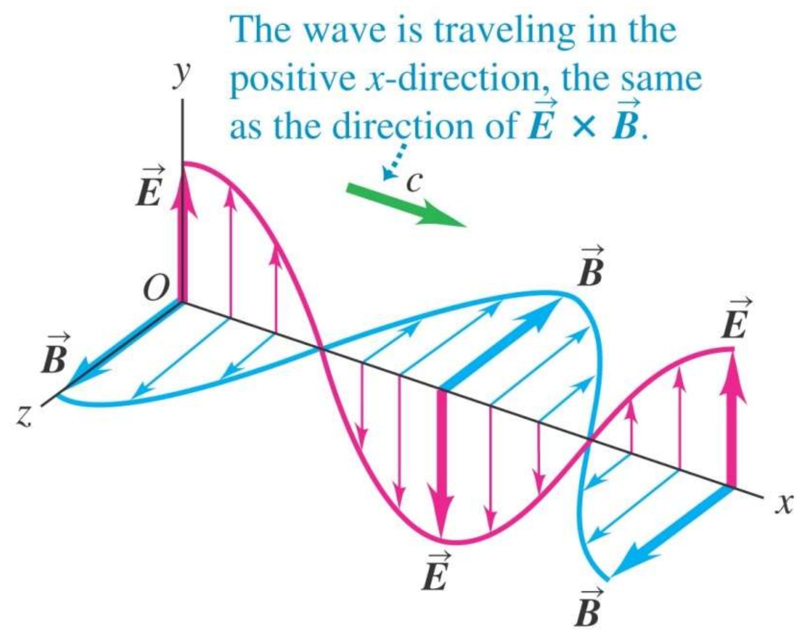
A Linearly Polarized EM Wave

For an electromagnetic wave, the direction of the *electric* field vector $\vec{\mathbf{E}}(x,t)$ gives the **polarization** of the wave.

An transverse electromagnetic wave with polarization in the y-direction:

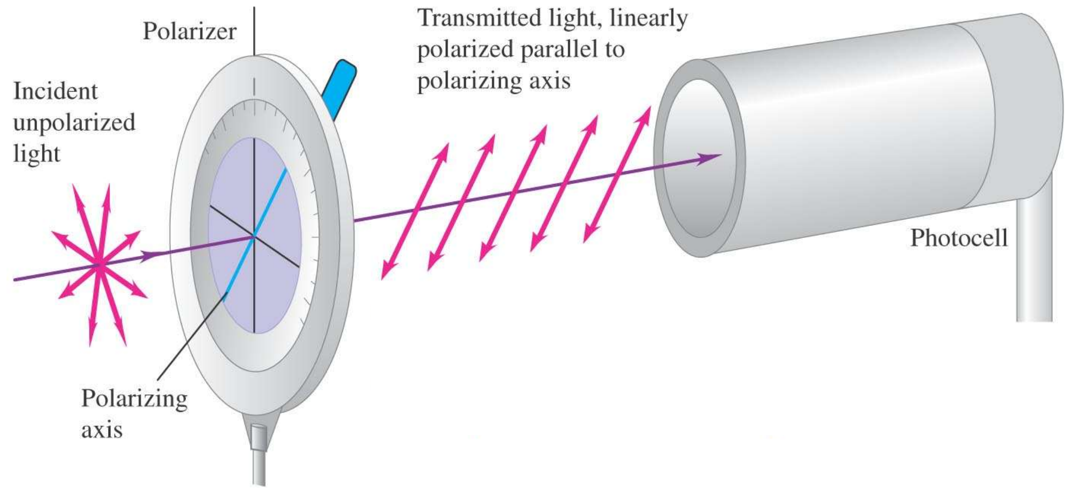
$$\begin{cases} \vec{\mathbf{E}}(x,t) = E_{\text{max}} \cos(kx - \omega t)\, \hat{\mathbf{j}} \\ \vec{\mathbf{B}}(x,t) = B_{\text{max}} \cos(kx - \omega t)\, \hat{\mathbf{k}} \end{cases}$$



A polarized wave in a well defined direction is called a *linearly polarized* wave.

The Action of a Polarizing Filter

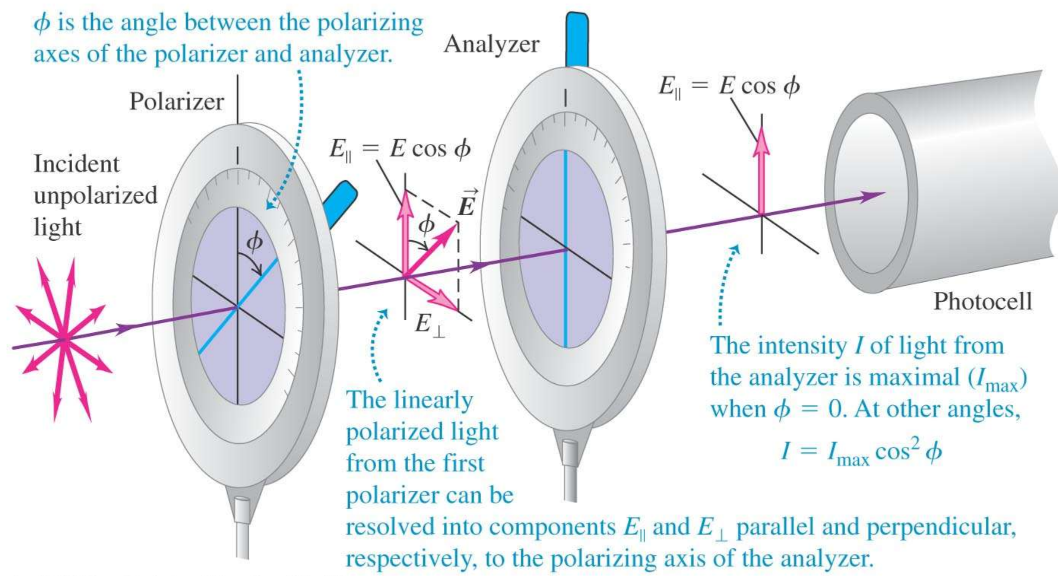
Unpolarized incident light will be linearly polarized parallel to the polarizing axis after transmission.



We can analyze the intensity of the transmitted light passing thru the *second* polarizer (an analyzer):

Only $E_{||}$ will be transmitted,

$$E_{trans} = E_{||} = E\cos\phi$$

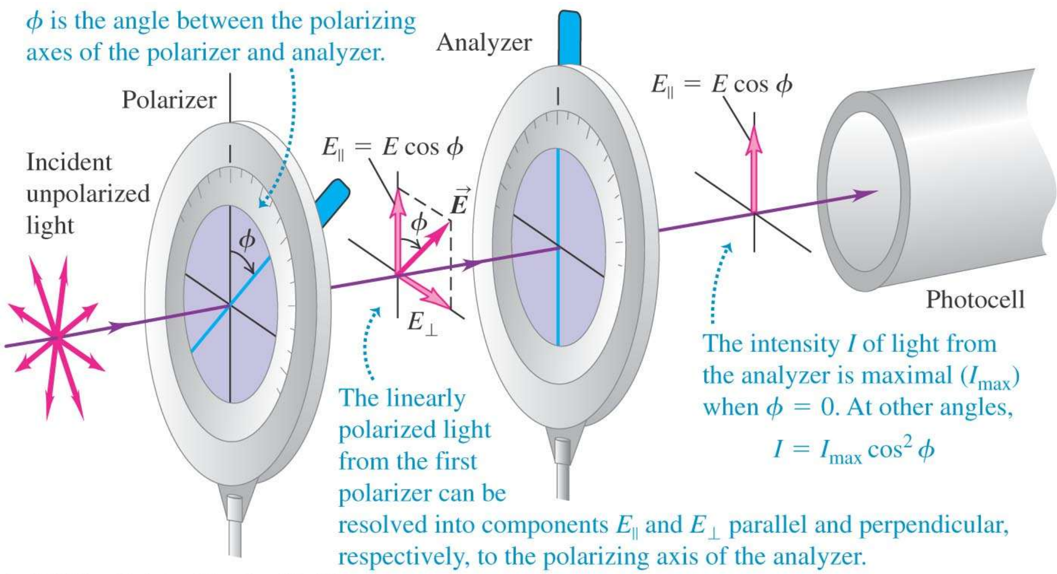


The Action of a Polarizing Filter

Since intensity (I) is proportional to E^2,

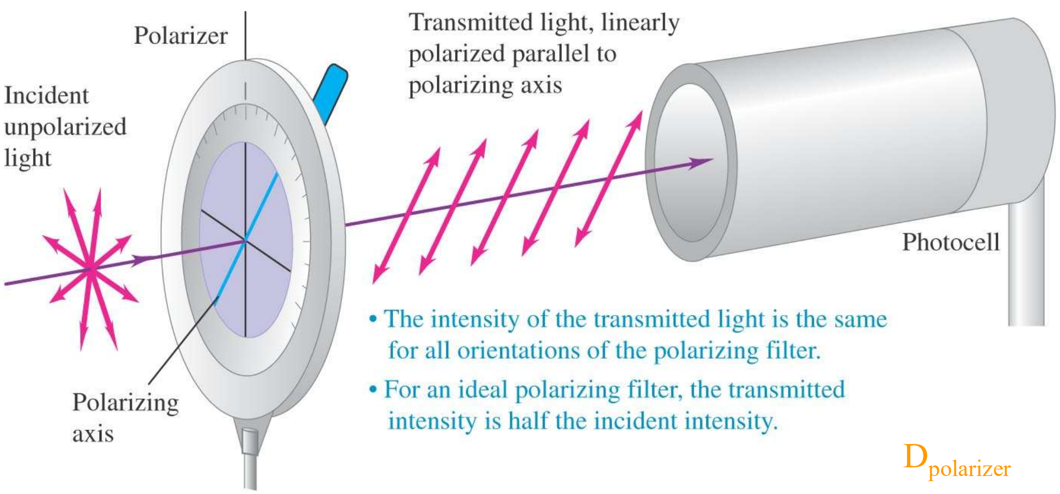
$$I_{trans} = I_{\max} \cos^2 \phi$$

(Malus's Law)
Transmitted intensity of *linearly* polarized light through a polarizer



