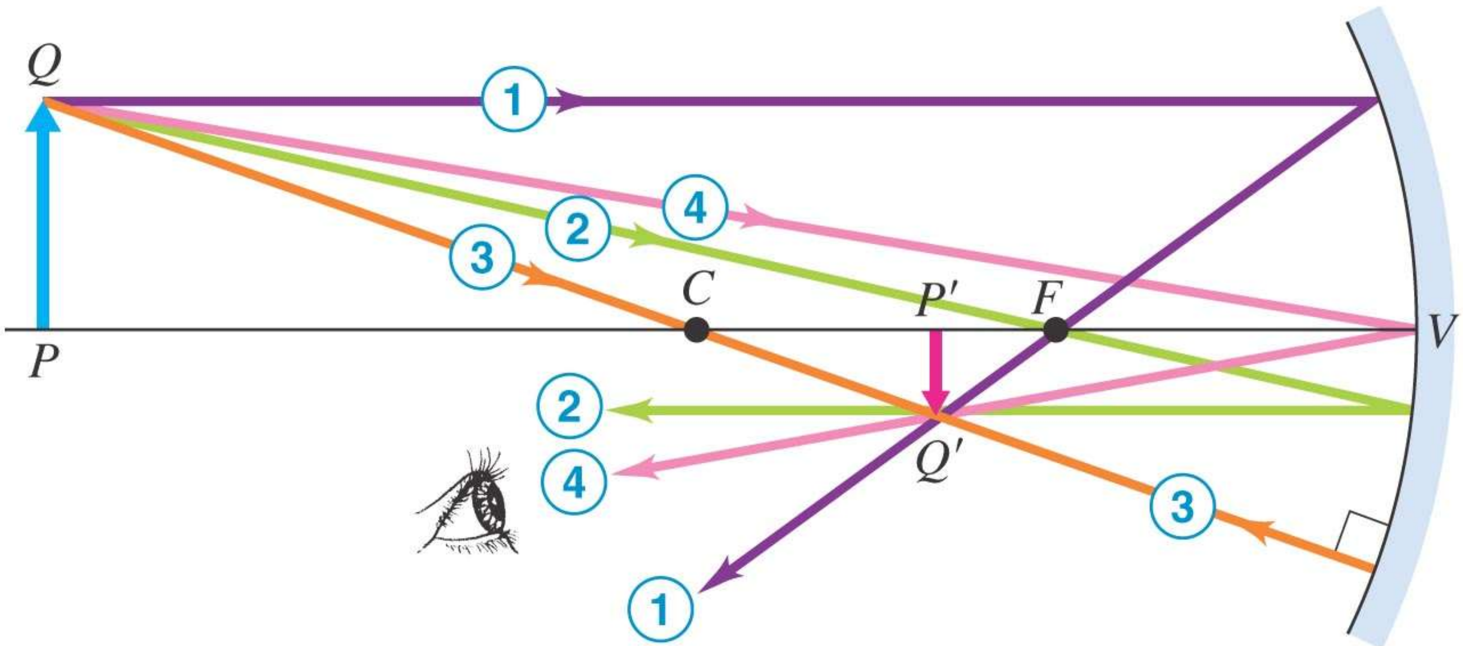
Geometric Methods: Rays Tracing

Principal rays for concave mirror



(1) Ray parallel to axis reflects through focal point.

(2) Ray through focal point reflects parallel to axis.

(3) Ray through center of curvature intersects the surface normally and reflects along its original path.

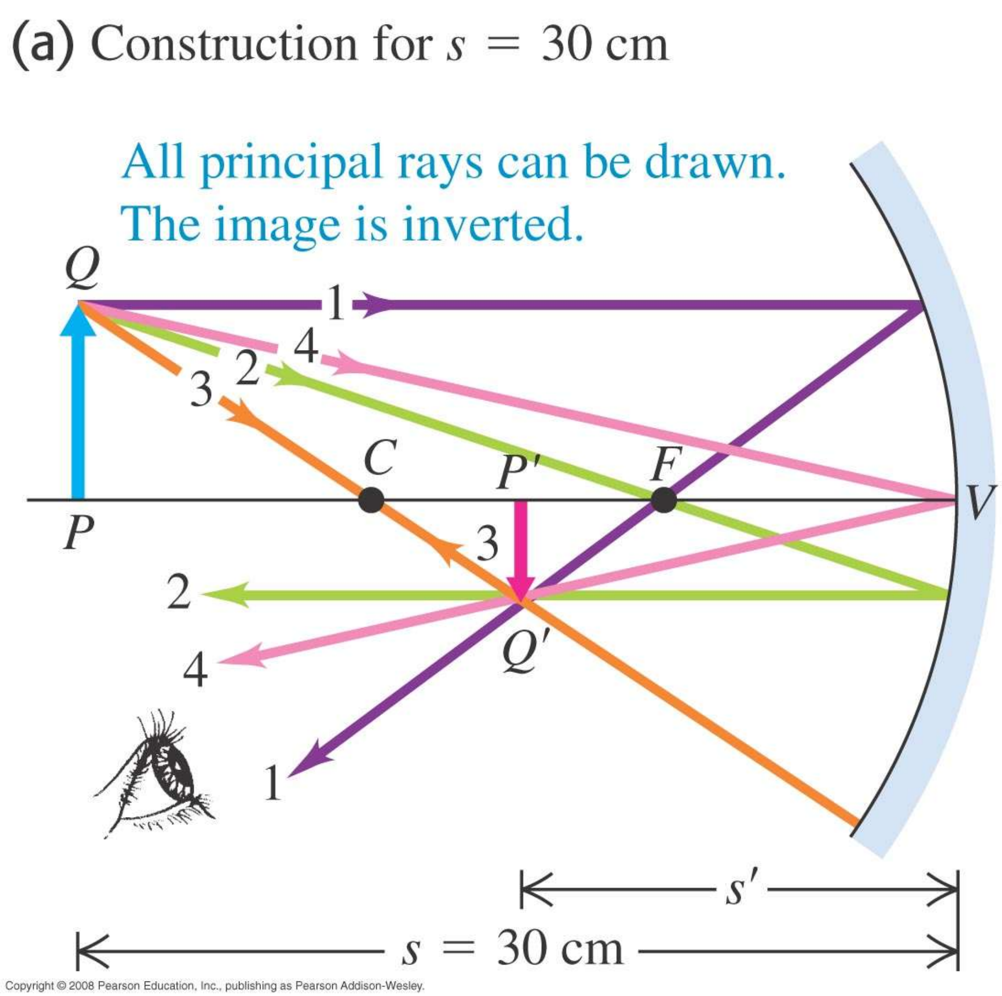
(4) Ray to vertex reflects symmetrically around optic axis.

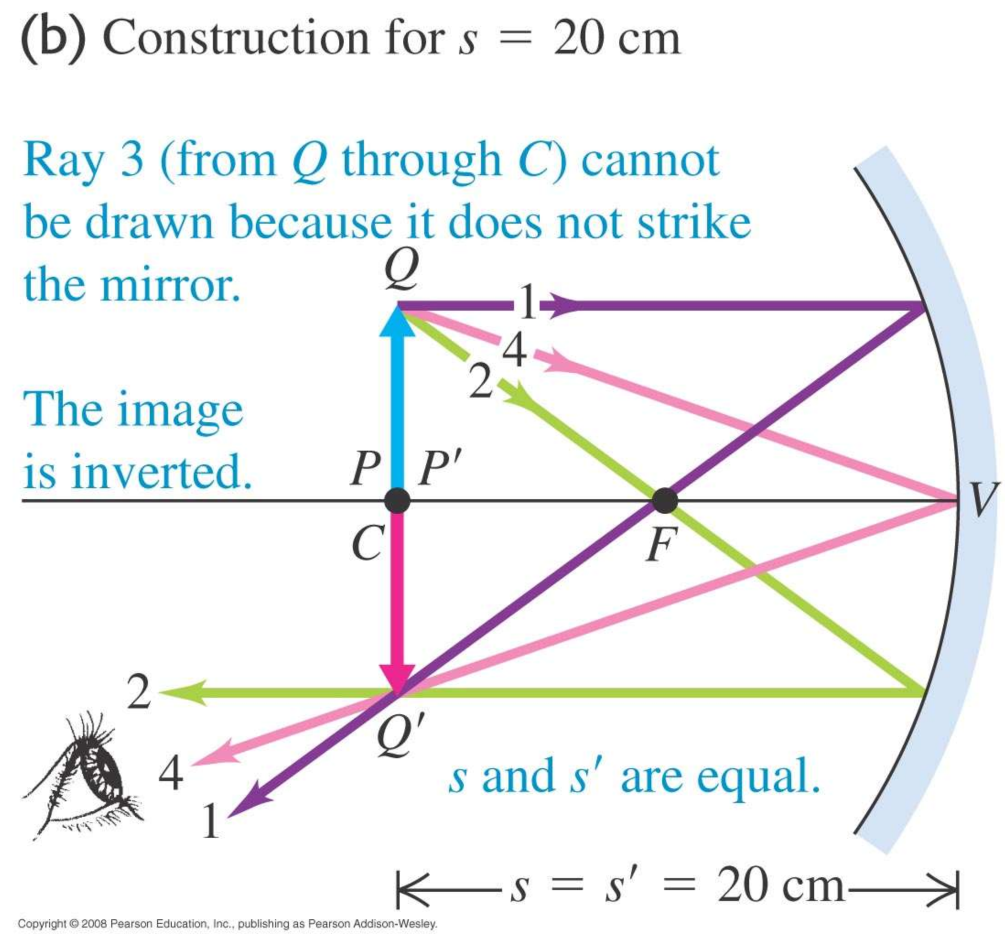
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Example 34.4: Concave Mirror