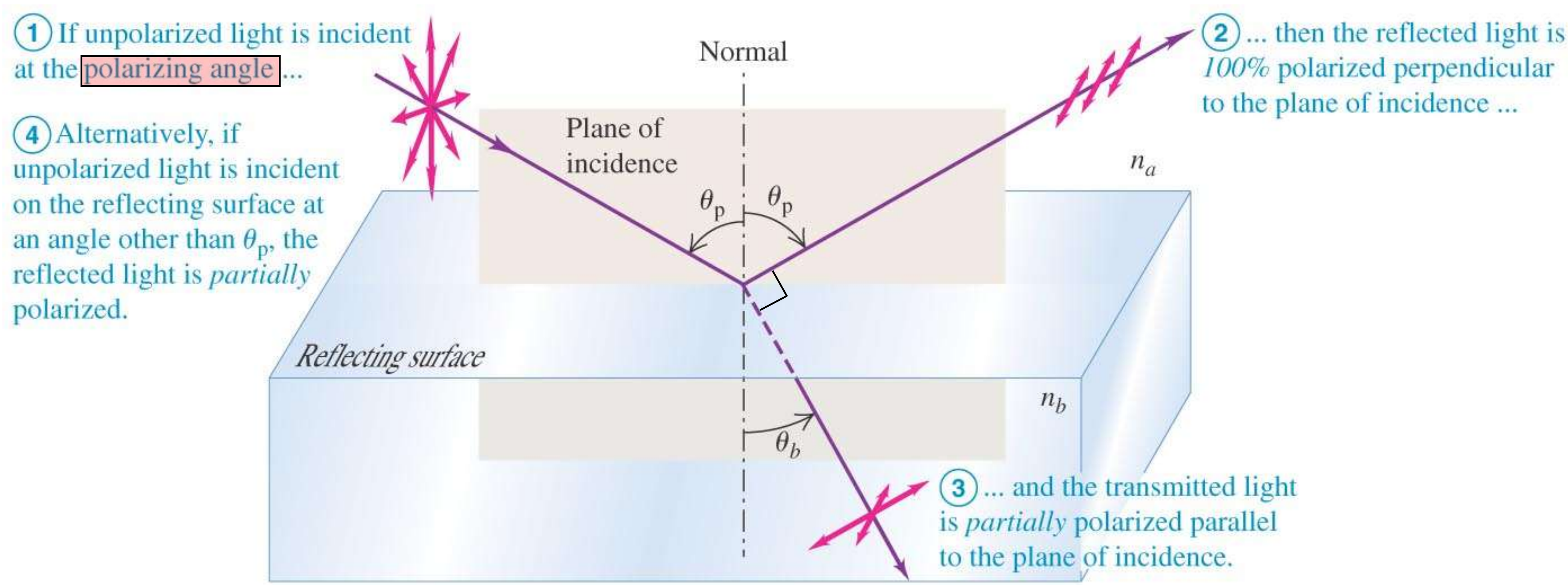
For an *upolarized* light, $\vec{\mathbf{E}}$ is in all directions,

$$I_{trans} = I_{\max} \int_0^{2\pi} \cos^2 \phi = \frac{1}{2} I_{\max}$$



$D_{polarizer}$

Polarization by Reflection



At the special angle (**polarizing angle** or **Brewster's angle**) θ_p, the electric field component *parallel* to the "plane of incidence" will not be reflected !

This occurs when $\theta_b = \pi/2 - \theta_p$

applet

Brewster’s Angle Weblink

http://physics.bu.edu/~duffy/semester2/c27_brewster.html

Polarization by Reflection

From Snell's law, we have,

$$n_a \sin\theta_p = n_b \sin\theta_b$$

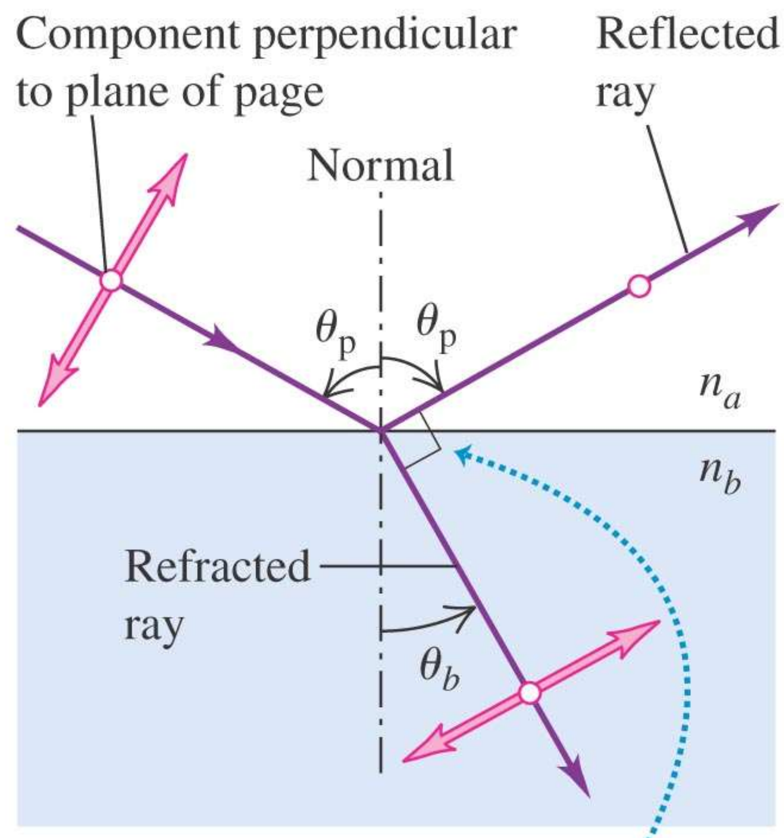
Then, using the condition for θ_p: $\theta_b = \pi/2 - \theta_p$

$$n_a \sin\theta_p = n_b \sin(\pi/2 - \theta_p)$$

This gives $n_a \sin\theta_p = n_b \cos\theta_p$

$$\Rightarrow \quad \boxed{\tan\theta_p = n_b/n_a}$$

(Brewster's Law)



When light strikes a surface at the polarizing angle, the reflected and refracted rays are perpendicular to each other and

$$\tan\theta_p = \frac{n_b}{n_a}$$

Polarization by Reflection

$\tan\theta_p = n_b / n_a$

For water/air (1.33/1) interface, $\theta_p \simeq 53^\circ$

For glass/air (1.5/1) interface, $\theta_p \simeq 56^\circ$

(with polarizer $||$ to window plane)

(with polarizer $\perp$ to window plane)

Polarization by Reflection

$\tan\theta_p = n_b / n_a$

For water/air (1.33/1) interface, $\theta_p \simeq 53^\circ$

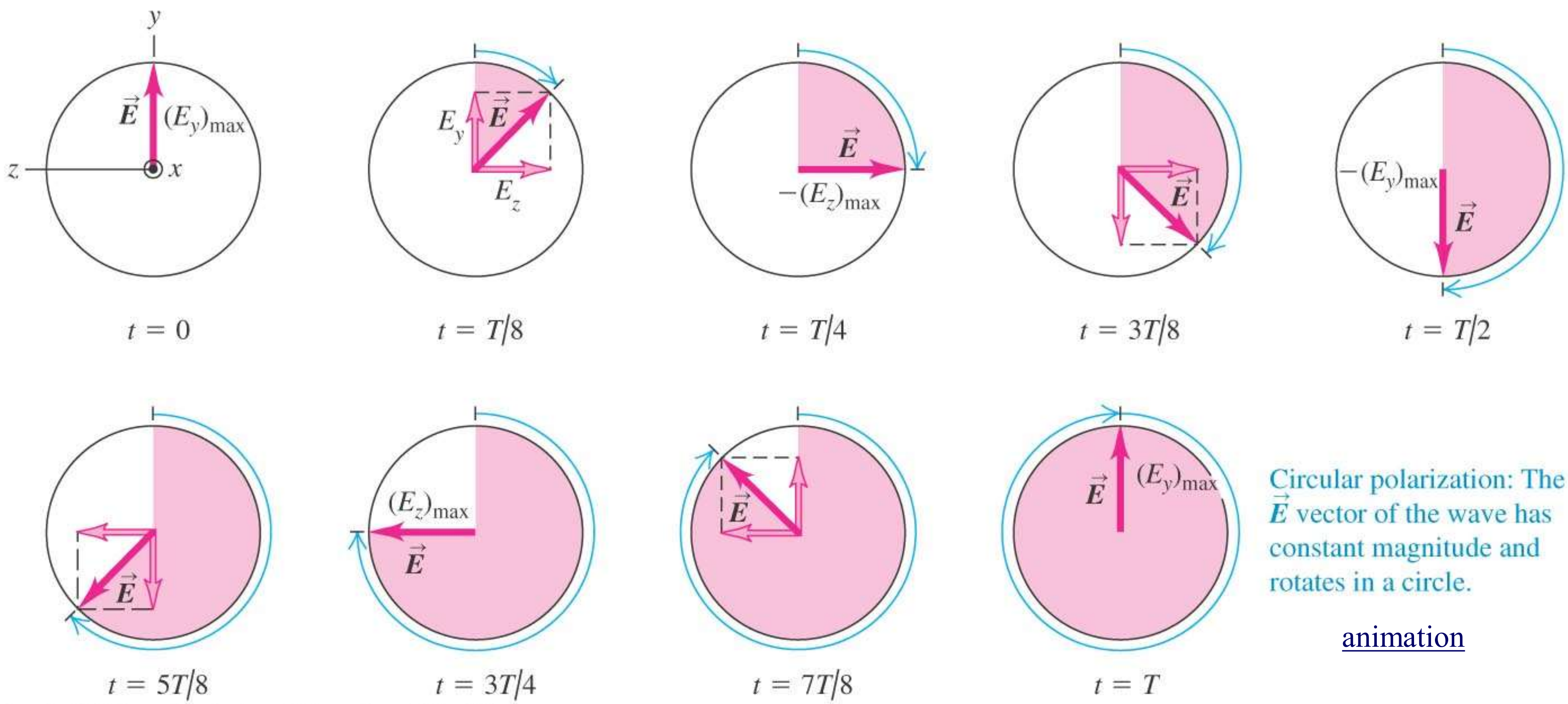
For glass/air (1.5/1) interface, $\theta_p \simeq 56^\circ$

(with polarizer $\parallel$ to reflection plane)

(with polarizer $\perp$ to reflection plane)

Circular and Elliptical Polarization

An **elliptical polarized wave** resulted when we have the *superposition* of two linearly polarized EM waves. In the special case when the two waves have *equal amplitudes* and are separated in phase by a *quarter-cycle*, a **circular polarized wave** will result. The resultant electric field vector $\vec{\mathbf{E}}$ will appear to rotate in a circle.



Circular polarization: The $\vec{E}$ vector of the wave has constant magnitude and rotates in a circle.

animation

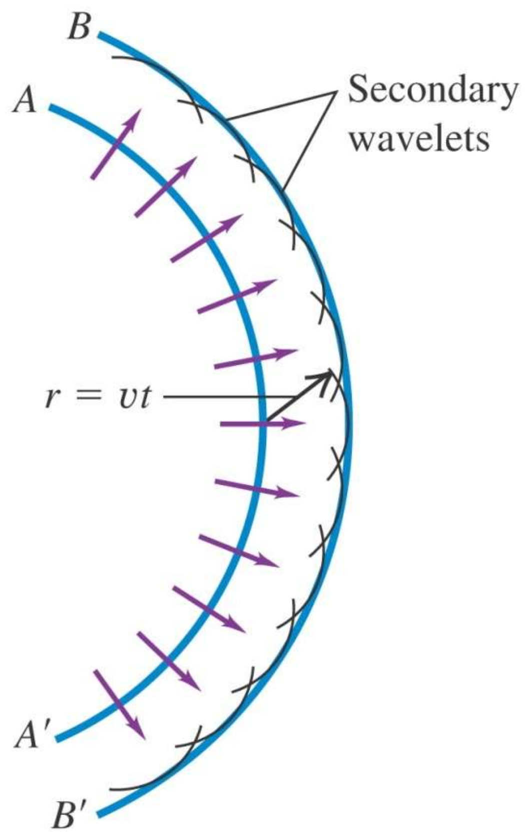
Circular Polarization Weblink

http://webphysics.davidson.edu/physlet_resources/dav_optics/examples/polarization.html