$P > C > F$

$P = C$

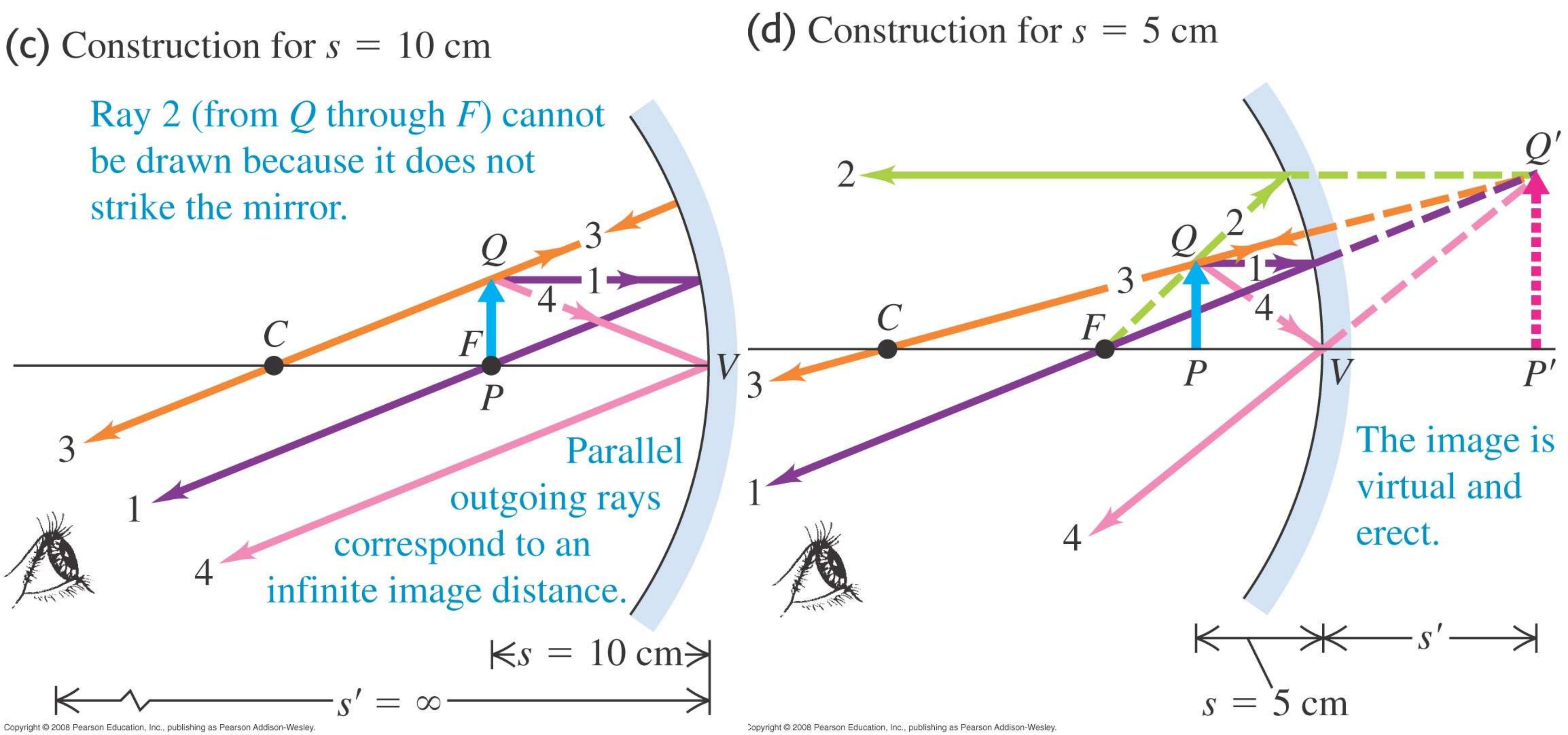




Example 34.4: Concave Mirror

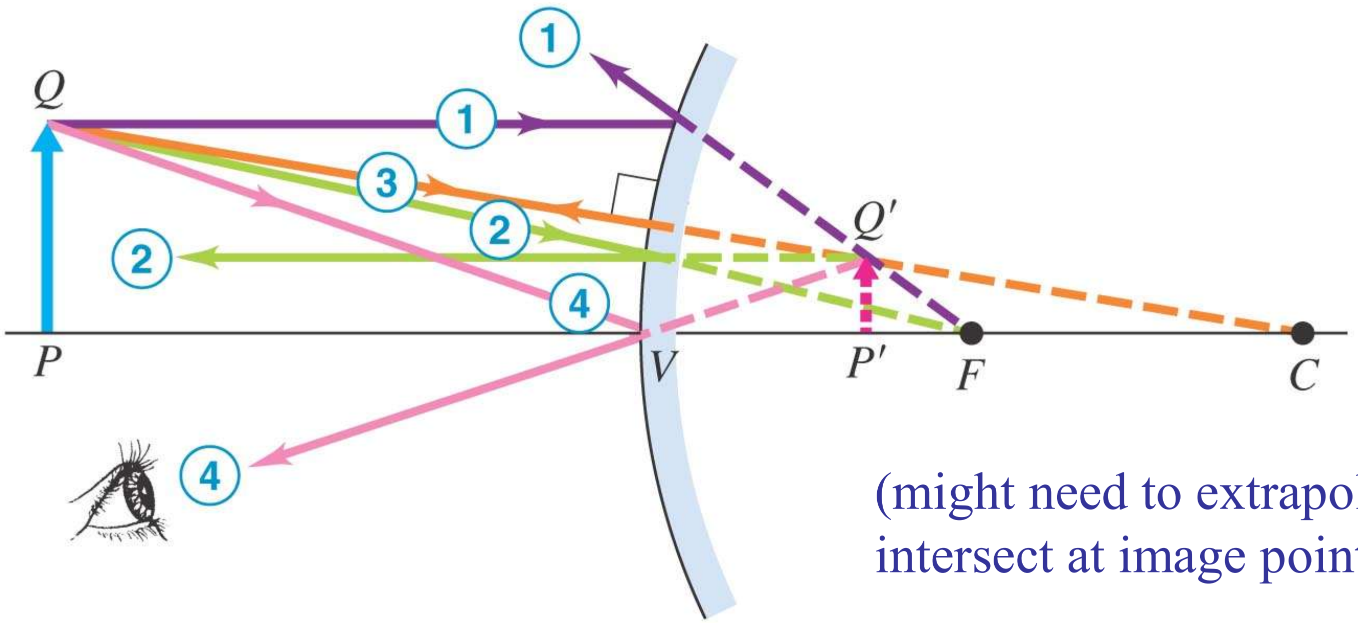
$P = F$

$P < F < C$



Geometric Methods: Rays Tracing

Principal rays for convex mirror



(might need to extrapolate lines to intersect at image point)

1. Reflected parallel ray appears to come from focal point.
2. Ray toward focal point reflects parallel to axis.
3. As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
4. As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

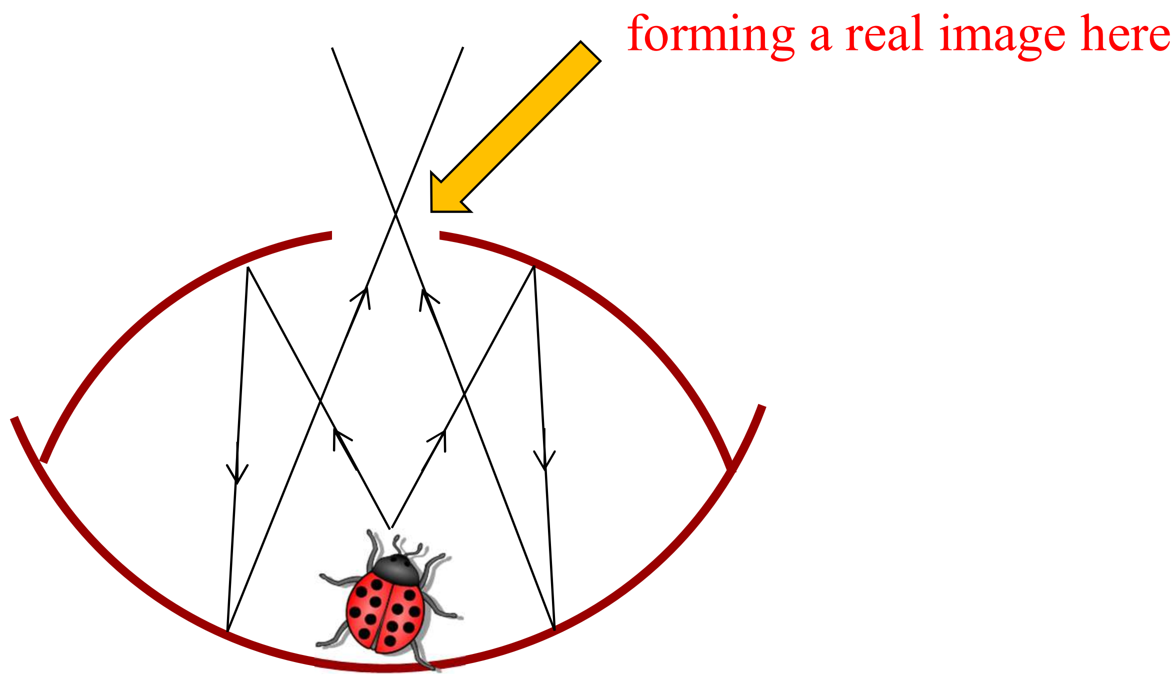
Example 34.4: Concave Mirror

Mirrors & Thin Lens Applet (by Fu-Kwun Hwang)

http://www.physics.metu.edu.tr/~bucurgat/ntnujava/Lens/lens_e.html

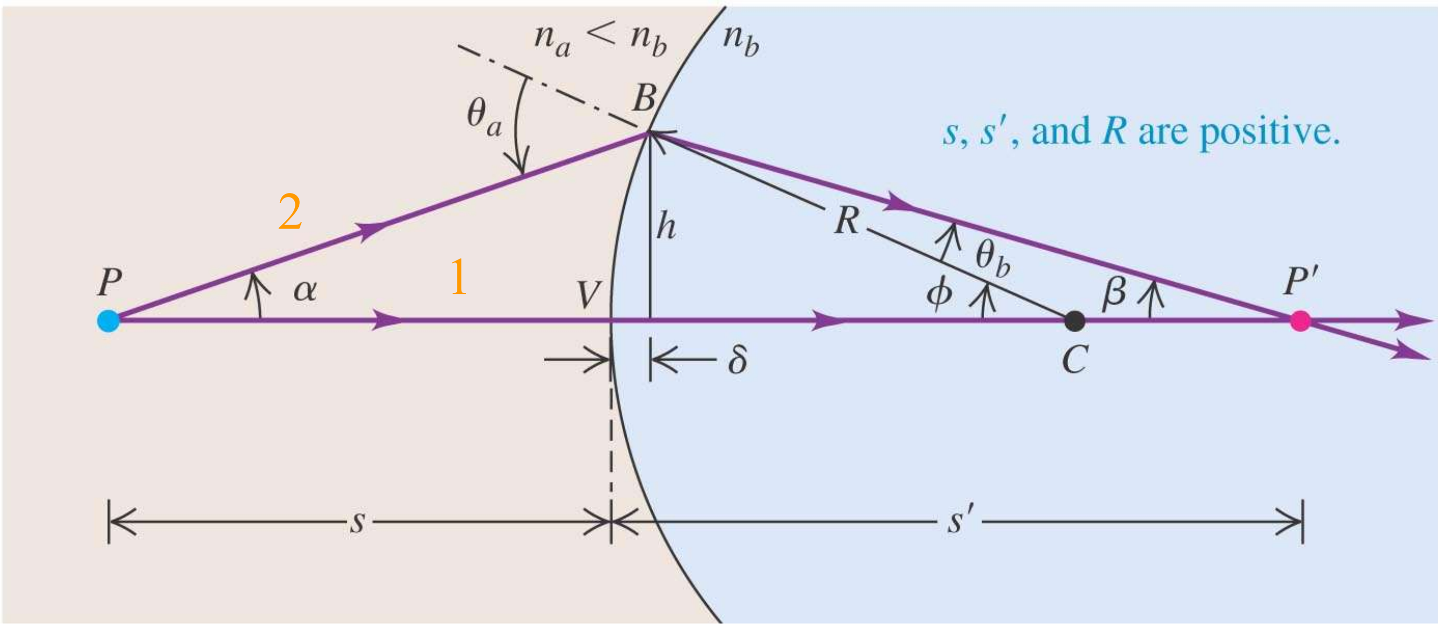
This applet is for both mirrors and thin lens. Use the drop down menu to choose.

Demo with Two Circular Mirrors



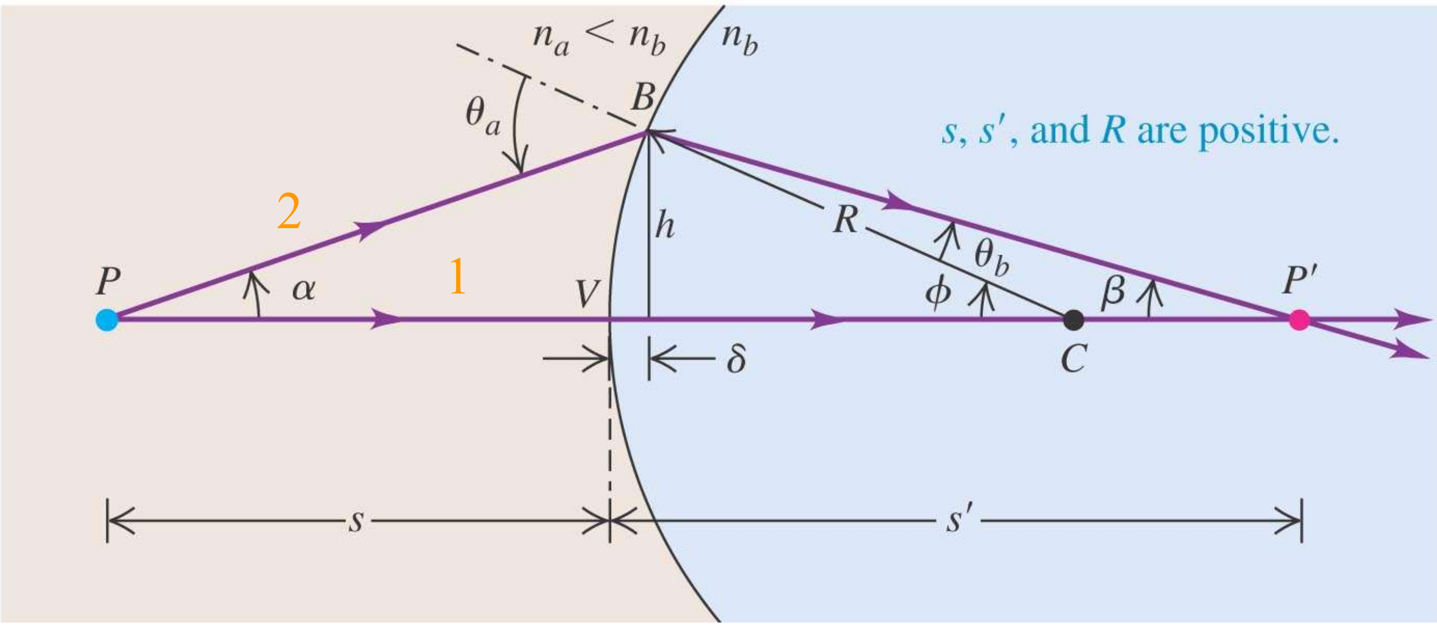
$D_{sp\ mirror}$ $D_{double\ mirror}$

Refraction at a Spherical Surface



- Ray 1 from P going through V (normal to the interface) will not suffer any deflection.
- Ray 2 from P going toward B will be refracted into n_b according to Snell's law.
- Image will form at P' where these two rays converge.

Refraction at a Spherical Surface



At B, Snell's law gives,

$$n_a \sin\theta_a = n_b \sin\theta_b$$

From $\triangle PBC$, $\theta_a = \alpha + \phi$

From $\triangle P'BC$, $\phi = \beta + \theta_b$

$$\rightarrow \theta_b = \phi - \beta$$

From trigonometry, we also have,

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta}$$

$$\tan\phi = \frac{h}{R-\delta}$$

Refraction at a Spherical Surface

Again, consider only paraxial rays so that the incident angles are small, we can use the small angle approximations: **sin** $\theta \sim$ **tan** $\theta \sim \theta$.

With this, Snell's law becomes: $n_a\theta_a = n_b\theta_b$

Substituting θ_a and θ_b from previous slide, we have,

$$n_a(\alpha+\phi) = n_b(\phi-\beta)$$

$$n_a\alpha + n_a\phi = n_b\phi - n_b\beta$$

$$n_a\alpha + n_b\beta = (n_b - n_a)\phi$$

With the small angle approximations, the trig relations reduce to,

$$\alpha \simeq \tan\alpha = \frac{h}{s} \qquad \beta \simeq \tan\beta = \frac{h}{s'} \qquad \phi \simeq \tan\phi = \frac{h}{R}$$

Refraction at a Spherical Surface

Substituting these expressions for α, β, and ϕ into Eq. and eliminating the common factor h, we then have,

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

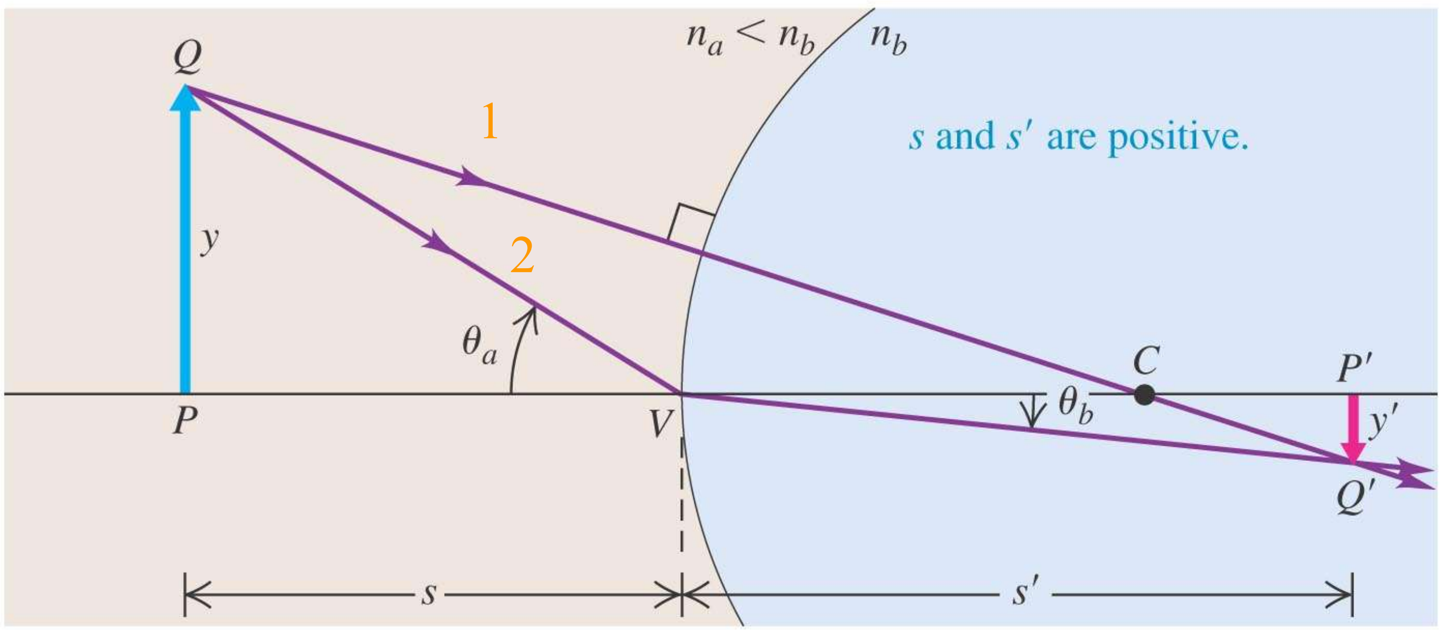
(object-image relationship, spherical refracting surface)

Similar to Spherical Mirrors, for Refracting Spherical Surfaces, the sign convention for the radius of curvature, R, is the same:

- R is + when the center of curvature C is on the same side as the outgoing light and – otherwise.

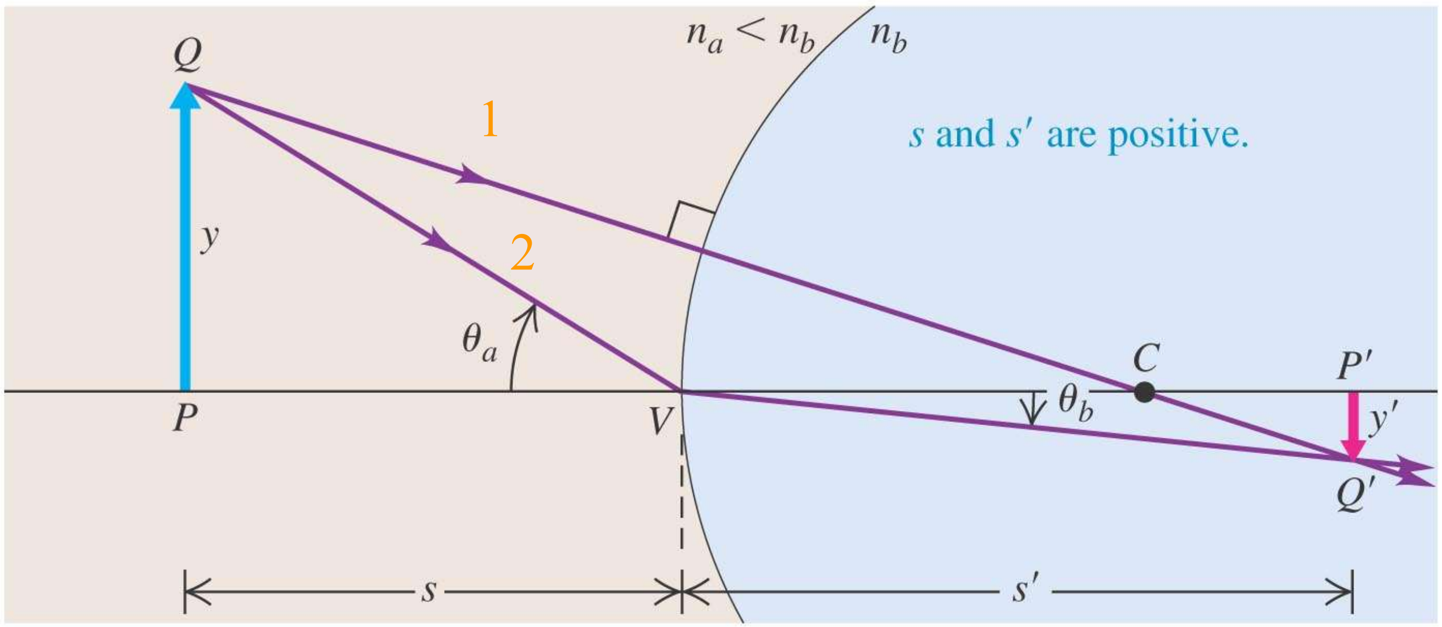
Refraction at a Spherical Surface

To calculate the lateral magnification m, we consider the following rays:



- Ray 1 from Q going toward C (along the normal to the interface) will not suffer any deflection.
- Ray 2 from Q going toward V will be refracted into n_b according to Snell's law.