https://www.youtube.com/watch?v=jY9hnDzA6Ps

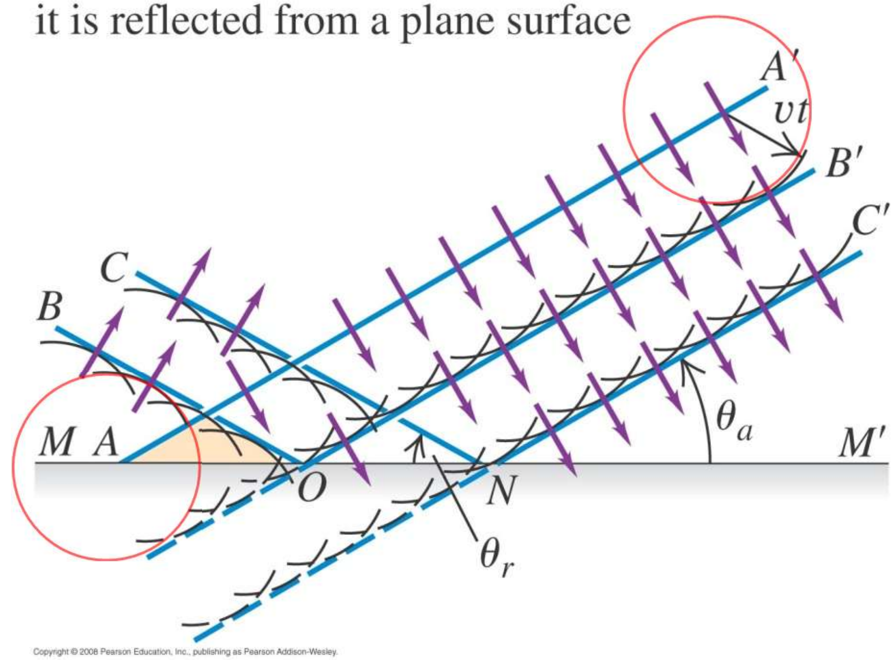
Huygens’ Principle

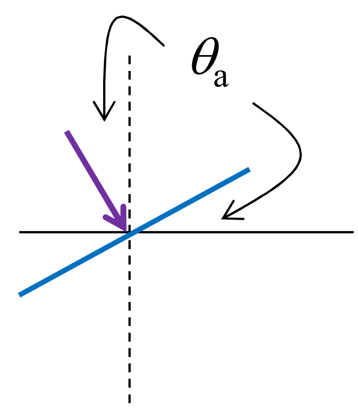
Christiaan Huygens (1629-195): The Huygens’ Principle can be used to predict the spreading of light wave. It is a geometrical construction using every point on a wave front as the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.



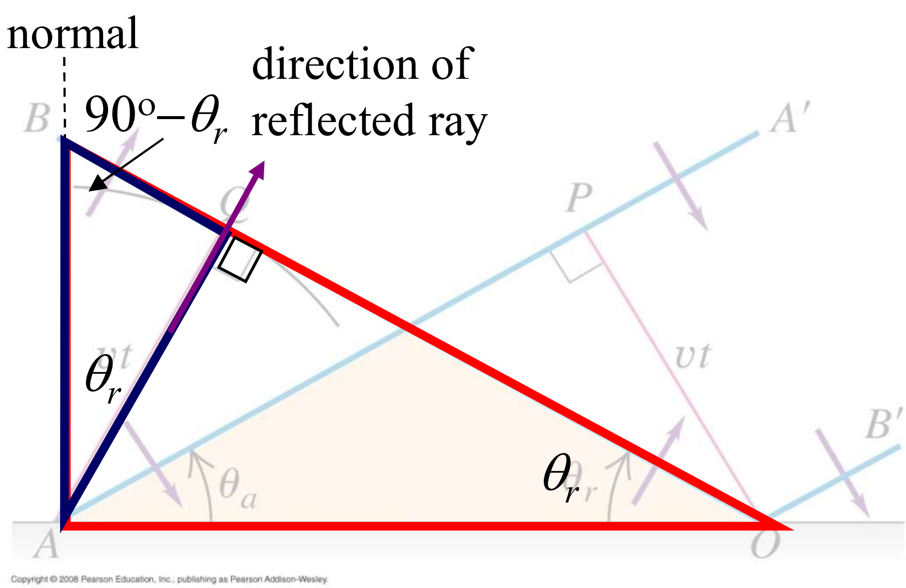
Application to the Law of Reflection

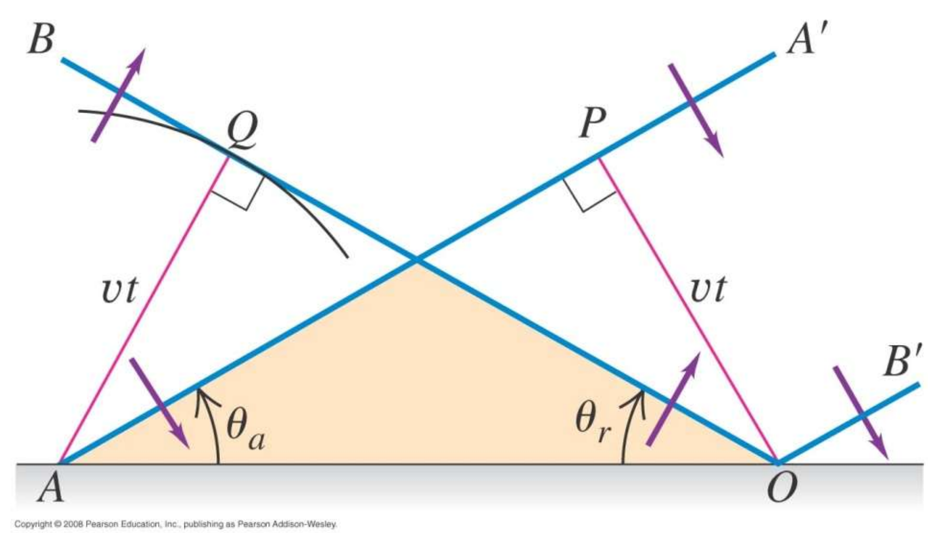
Successive positions of a plane wave AA' as it is reflected from a plane surface





Huygens' Principle Applies to Reflection





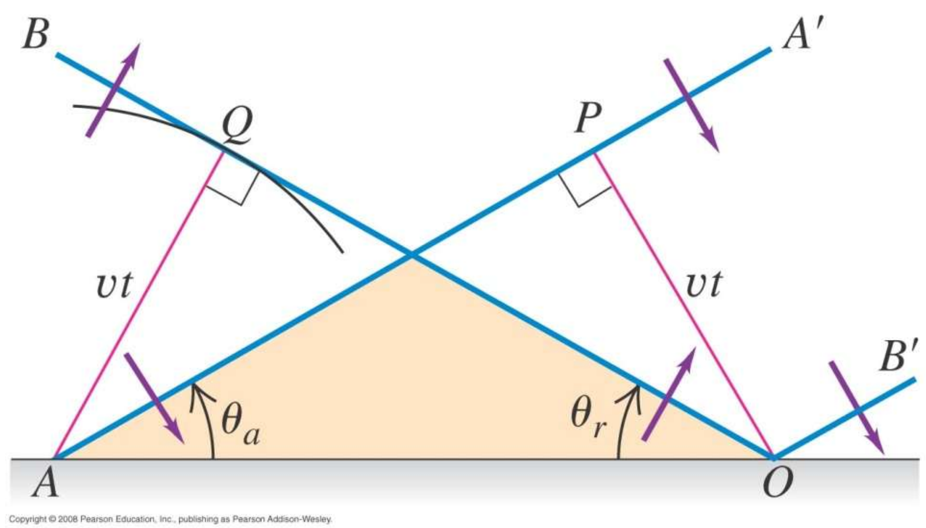
Since

$$AQ = OP$$

$$\angle Q = \angle P = 90^\circ \quad \Rightarrow \quad \triangle AQO \sim \triangle OPA \quad \text{and} \quad \theta_r = \theta_a$$

$$AO = AO$$

Huygens' Principle Applies to Reflection



Since

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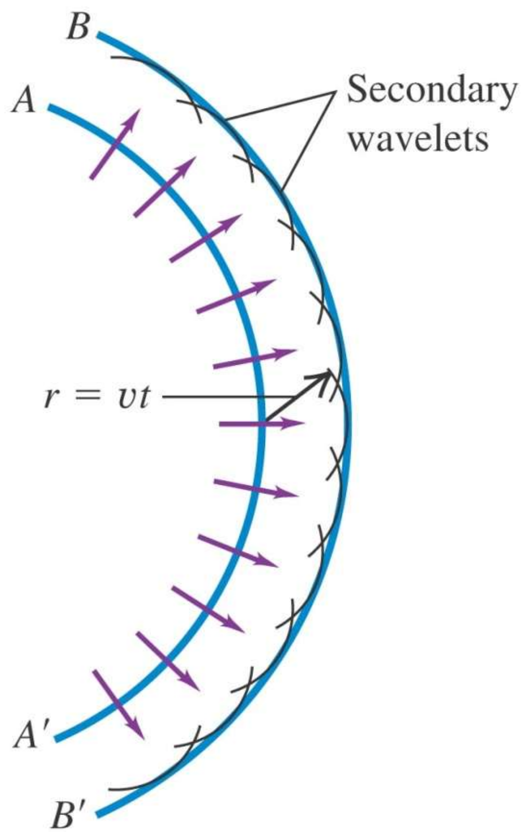
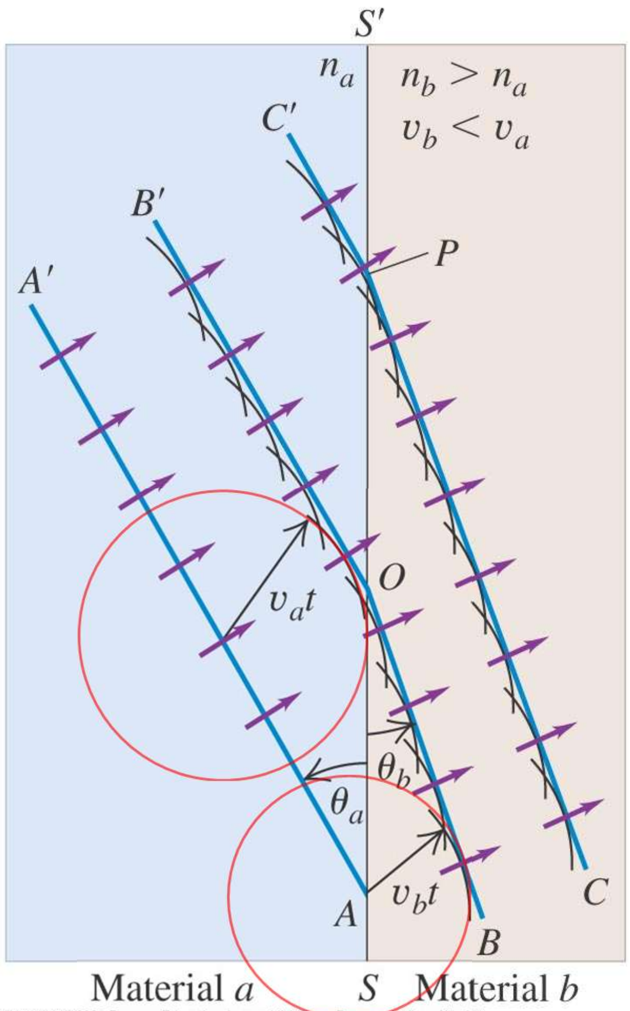
Huygens' Principle

Christiaan Huygens (1629-195): The Huygens' Principle can be used to predict the spreading of light wave. It is a geometrical construction using every point on a wave front as the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.



Huygens' Principle Application to Refraction

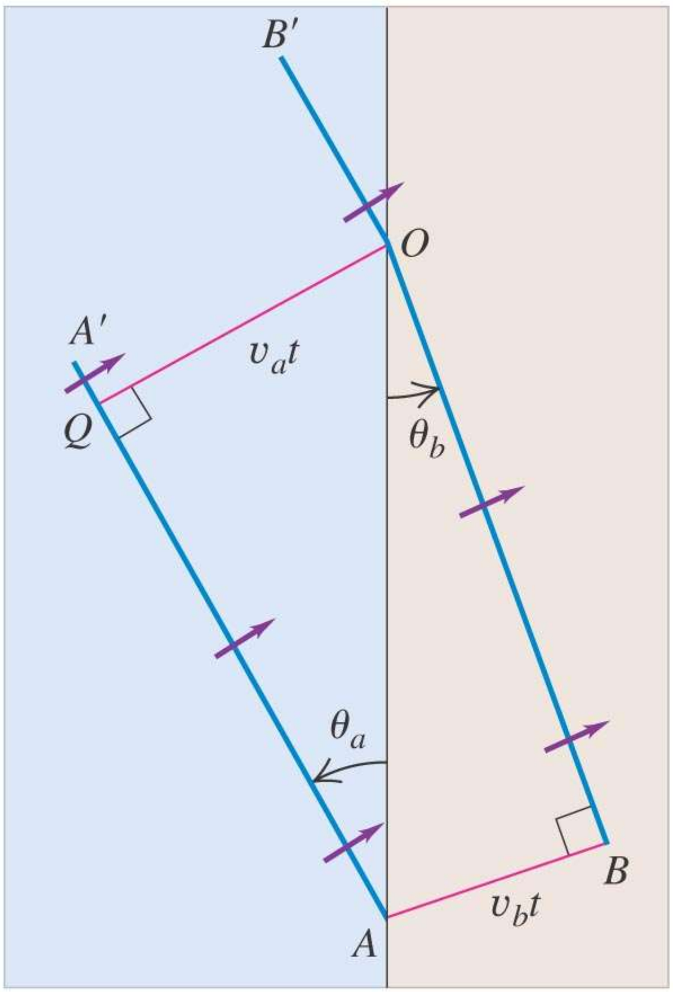
(a) Successive positions of a plane wave AA' as it is refracted by a plane surface



Note: Wave *slow* down in material *b* !

Distance traveled by wave in *b* is shorter → smaller wavelets

Huygens' Principle Application to Refraction



In time *t*, the wavelet at Q travels to O and the wavelet at A travels to B.

$$\Rightarrow \begin{cases} QO = v_a t \\ AB = v_b t \end{cases}$$

Using $\triangle AQO$ and $\triangle OBA$, we have,

$$\begin{cases} \sin\theta_a = \dfrac{v_a t}{AO} \\ \sin\theta_b = \dfrac{v_b t}{AO} \end{cases}$$

Substituting $v_a = c/n_a$ and $v_b = c/n_b$ and dividing,

$$\Rightarrow \frac{\sin\theta_a}{\sin\theta_b} = \frac{c/n_a}{c/n_b} = \frac{n_b}{n_a} \quad \Rightarrow \quad n_a \sin\theta_a = n_b \sin\theta_b$$

(Snell's Law)

PHYS 262

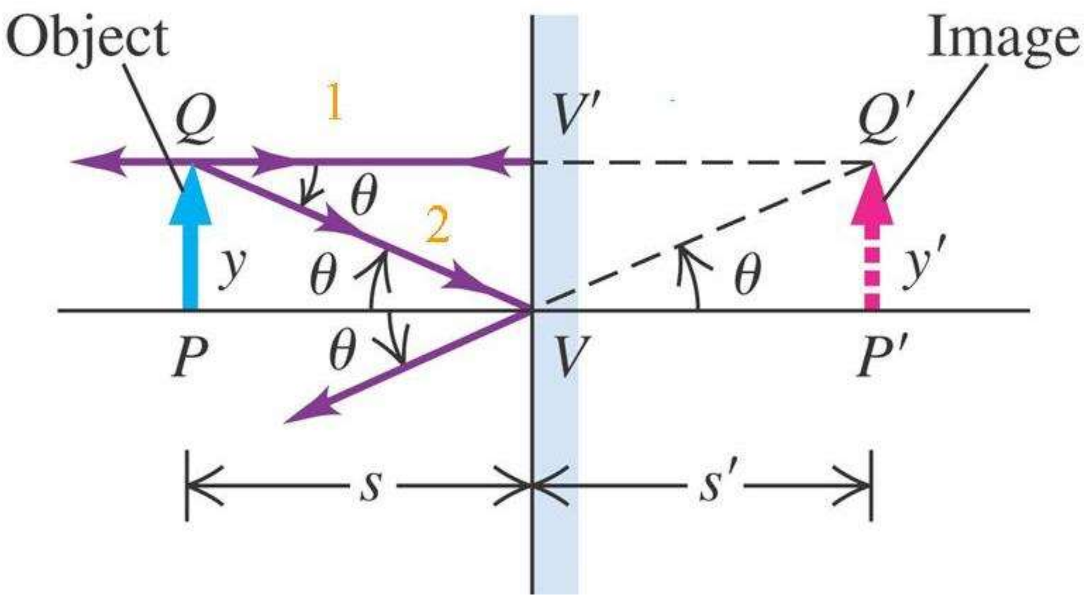
George Mason University

Prof. Paul So

Chapter 34: Geometric Optics

- Reflection & Refraction at a Plane Surface
- Reflection & Refraction at a Spherical Surface
- Thin Lenses
- Optical Instruments

Images Formed by Flat Mirrors



Rays tracing to find image:

- Originate rays from a point on an object.
- Follow thru their reflections
- Image is located at where reflected rays converge to or seem to diverge from. (follow ray 1 and 2)

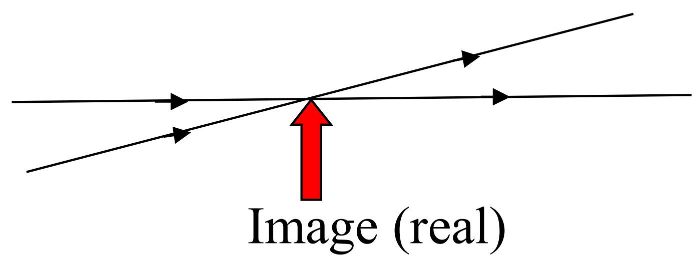
Definitions:

S → Object Distance
S' → Image Distance

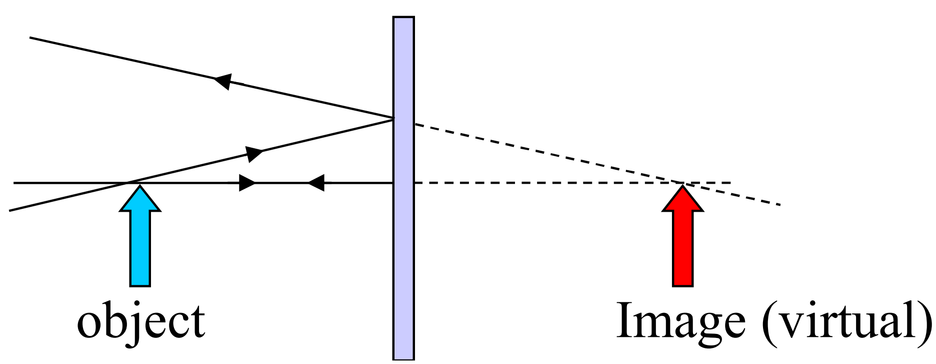
$$m = \frac{image\ height}{object\ height} = \frac{y'}{y}$$ → Lateral Magnification

Real and Virtual Images

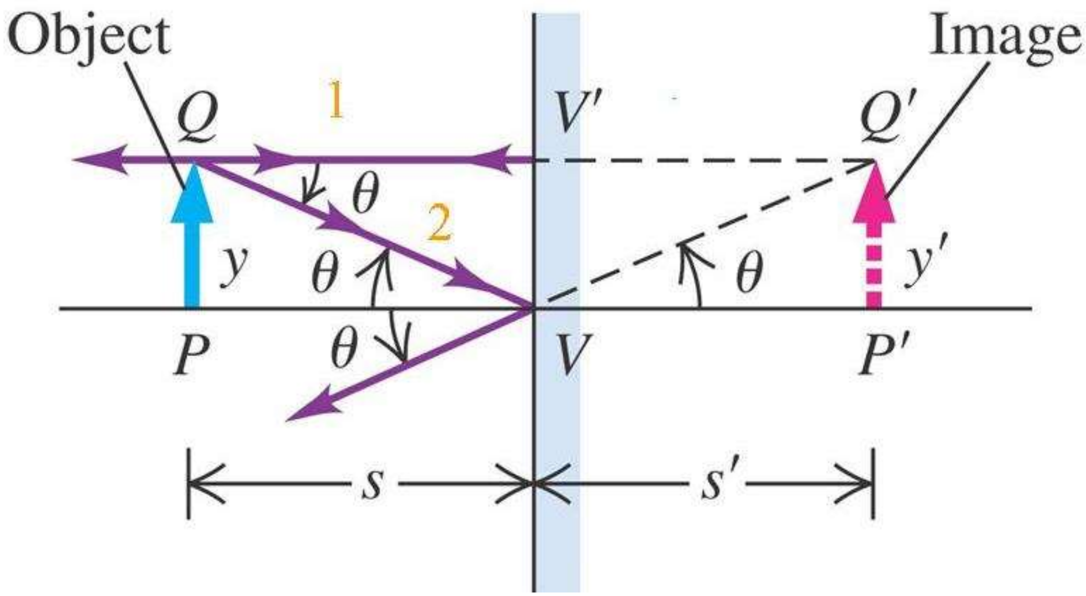
- Image can be **real** or **virtual**
  - **Real Image**: rays actually first converge then diverge from the image point.



  - **Virtual Image**: rays do not actually pass thru the image point but they appear to be diverging from it.