Refraction at a Spherical Surface



From geometry, we have the following relations,

$$\tan\theta_a = y/s \qquad \tan\theta_b = -y'/s'$$

From Snell's law, we have, $n_a \sin\theta_a = n_b \sin\theta_b$

Refraction at a Spherical Surface

Using the small angle approximation again (sin $\theta \sim$ tan θ), the Snell's law can be rewritten as,

$$n_a \sin\theta_a = n_b \sin\theta_b \xrightarrow{\sin\theta \simeq \tan\theta} n_a \frac{y}{s} = -n_b \frac{y'}{s'}$$

Substituting these into the definition for lateral magnification, we have,

$$m = \frac{y'}{y} \quad \rightarrow \boxed{m = -\frac{n_a s'}{n_b s}}$$

(lateral magnification,
spherical refracting surface)

Refraction at a Flat Surface

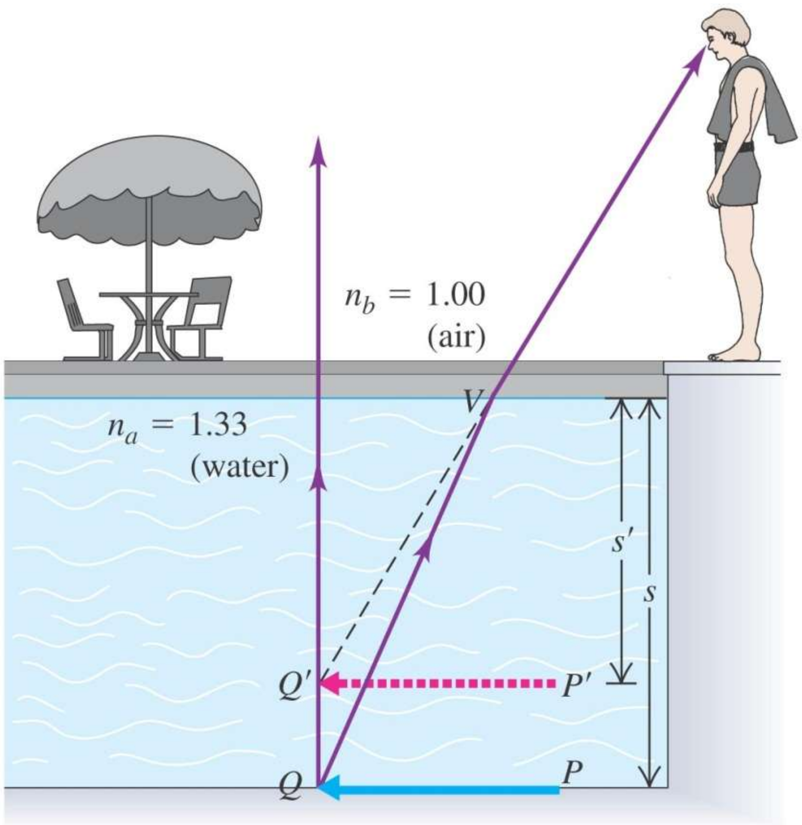
For a flat surface, we have $R = \infty$. Then, the Object-Image relation can be reduced simply as,

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{\infty} = 0$$

virtual

$$\rightarrow \quad \frac{n_a}{s} = -\frac{n_b}{s'} \quad \rightarrow \quad \frac{n_a s'}{n_b s} = -1$$

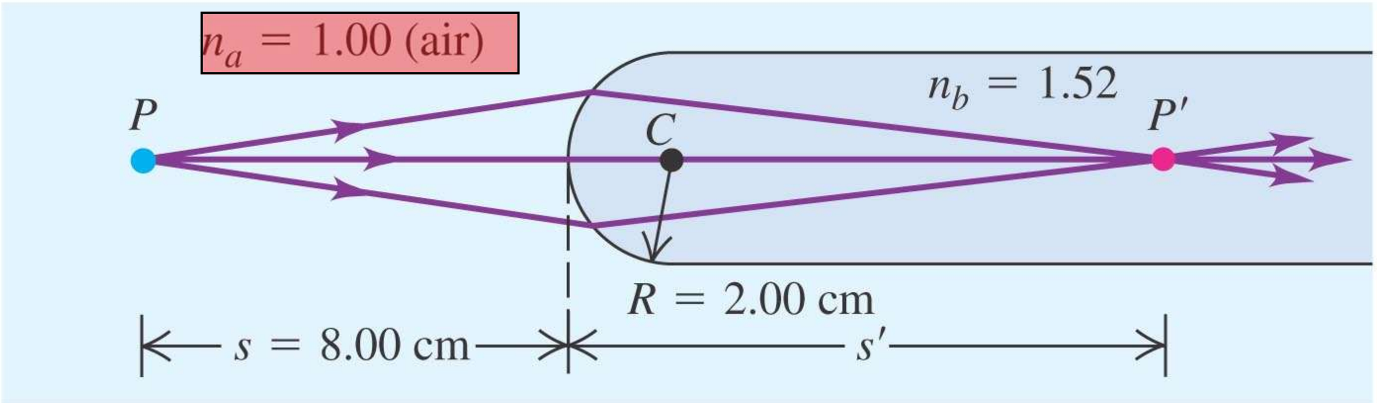
Combing this with our result for lateral magnification, we have, $m = 1$ (upright) so that, the image is *unmagnified* and *upright*.



Example 34.5 & 34.6

- Images formed by a spherical surface can be *real* (+) or *virtual* (-) depending on n_a, n_b, s, and R.

Ex 34.5:



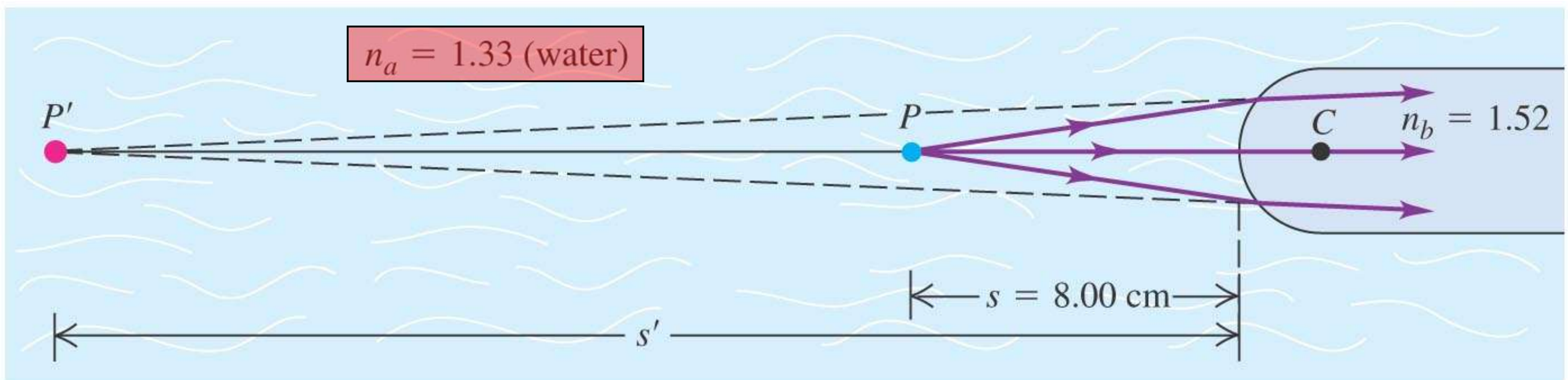
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \rightarrow \frac{1.00}{8.00cm}+\frac{1.52}{s'}=\frac{0.52}{+2.00cm} \rightarrow s'=+11.3cm$$

Example 34.5 & 34.6

- Images formed by a spherical surface can be *real* (+) or *virtual* (-) depending on n_a, n_b, s, and R.

Ex 34.6:

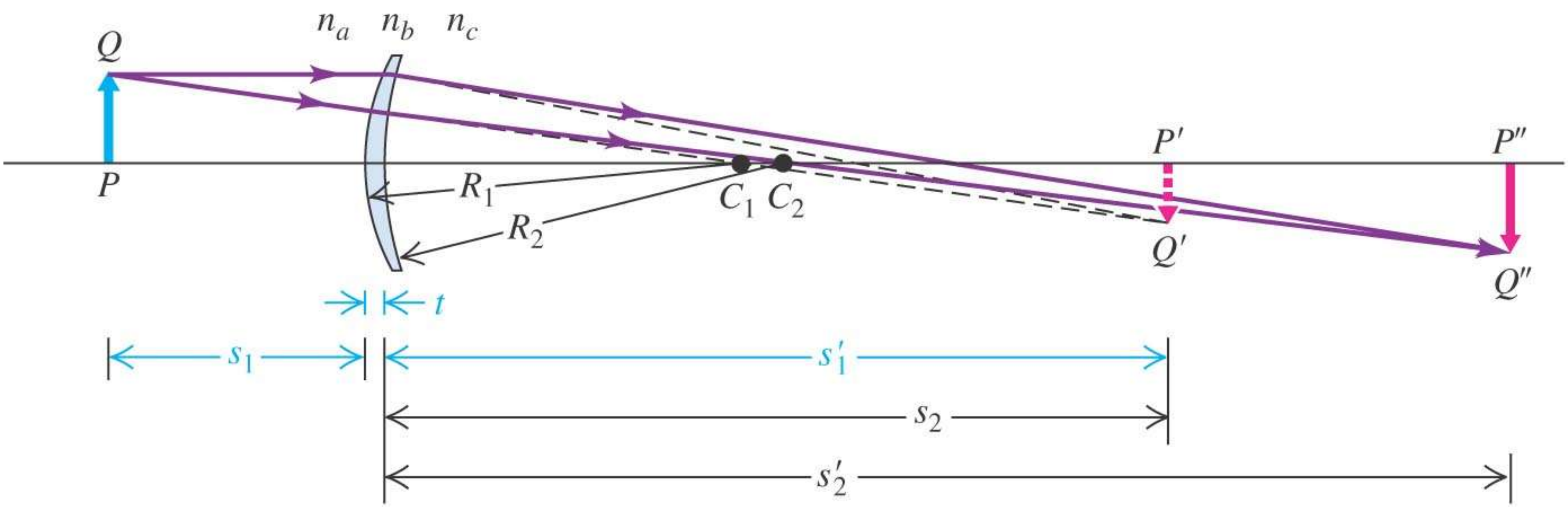


Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \quad\rightarrow\quad \frac{1.33}{8.00cm}+\frac{1.52}{s'}=\frac{0.19}{+2.00cm} \quad\rightarrow\quad s'=-21.3cm$$

Thin Lenses

Consider a *thin* lens as two closely spaced spherical surfaces.

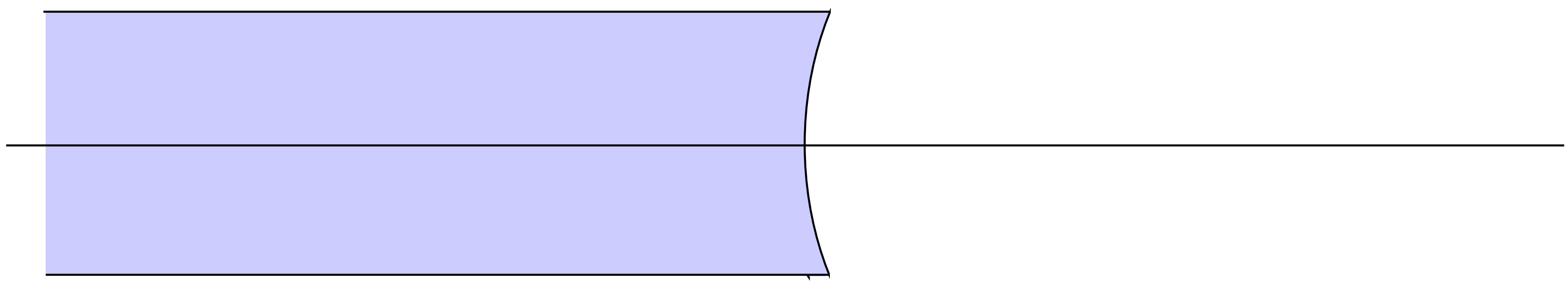


Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

- "thin" means that $t \ll$ other lengths
- For images produced by these two refracting surfaces, we will use the image *Q'* from the *first* surface as the *object* for the *second* surface

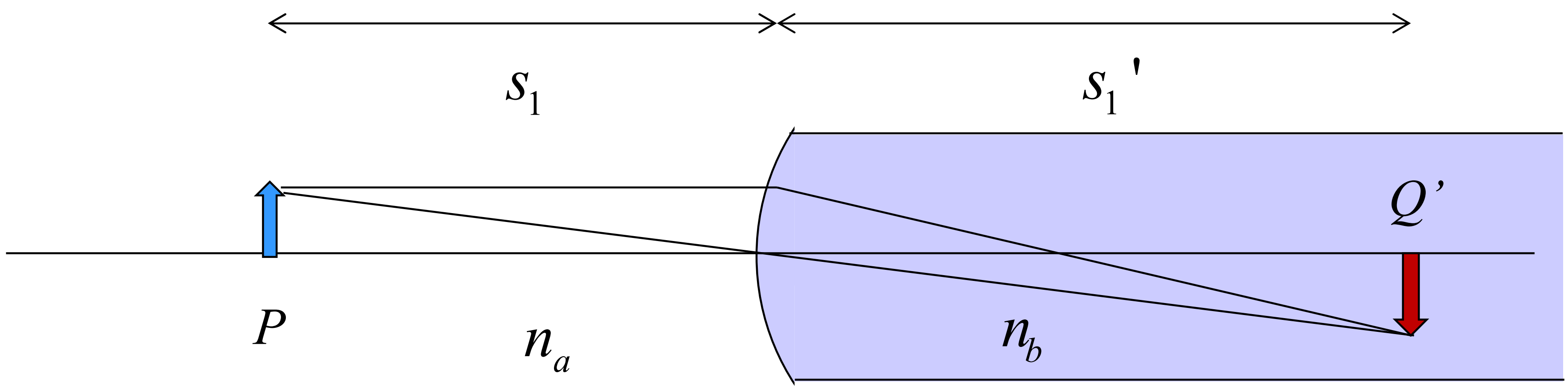
Thin Lenses

Consider a *thin* lens as two closely spaced spherical surfaces.