Images Form by Flat Mirrors



Since $\triangle PQV \sim \triangle VV'Q$ *and* $\triangle VV'Q \sim \triangle VV'Q'$ *and* $\triangle VV'Q' \sim \triangle P'Q'V$ are all congruent,

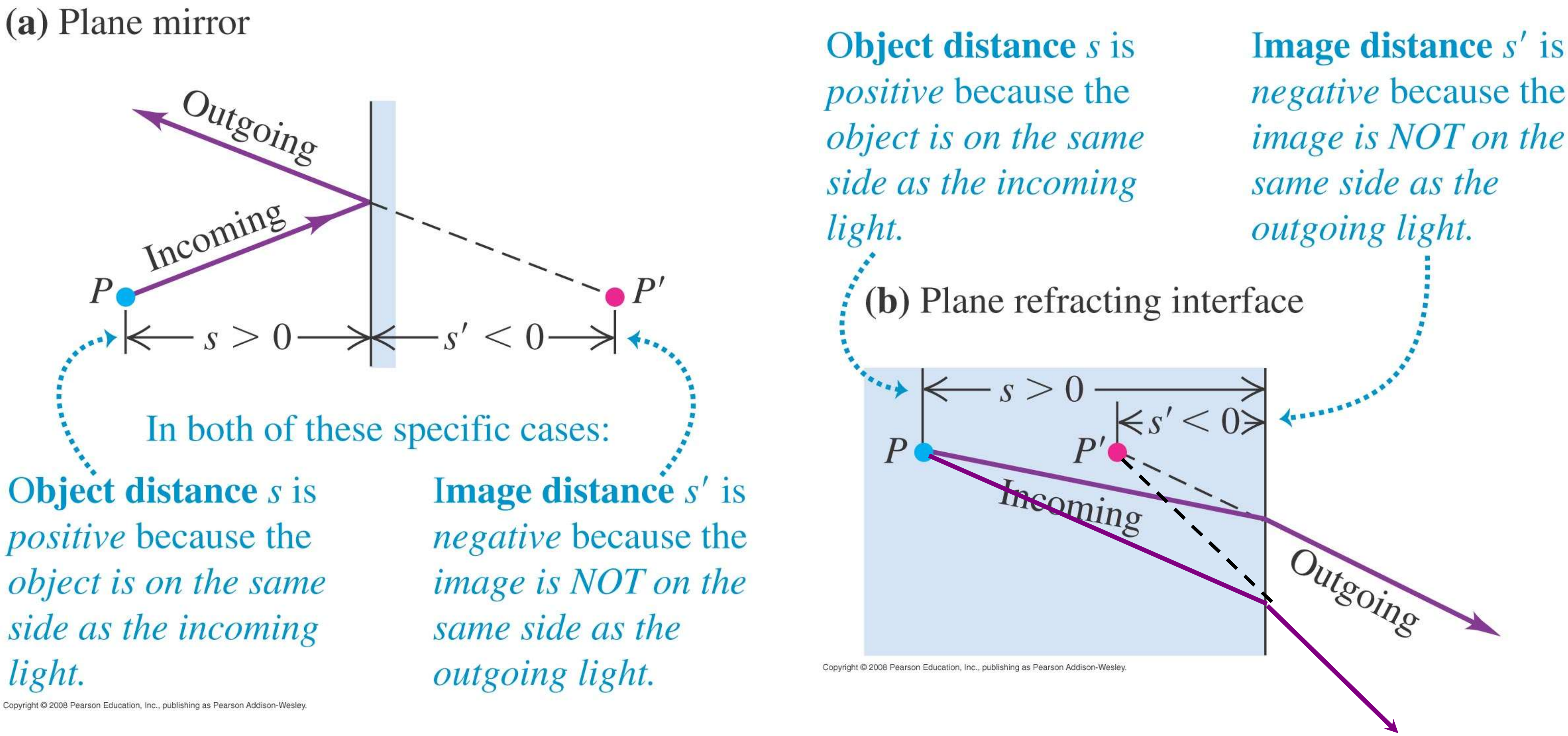
$$\Rightarrow \begin{cases} s = -s' \\ y = y' \quad (m = 1) \end{cases} \quad \text{(for flat mirrors)}$$

Sign Rules

1. Object Distance:
   - *s* is + (real) if the object is on the same side as the *incoming* light (for both reflecting and refracting surfaces) and *s* is – (virtual) otherwise.
2. Image Distance:
   - *s'* is + (real) if the image is on the same side as the *outgoing* light and is – (virtual) otherwise.
3. Object/Image Height:
   - *y* (*y*') is + if the image (object) is erect or upright. It is – if it is inverted.

* Incoming is on the side where the "original" object is located

Sign Rules & Incoming/Outgoing

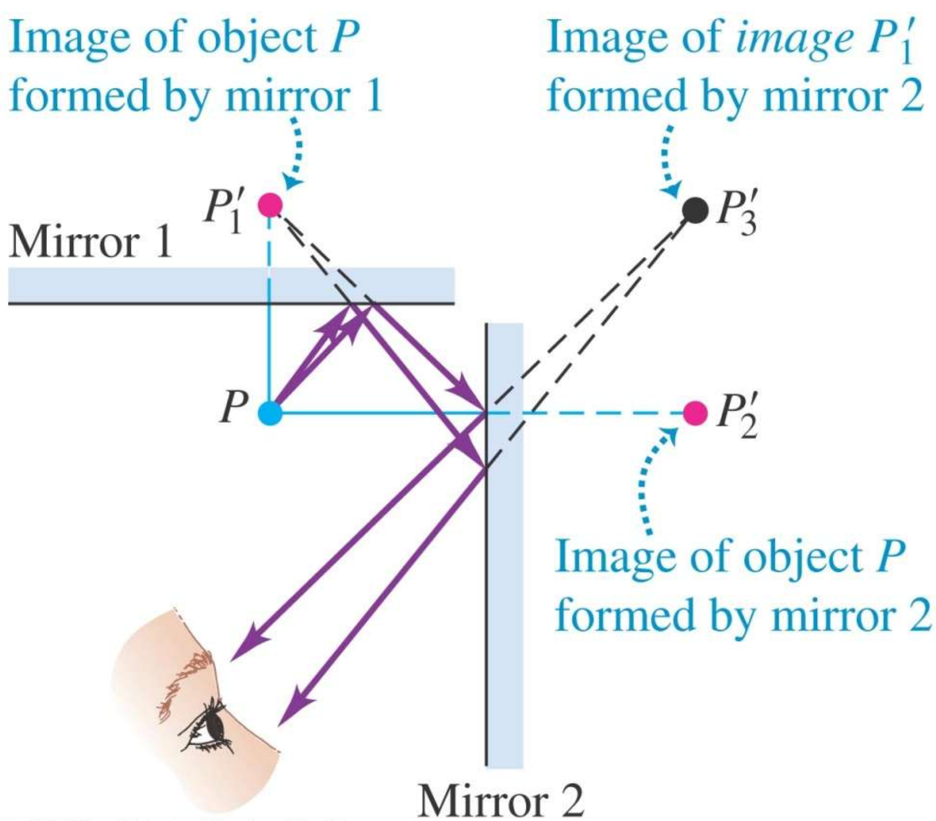


notes

- Clarify the image point issue (slide #7):
  - For a flat refracting surface, if observed in the other medium (light arrives at eye after passing thru interface), the object will appear to be at P'.
  - If observed in the same medium (light arrives at eye without passing thru interface), the object will stay at P.

Images by Multiple Reflections/Refractions

When there are multiple reflecting and/or refracting surfaces, image formed by previous surface serves as object for the next surface.

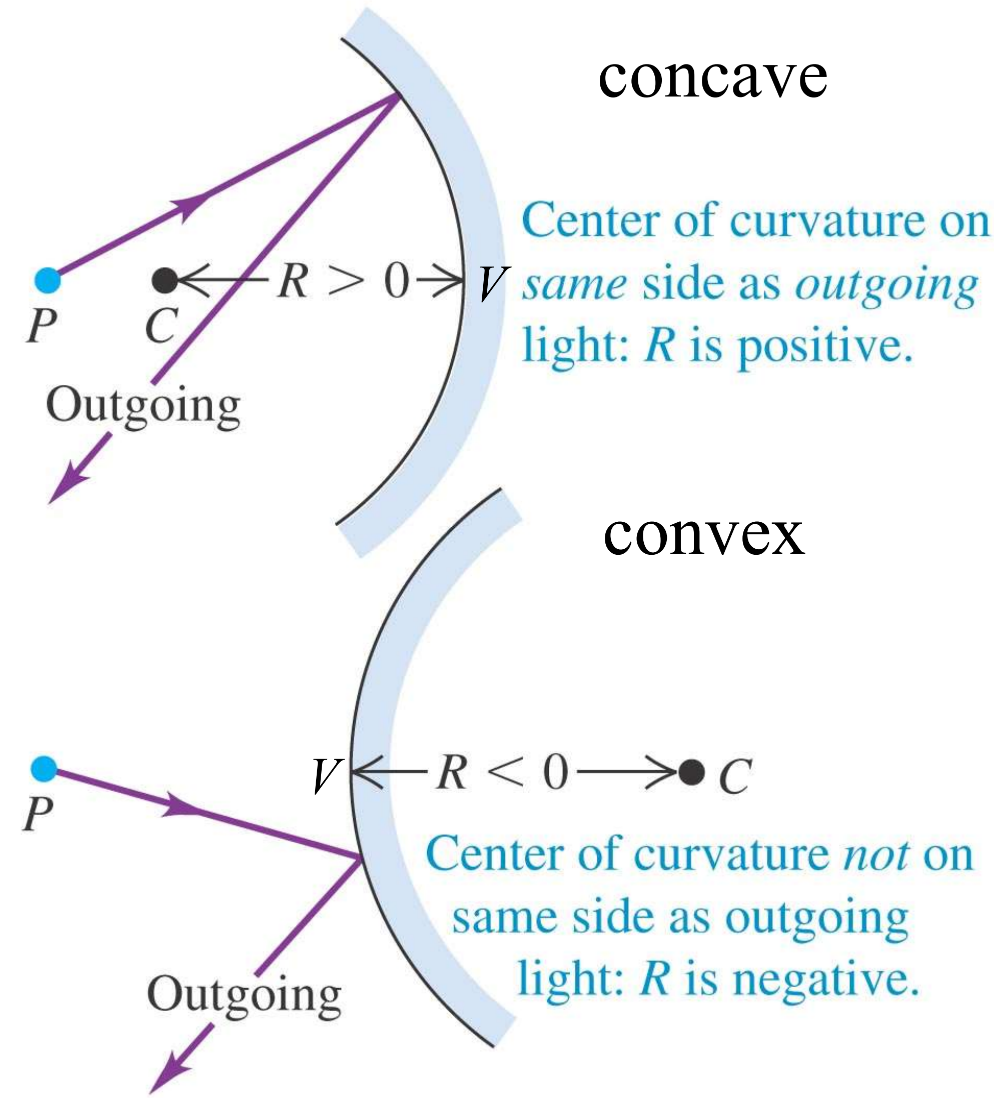


Reflection at a Spherical Surface

Sign Rule (#4) for the *radius of curvature* of a spherical surface:

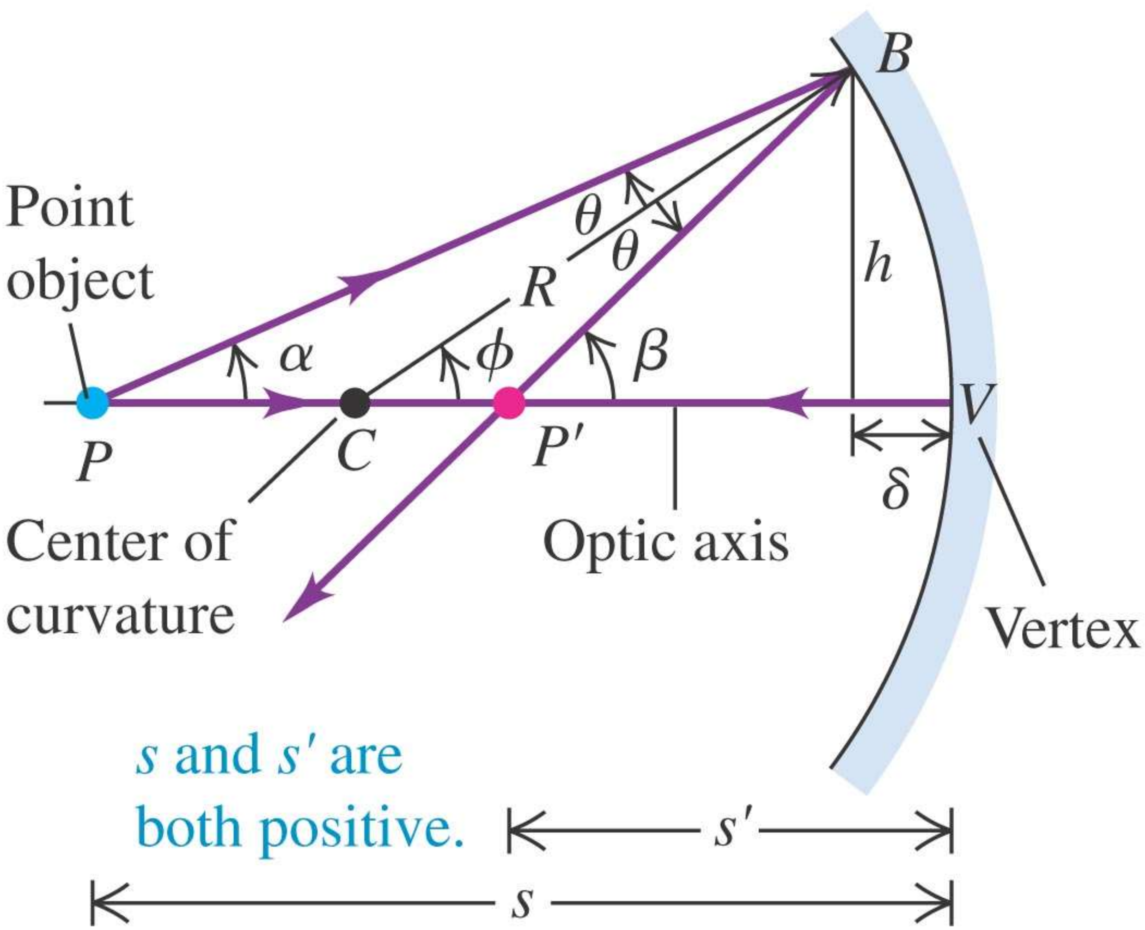
→The radius of curvature *R* is + when the center of curvature *C* is on the *same* side as the *outgoing* light (concave) and – otherwise.

(*CV* is called the optical axis.)



Reflection at a Spherical Surface

(a) Construction for finding the position P' of an image formed by a concave spherical mirror



From $\triangle PBC$, $\alpha + \theta = \phi$

From $\triangle CBP'$, $\phi + \theta = \beta$

Subtracting the two eqs and solve for θ:

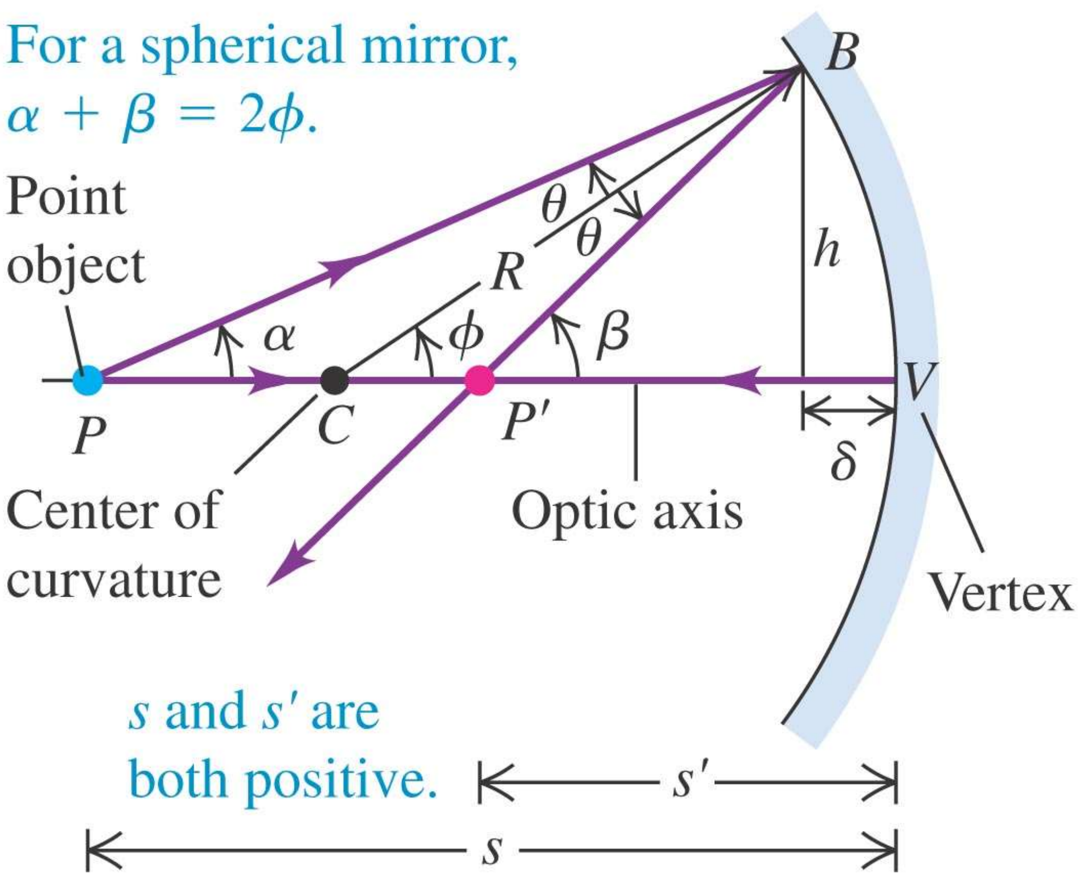
$$\alpha + \theta - (\phi + \theta) = \phi - \beta$$

$$\alpha - \phi = \phi - \beta$$

$$\alpha + \beta = 2\phi$$

Reflection at a Spherical Surface

(a) Construction for finding the position P' of an image formed by a concave spherical mirror



Relating the angles to the physical distances, we have:

$$\tan\alpha = \frac{h}{s-\delta} \qquad \tan\beta = \frac{h}{s'-\delta}$$

$$\tan\phi = \frac{h}{R-\delta}$$

Reflection at a Spherical Surface

Paraxial Assumption: We consider only rays which are *nearly parallel* to the *optical axis* and *close* to it. These rays are called **paraxial rays**.

With this approximation, the angles α, β, and ϕ will be small and δ can be neglected with respect to s, s', and R and

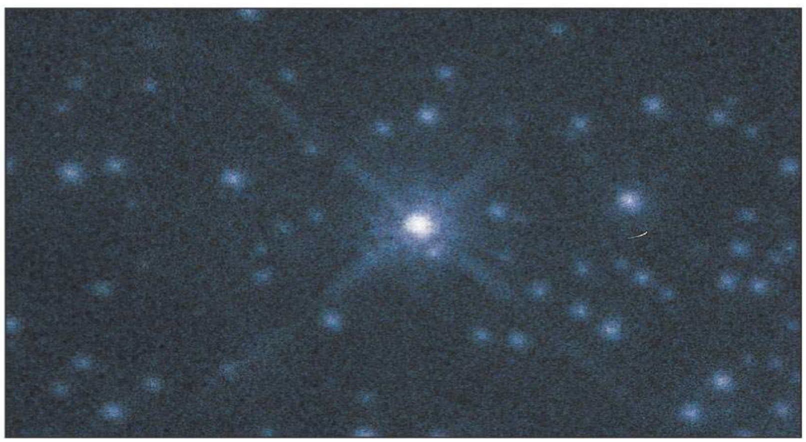
$$\alpha \simeq \tan\alpha = \frac{h}{s} \qquad \beta \simeq \tan\beta = \frac{h}{s'} \qquad \phi \simeq \tan\phi = \frac{h}{R}$$

Combining with $\alpha + \beta = 2\phi$ and eliminating h, we then arrive at,

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}}$$ (object-image relation, spherical mirror)

Focal Point and Focal Length

For an object (stars) very far away from the mirror $(s = \infty)$, the incoming rays can be considered to be parallel.



$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \quad \Rightarrow \quad s' = \frac{R}{2}$$

What will happen to these rays?

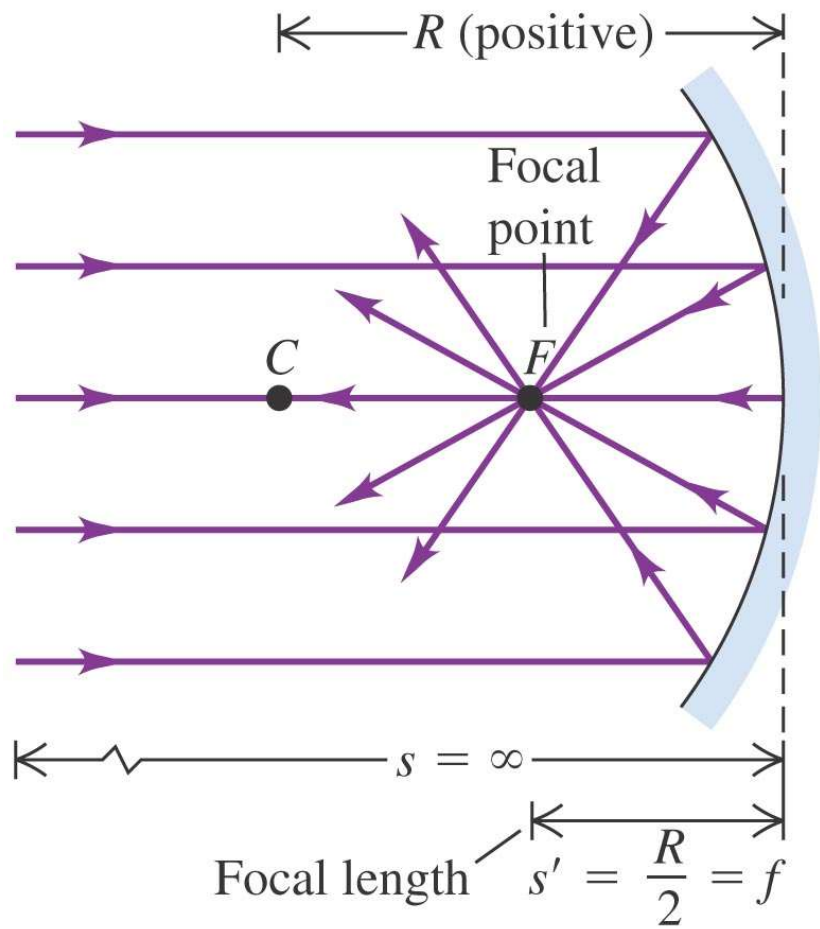
Focal Point and Focal Length

So, all parallel rays from ($s = \infty$) will converge to the *same* image point at $s' = R/2$. This special point is called the **focal point** F and the distance from the vertex of the mirror to F is call the **focal length** f,

$$f = \frac{R}{2}$$

(focal length of a spherical mirror)

(a) All parallel rays incident on a spherical mirror reflect through the focal point.



Focal Point and Focal Length

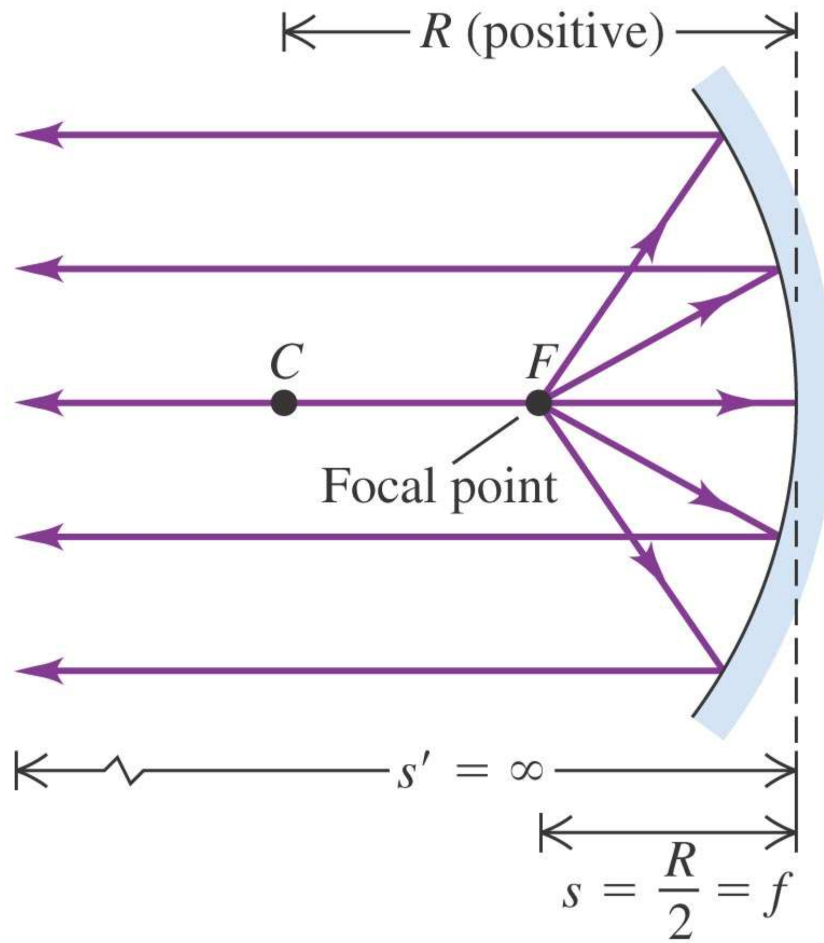
Let consider the reverse situation, light rays are emanating from the focal point F, where will the image be?

$$f = \frac{R}{2}$$

$$\frac{2}{R} + \frac{1}{s'} = \frac{2}{R} \quad \Rightarrow \quad \frac{1}{s'} = 0 \quad or \quad s' = \infty$$

Thus, as expected, the situation is time reversed, rays starting out from F will be reflected out toward infinity as parallel rays.

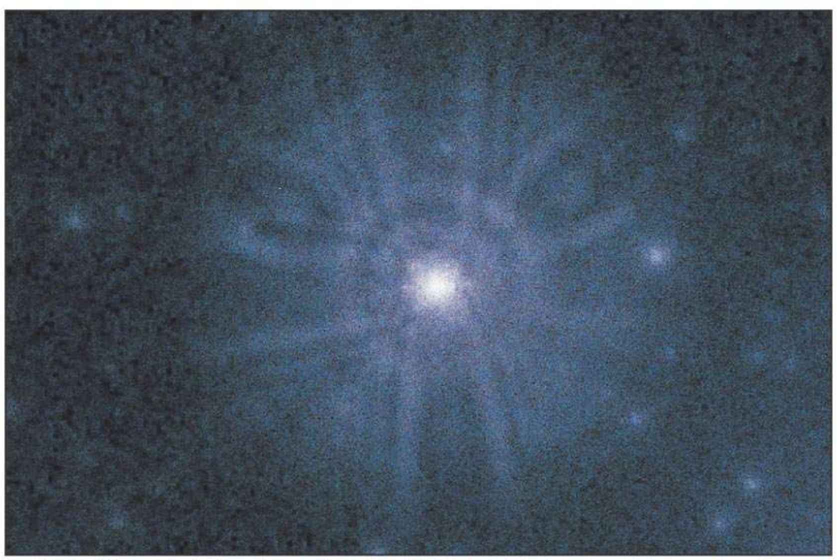
(b) Rays diverging from the focal point reflect to form parallel outgoing rays.



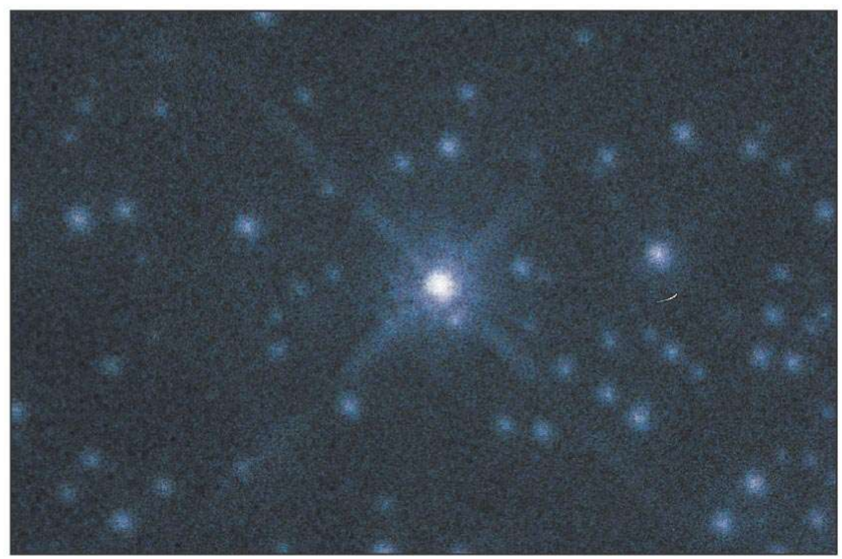
Spherical Aberration

Recall that this is only an *approximation*. The focal point is a sharp point only if we consider paraxial rays. For *non-paraxial rays*, they do *not* necessary converge to a precise point. The blurring of the focal point in an actual spherical mirror is called **spherical aberration**.

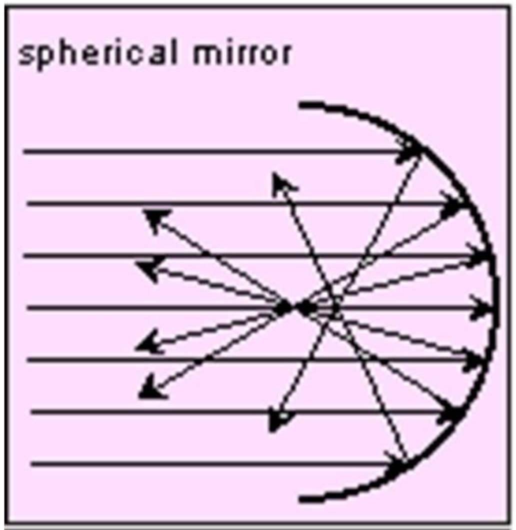
(b) A star seen with the original mirror



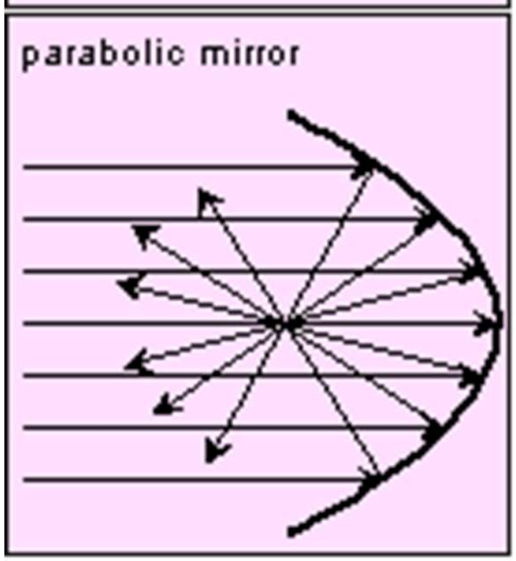
(c) The same star with corrective optics



Parabolic vs. Circular Mirrors



With spherical aberrations → only paraxial rays fall on Focus



No spherical aberrations → all parallel rays fall on Focus

James Webb Space Telescope

Carina Nebula

https://webbtelescope.org/contents/articles/webb-stats

Image Description and Credits

Carina Nebula (High resolution)

**Higher resolution is here: www.flickr.com/photos/nasawebbtelescope/52259221868/in/da... **
A star is born!

Behind the curtain of dust and gas in these "Cosmic Cliffs" are previously hidden baby stars, now uncovered by Webb. We know — this is a show-stopper. Just take a second to admire the Carina Nebula in all its glory: nasa.gov/webbfirstimages/

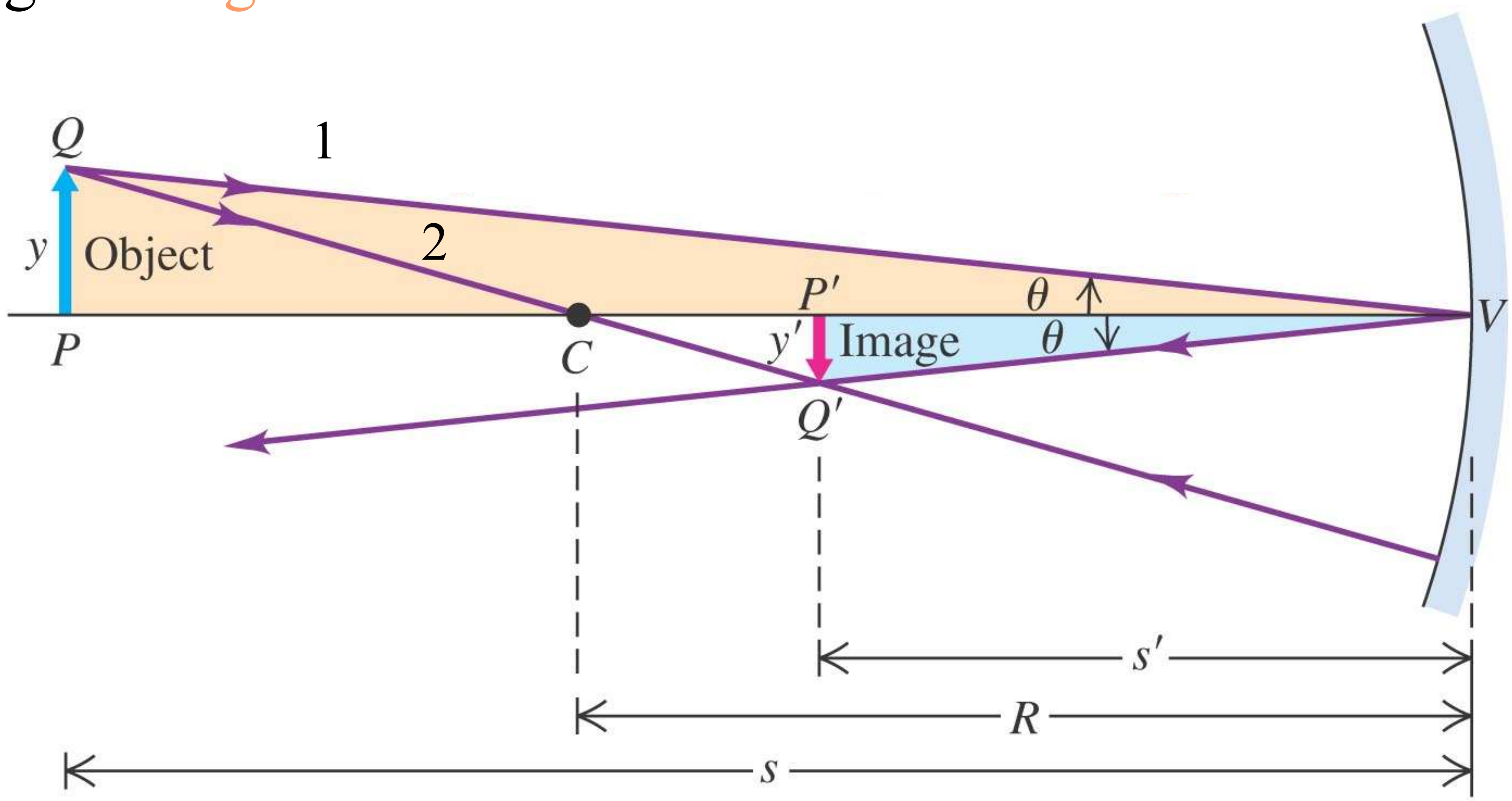
Webb's new view gives us a rare peek into stars in their earliest, rapid stages of formation. For an individual star, this period only lasts about 50,000 to 100,000 years.