Thin Lenses

LEFT refracting surface of thin lenses



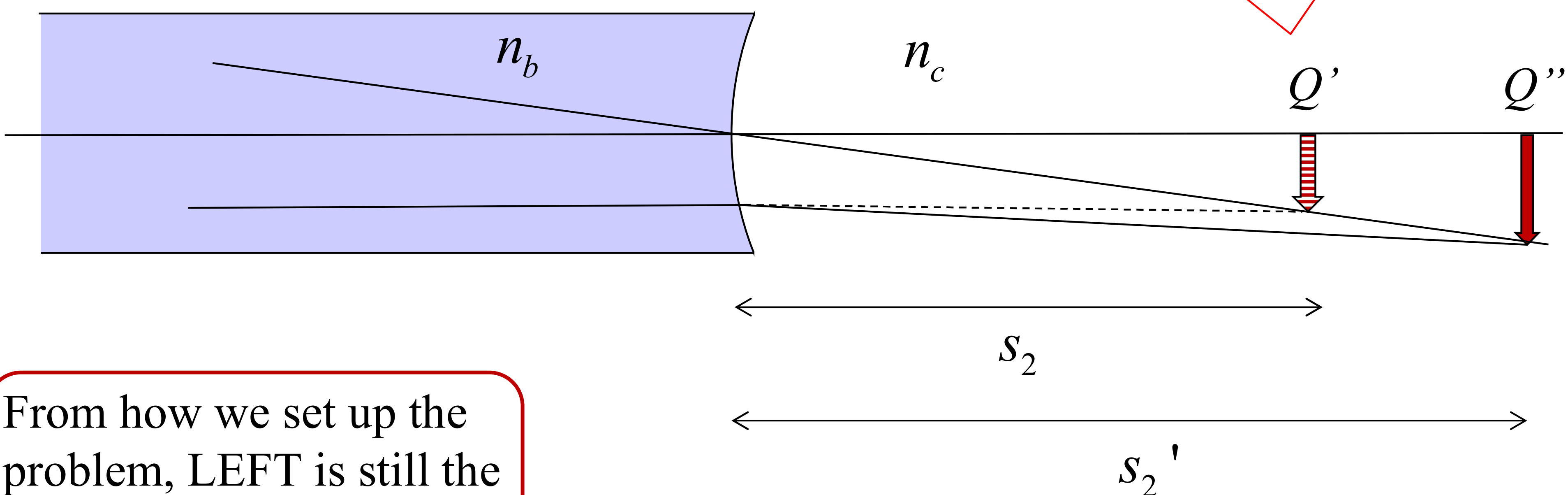
$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$$

Thin Lenses

RIGHT refracting surface of thin lenses

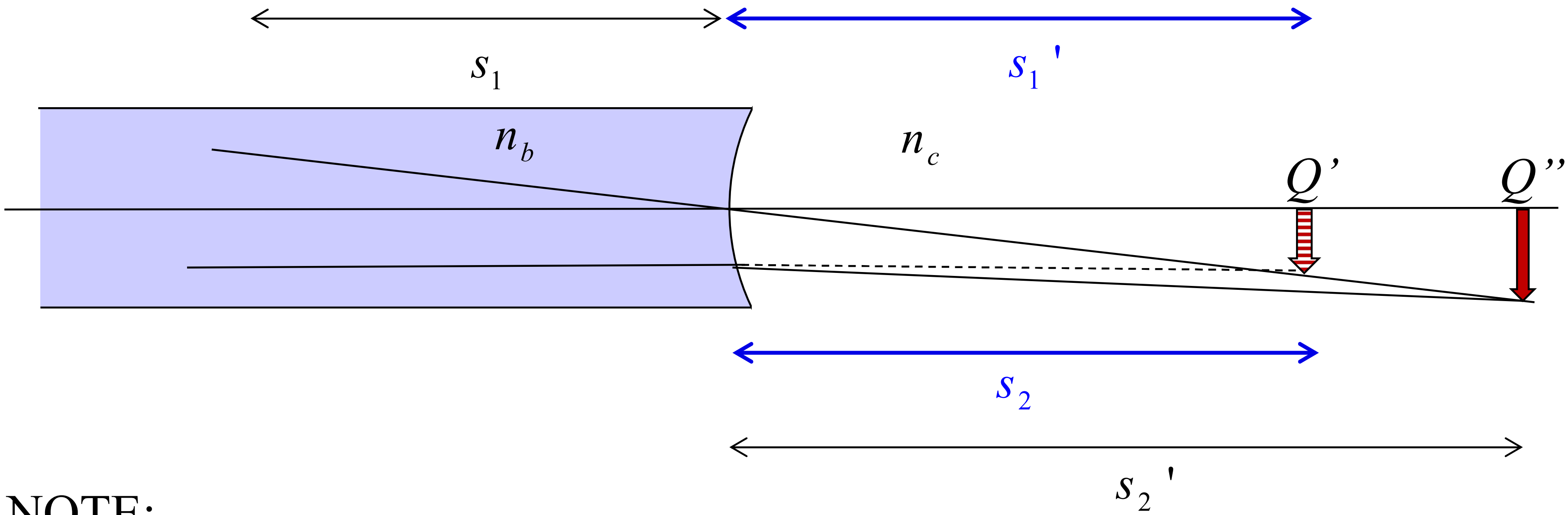
$$\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$$

Note: *Q*' is now a virtual object for the right refracting surface



From how we set up the problem, LEFT is still the **incoming** side

Thin Lens



NOTE:

For the situation indicated here, *Q'* is on the side of the "out-going" light. By the sign convention, we have: $s_1\,'\,(\textit{image dist}\,1) > 0$ and $s_2\,(\textit{object dist}\,2) < 0$

But, since they represent the same physical distance to *Q'*, for consistency, we need to have:

$$s_2 = -s_1\,'$$

Thin Lens

$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$$

(left surface)

$$\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$$

(right surface)

Thin Lens

$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$$

(left surface)

$$\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$$

(right surface)

NOTES:

- Since both C_1 and C_2 are on the outgoing side of light, R_1 and R_2 are + by convention.
- Since the material outside of the lens is typically air or vacuum, we take n_a and $n_c = 1$.
- For simplicity, we will call n_b (for the lens itself) n.
- We also apply the image-to-object consistency relation: $s_2 = - s_1$'

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

$$-\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

Thin Lens

To eliminate s_1' by adding these two equations:

$$\frac{1}{s_1}+\frac{1}{s_2'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Calling our original object distance s_1 simply as s and our final image distance s_2' simply as s', we have:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \text{(object-image relation, thin lens)}$$

where,

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{(lensmaker's equation)}$$

Converging & Diverging lens

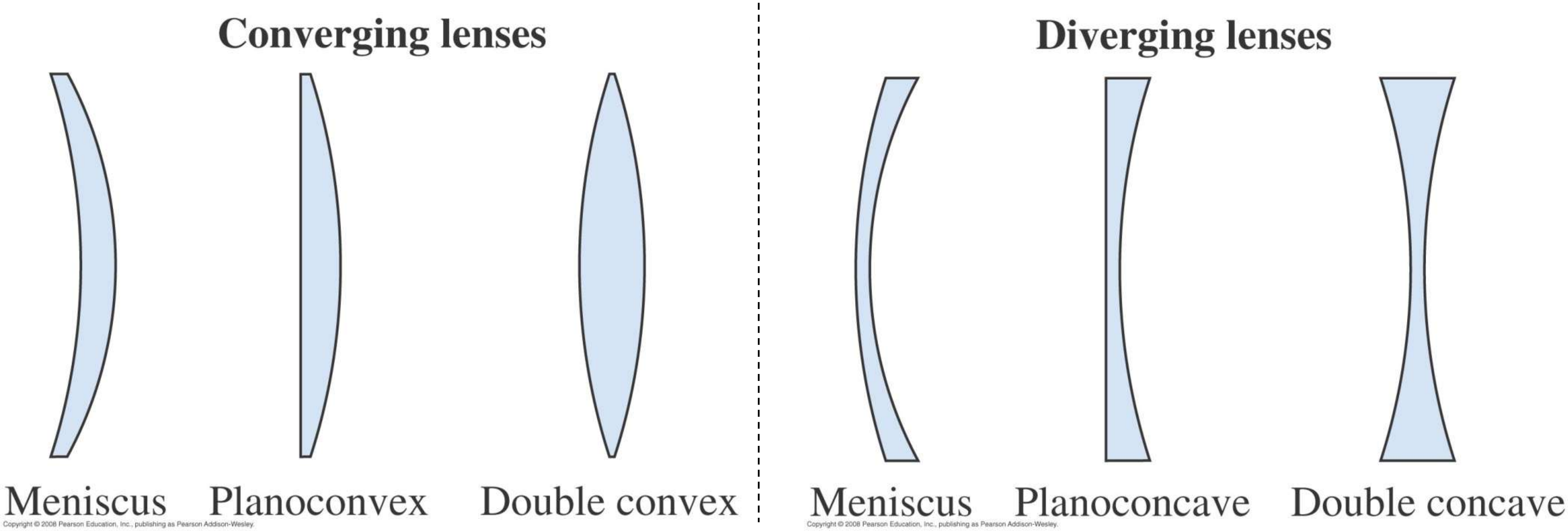
Depending on the values of n_{lens}, $n_{outside}$, R_1 and R_2 , f can be positive or negative !

$$\frac{1}{f} = \frac{(n_{lens} - n_{outside})}{n_{outside}} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

D_{lens}

f → positive

f → negative



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Sign Rules for Mirrors & Lens

1. Object Distance:
   - *s* is + if the object is on the same side as the incoming light (for both reflecting and refracting surfaces) and *s* is – otherwise.
2. Image Distance:
   - *s'* is + if the image is on the same side as the outgoing light and is – otherwise.
3. Object/Image Height:
   - *y* (*y*') is + if the image (object) is erect or upright. It is – if it is inverted.
4. Radius of Curvature:
   - *R* is + when the center of curvature C is on the same side as the outgoing light and – otherwise.
5. Focus Length: (+ concave, - convex)

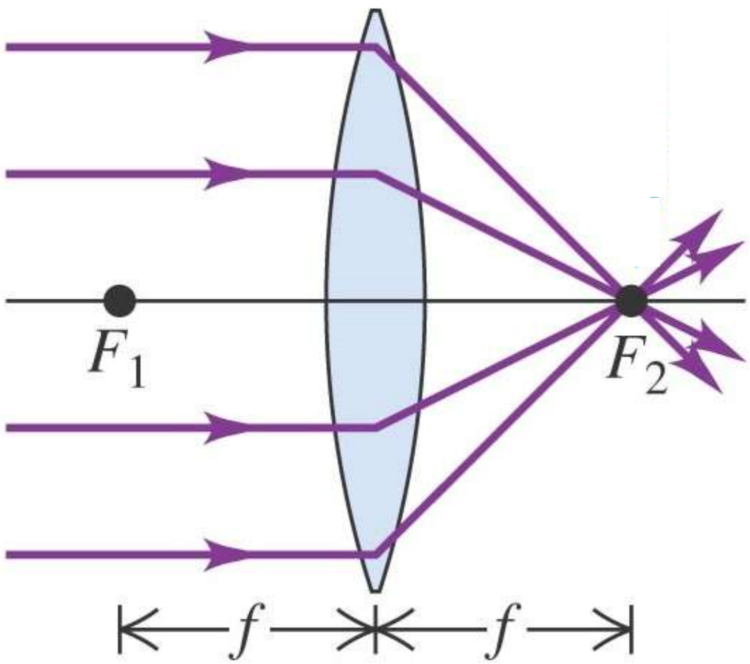
   (+ converging, - diverging)

Focal Points of a Converging Lens

Consider a far away object ($s = \infty$) so that the incoming rays are parallel to the optical axis, these rays will all converge at one point (the *right* focal point F_2) at a distance f (the focal length) to the right of the lens,

$$\boxed{f > 0 \quad (converging)}$$

$$\frac{1}{\infty} + \frac{1}{s'} = \frac{1}{f} \quad \rightarrow \quad s' = f$$



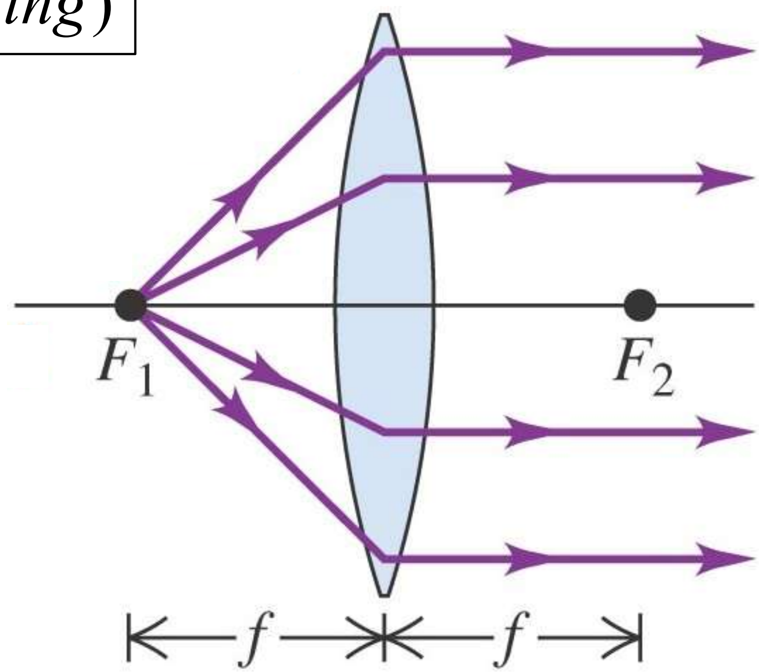
This gives the focusing capability of a thin lens.

Focal Points of a Thin Lens

Now, consider the reverse situation, if the outgoing rays are all parallel to the optical axis so that the image is at infinity $(s' = \infty)$, where is the object originally located?

$$\boxed{f > 0 \quad (converging)}$$

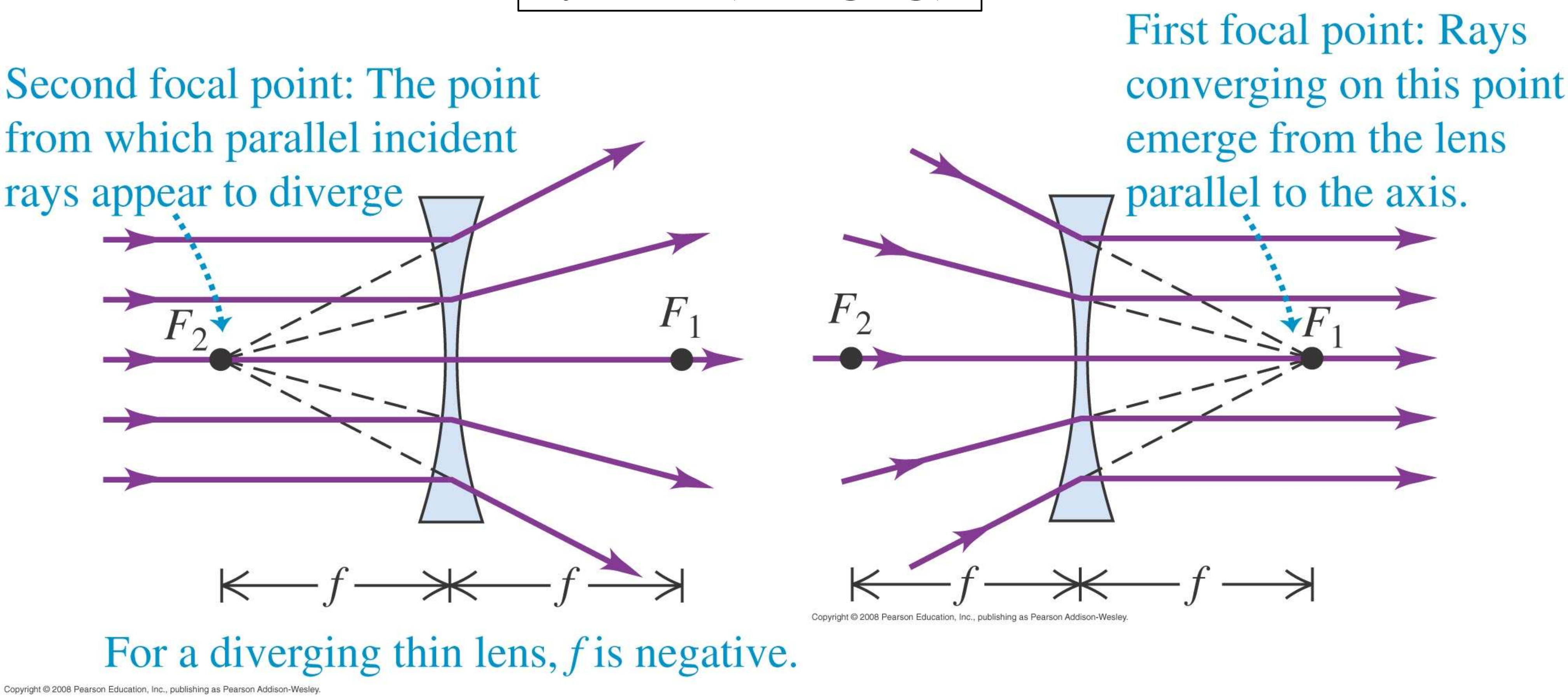
$$\frac{1}{s} + \frac{1}{\infty} = \frac{1}{f} \quad \rightarrow \quad s = f$$



This simple calculation indicates that all rays must originate from a single point (the *left* focal point F_1) at a distance f (the focal length) to the left of the lens,

Properties of a Diverging Lens

$$\boxed{f < 0 \quad (\textit{diverging})}$$

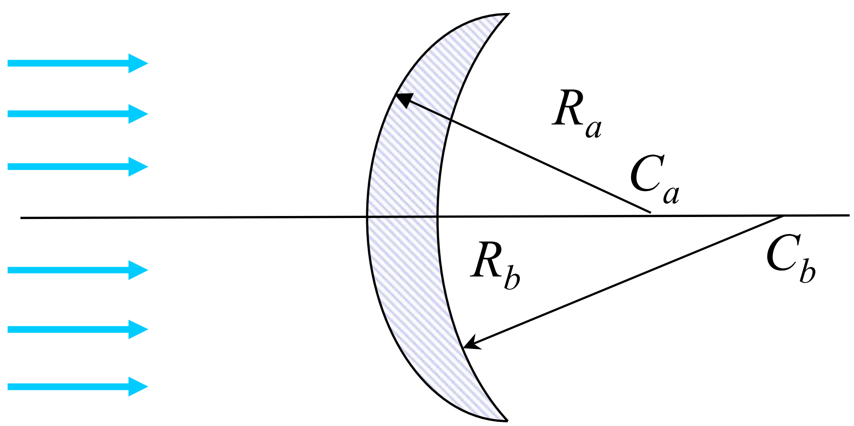


Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Symmetry of the Lensmaker's Equation

Because the Lensmaker's equation is symmetric with respect to the sign convention for R_a and R_b, the left *and* right focal lengths are the *same* irrespective of the difference in the values of R_a and R_b.

Example: Application of the Lensmaker's Equation:



$$\frac{1}{f}=(n-1)\left(\frac{1}{R_a}-\frac{1}{R_b}\right)$$

$$=(1.33-1)\left(\frac{1}{10}-\frac{1}{15}\right)=\frac{0.33}{30}=0.011$$

$$f=+90.9cm$$

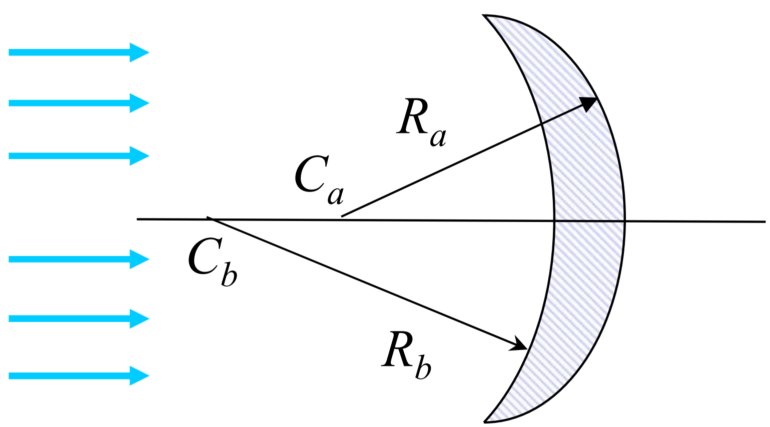
$R_a=+10.0cm$ (C_a same side as outgoing light)

$R_b=+15.0cm$ (C_a same side as outgoing light)

The lens is converging.

Symmetry of the Lensmaker's Equation

Now, we flip our lens around so that the refracting surface with R_b will be on the incoming light side,



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_b} - \frac{1}{R_a}\right)$$

$$= (1.33-1)\left(-\frac{1}{15} - \left(-\frac{1}{10}\right)\right)$$

$$= 0.33\left(-\frac{1}{15} + \frac{1}{10}\right) = \frac{0.33}{30} = 0.011$$

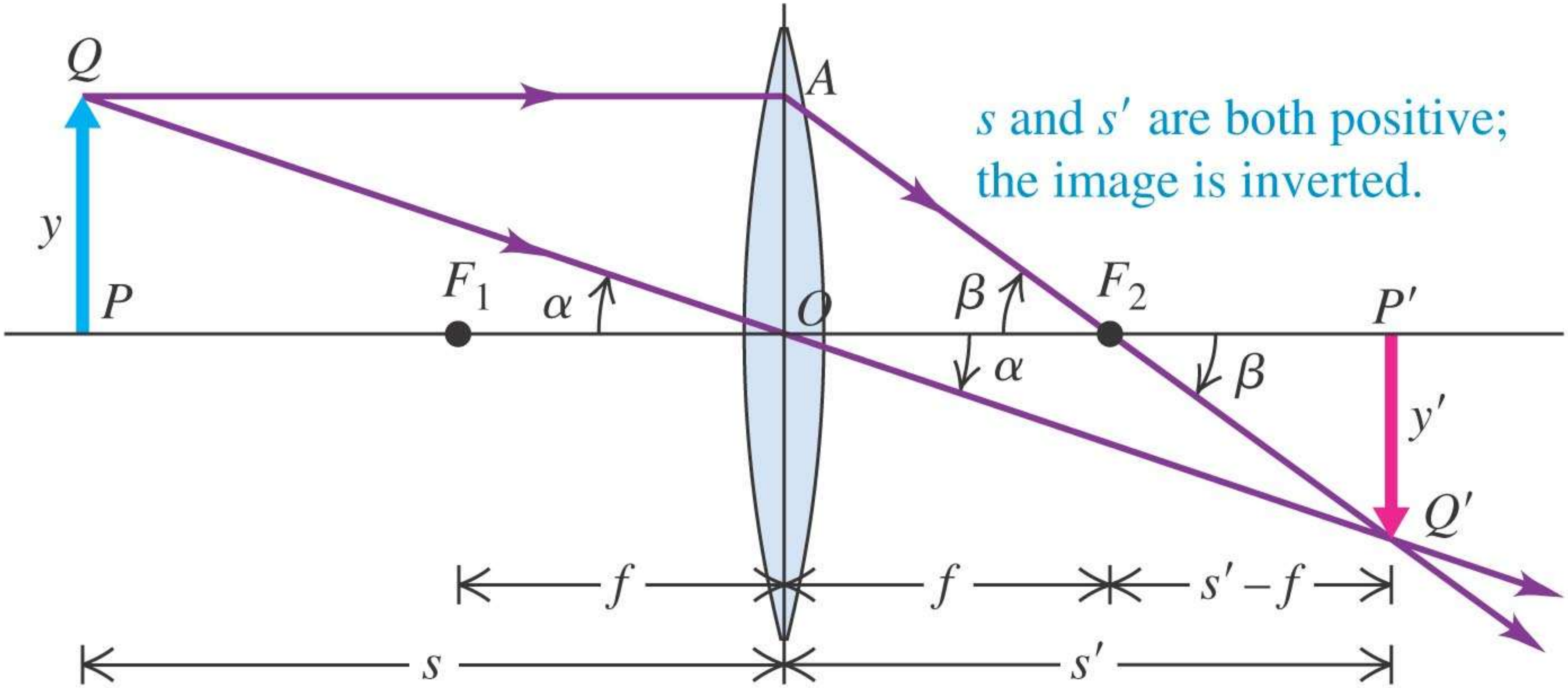
$$f = +90.9cm$$

$R_a = -10.0cm$ (C_a NOT on outgoing light side)

$R_b = -15.0cm$ (C_a NOT on outgoing light side)

Same result as previous calculation!

Lateral Magnification of Lenses

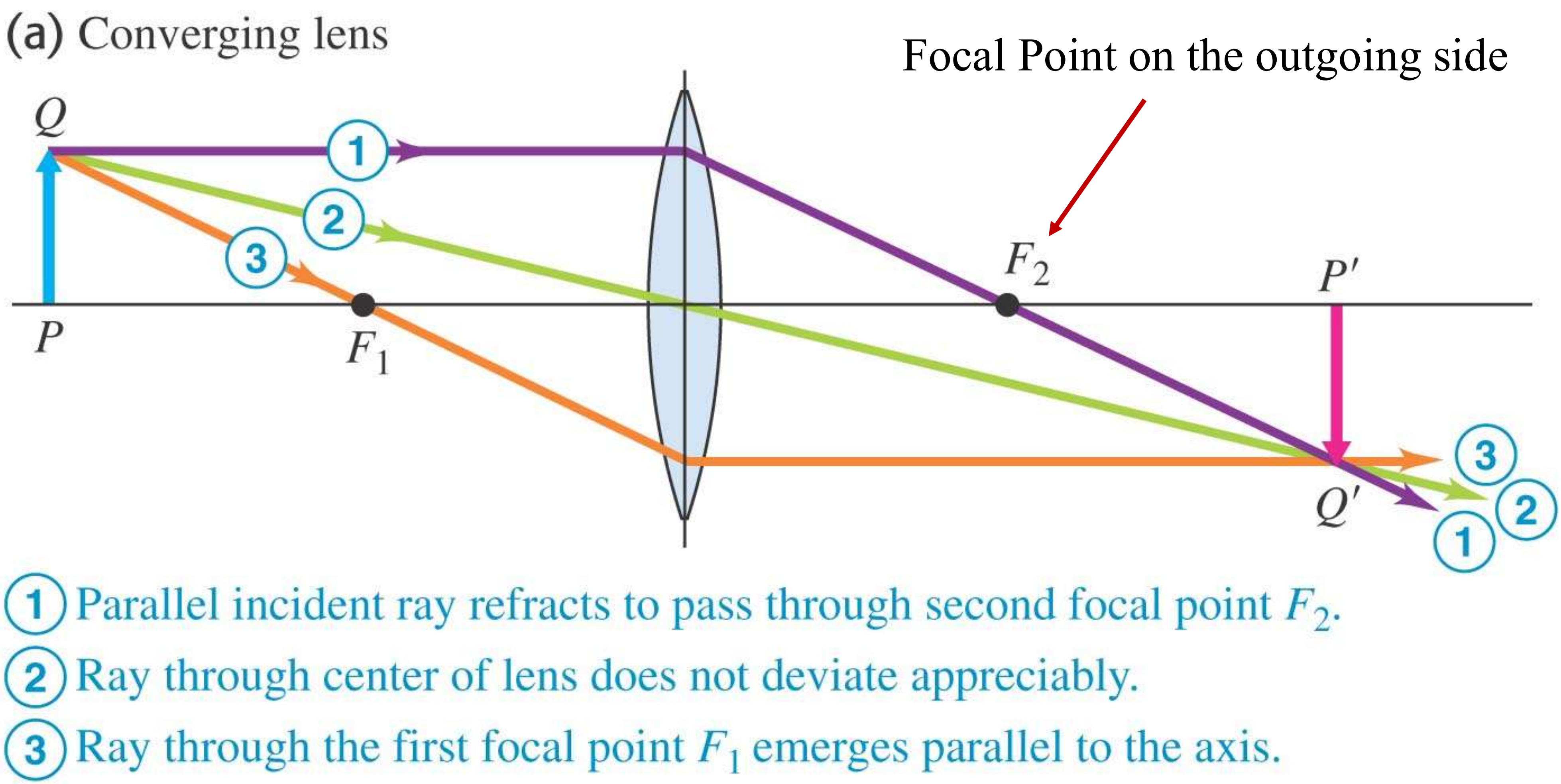


Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

$\triangle OPQ$ and $\triangle OP'Q'$ are similar, so that we have, $\dfrac{y}{s} = \dfrac{-y'}{s'}$

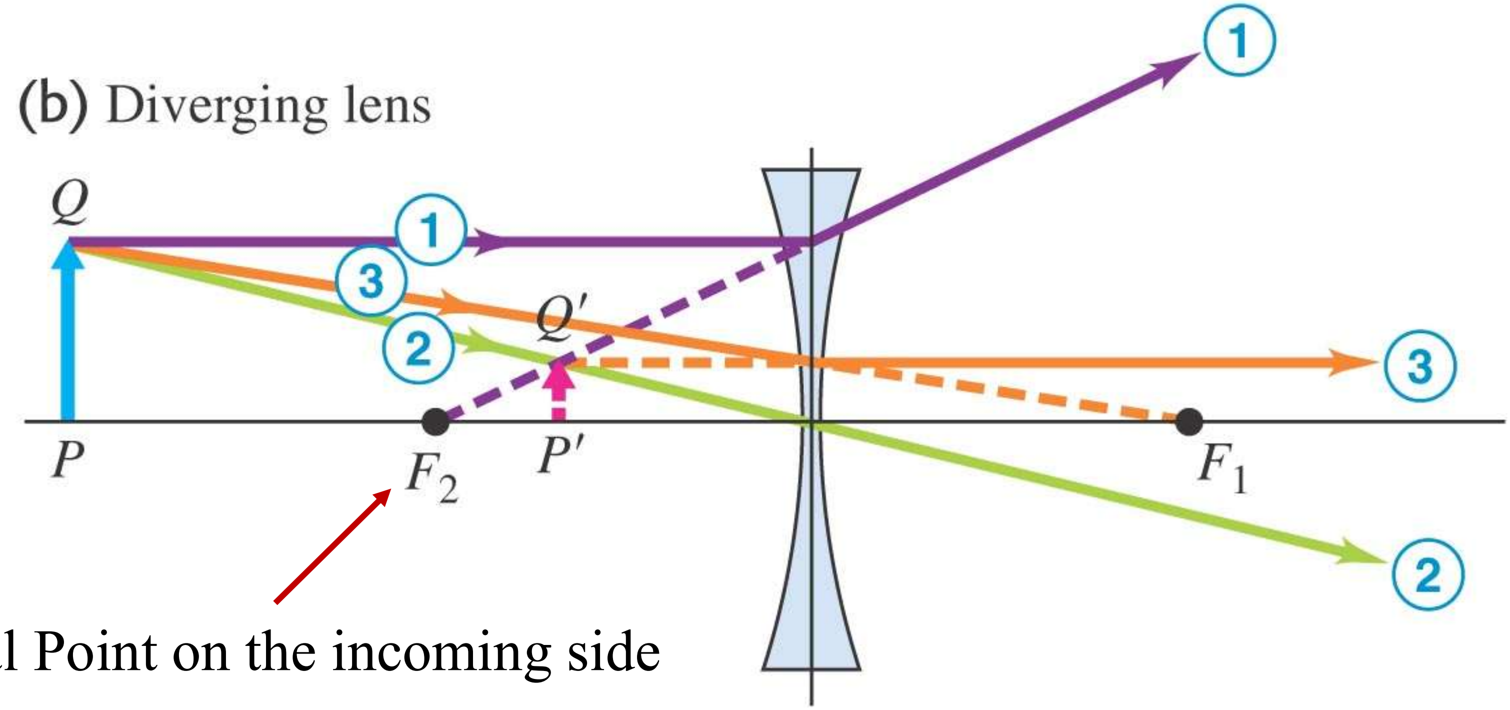
This gives, $\boxed{m = \dfrac{y'}{y} = \dfrac{-s'}{s}}$ (lateral magnification, thin lens)

Ray Tracing Methods for Lenses



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Rays Tracing Methods for Lenses



Focal Point on the incoming side

(1) Parallel incident ray appears after refraction to have come from the second focal point F_2.

(2) Ray through center of lens does not deviate appreciably.

(3) Ray aimed at the first focal point F_1 emerges parallel to the axis.

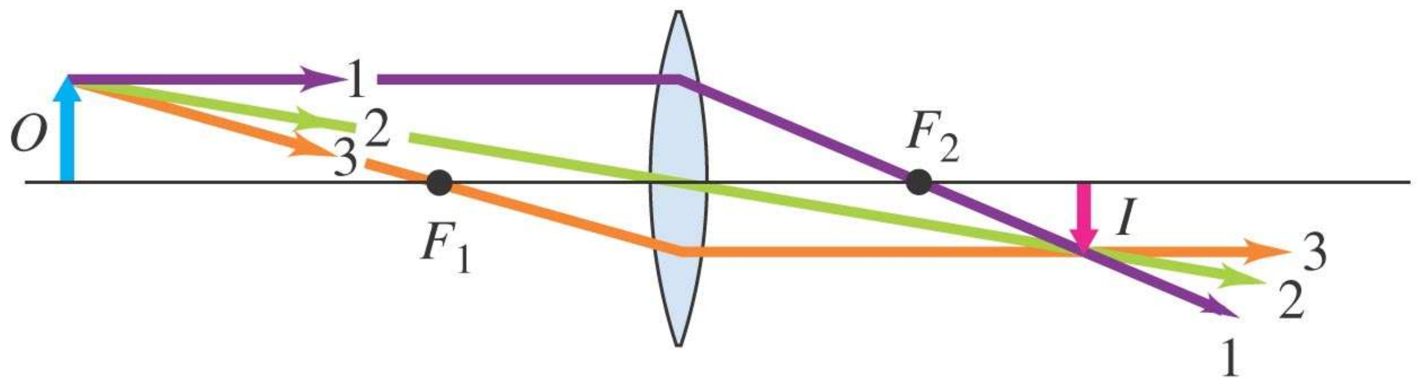
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Object-Image Relations: Thin Lenses

Thin Lens Applet (by Fu-Kwun Hwang)

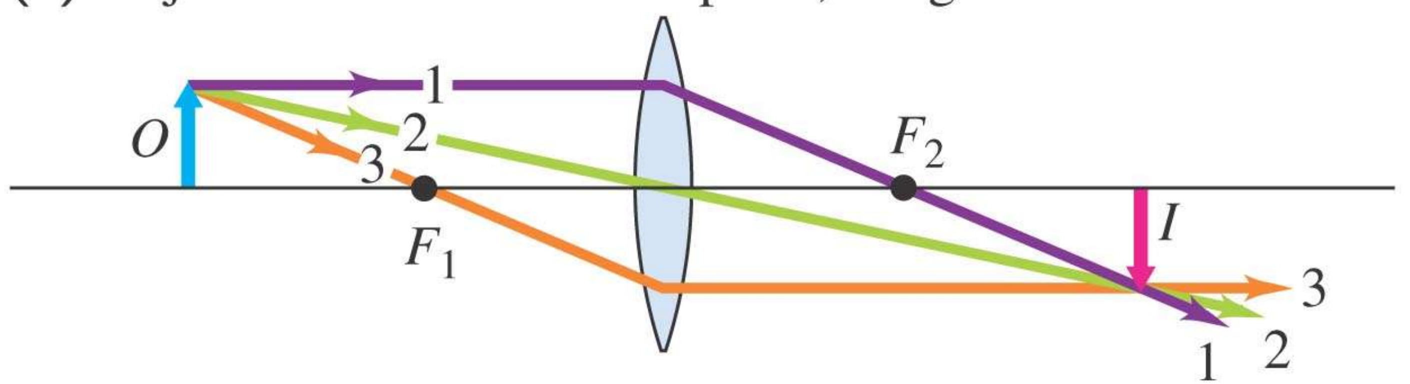
http://www.physics.metu.edu.tr/~bucurgat/ntnujava/Lens/lens_e.html

(a) Object O is outside focal point; image I is real.



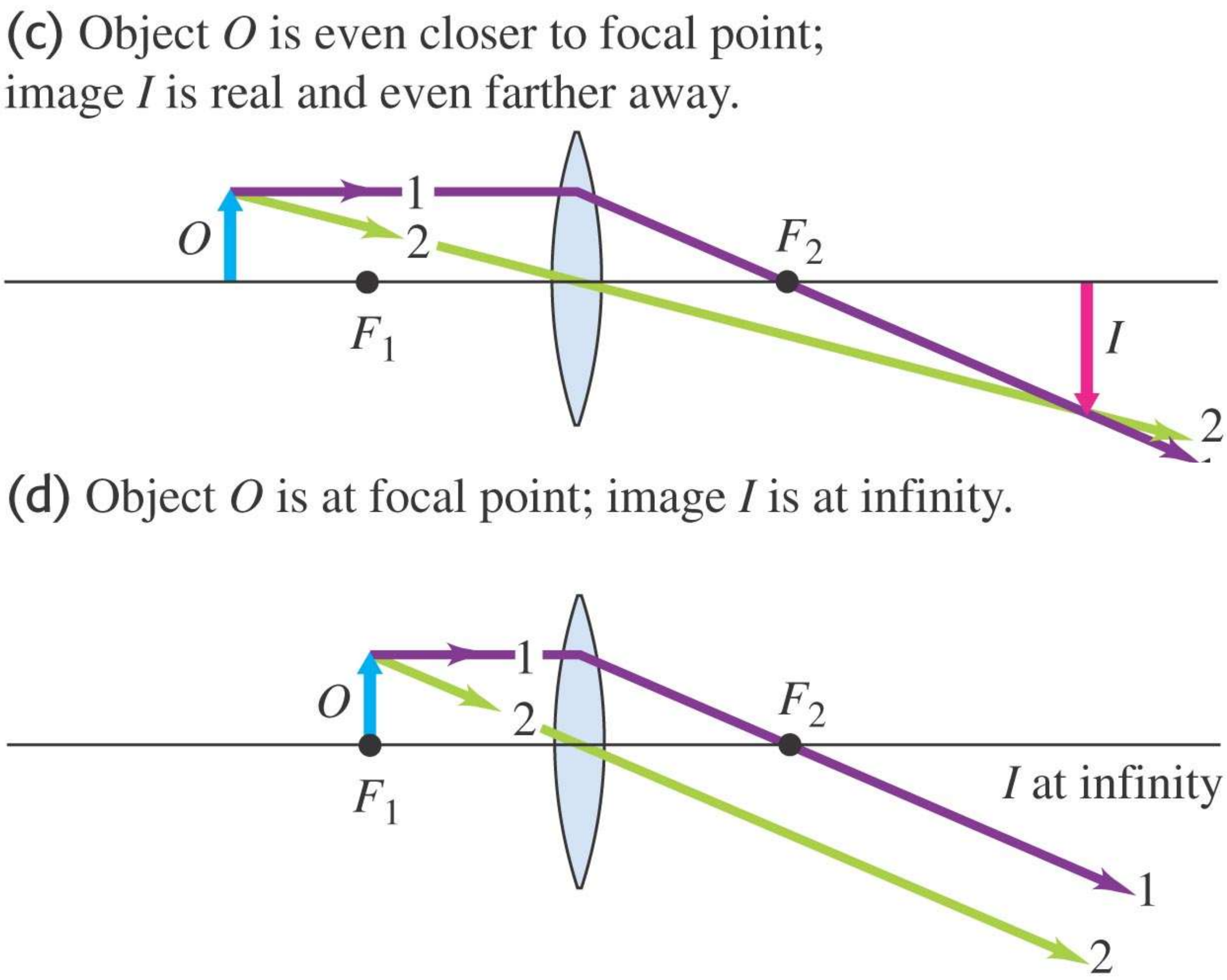
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

(b) Object O is closer to focal point; image I is real and farther away.



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

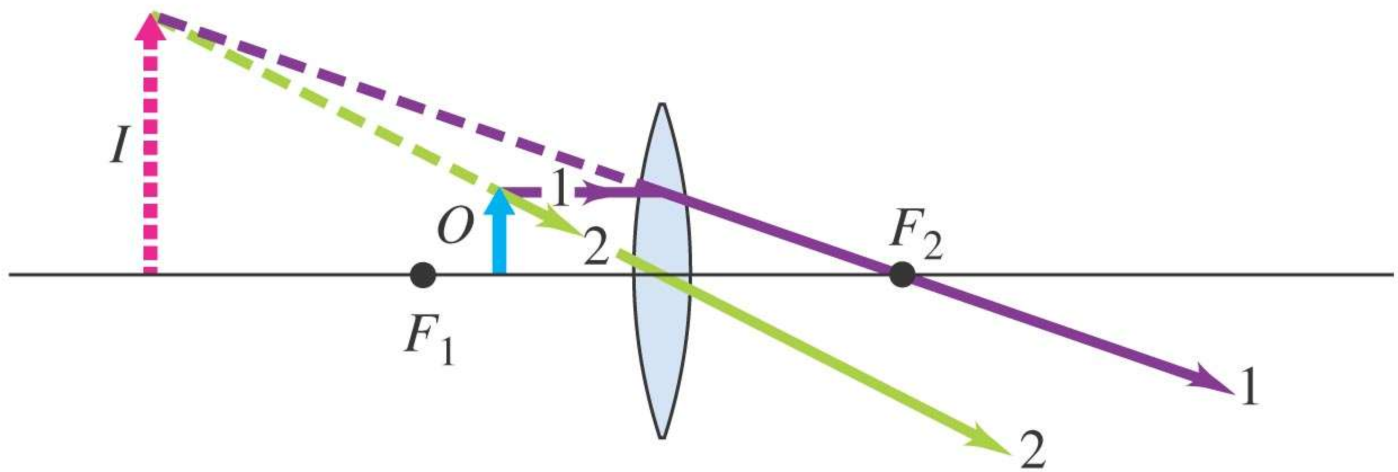
Object-Image Relations: Thin Lenses



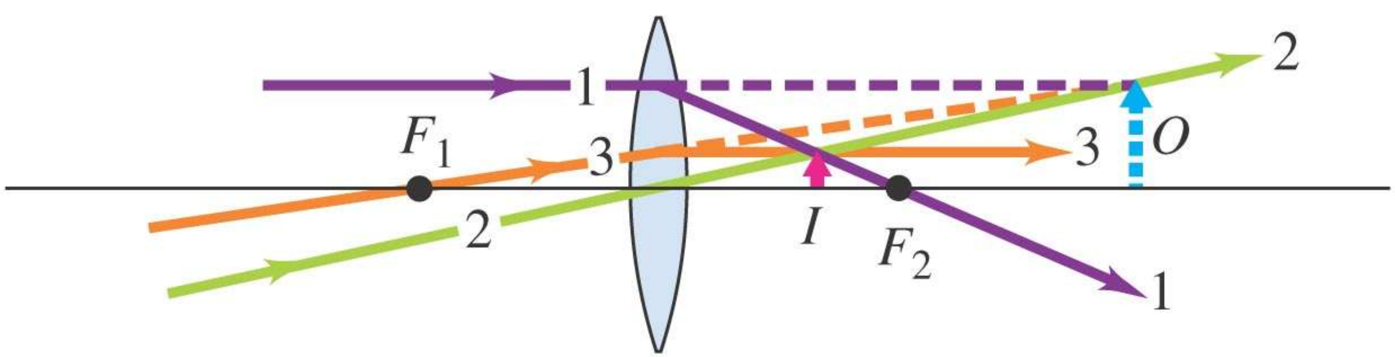
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Object-Image Relations: Thin Lenses

(e) Object *O* is inside focal point;
image *I* is virtual and larger than object.



(f) A virtual object *O* (light rays are *converging* on lens)



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.