Image Description:
The image is divided horizontally by an undulating line between a cloudscape forming a nebula along the bottom portion and a comparatively clear upper portion. Speckled across both portions is a starfield, showing innumerable stars of many sizes. The smallest of these are small, distant, and faint points of light. The largest of these appear larger, closer, brighter, and more fully resolved with 8-point diffraction spikes. The upper portion of the image is blueish, and has wispy translucent cloud-like streaks rising from the nebula below. The orangish cloudy formation in the bottom half varies in density and ranges from translucent to opaque. The stars vary in color, the majority of which have a blue or orange hue. The cloud-like structure of the nebula contains ridges, peaks, and valleys – an appearance very similar to a mountain range. Three long diffraction spikes from the top right edge of the image suggest the presence of a large star just out of view.

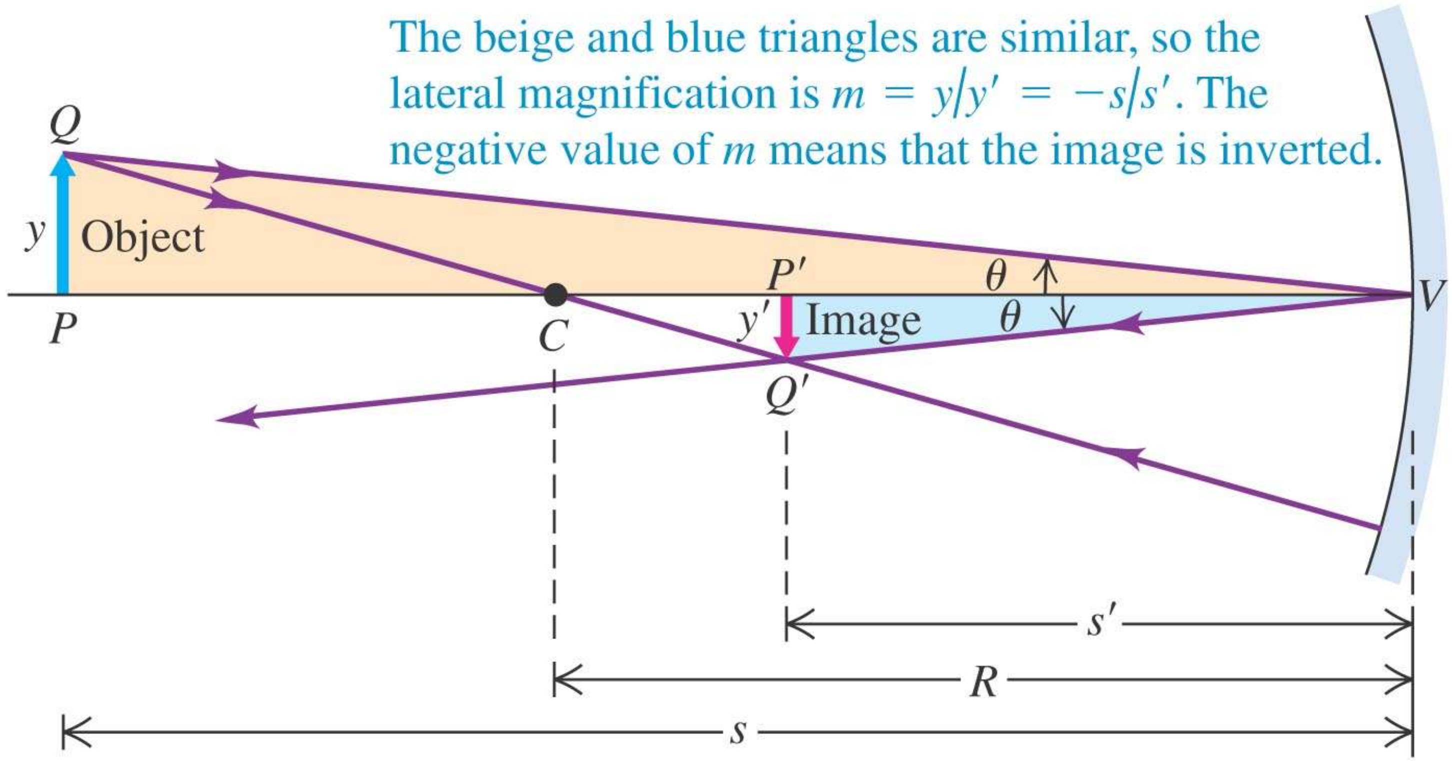
Credits: NASA, ESA, CSA, and STScI

Lateral Magnification of a Spherical Mirror

Following rays #1 and #2, we can form the following two similar triangles: beige and blue.



Lateral Magnification of a Spherical Mirror



The two similar triangles gives, $\dfrac{y}{s} = \dfrac{-y'}{s'}$

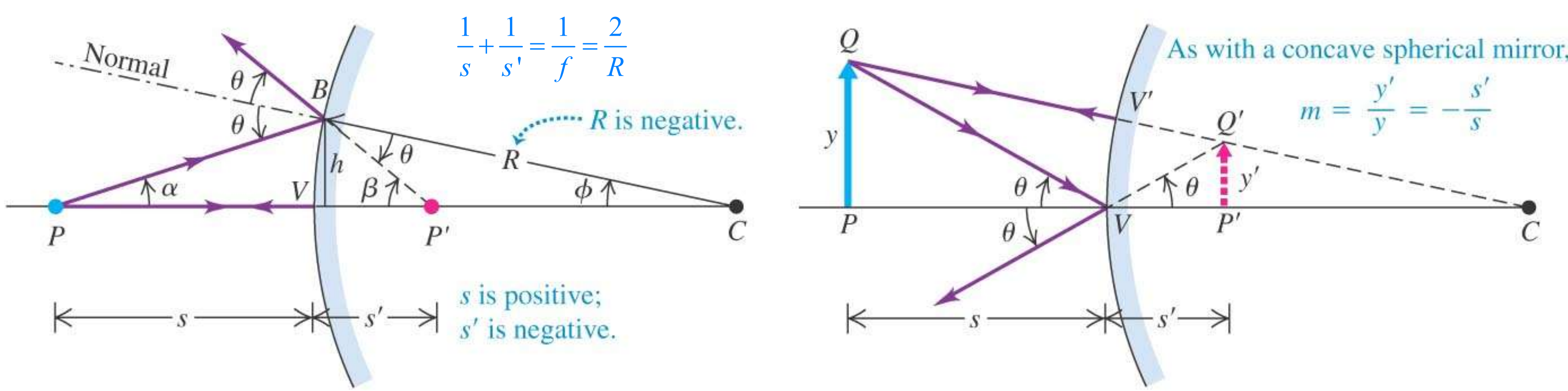
Substituting this into the definition for m, $m = \dfrac{y'}{y} = \dfrac{-s'}{s}$

Convex Spherical Mirrors

The geometric optics formulas for a convex mirror are the same as for a concave mirror except that *R* and *f* are *negative*.

(a) Construction for finding the position of an image formed by a convex mirror

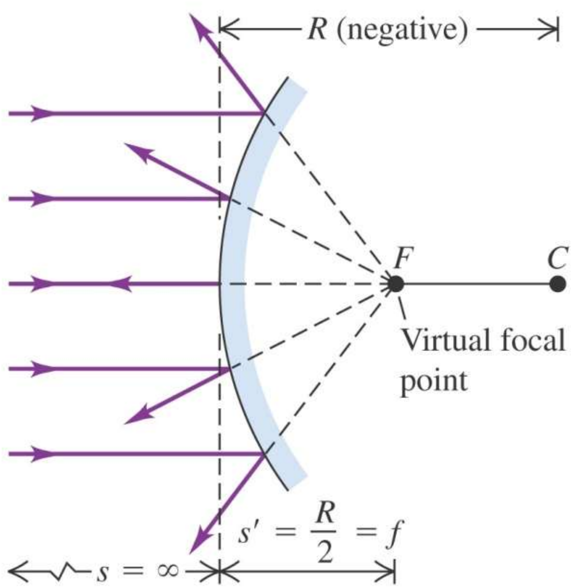
(b) Construction for finding the magnification of an image formed by a convex mirror



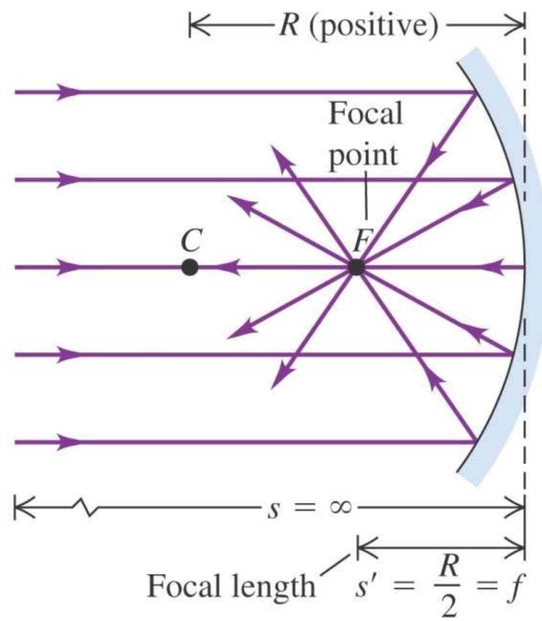
Convex Spherical Mirrors

With *R* and *f negative*, parallel rays falling upon a convex mirror will *diverge* as if emanating from a *virtual focal point F* behind the mirror.

convex



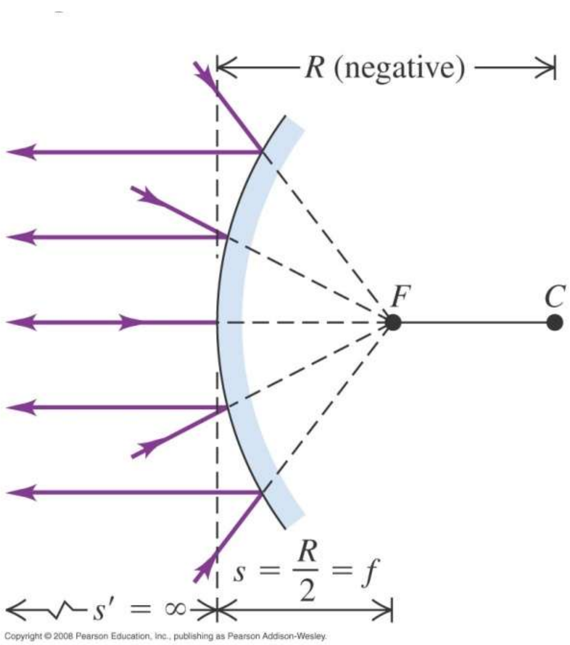
concave



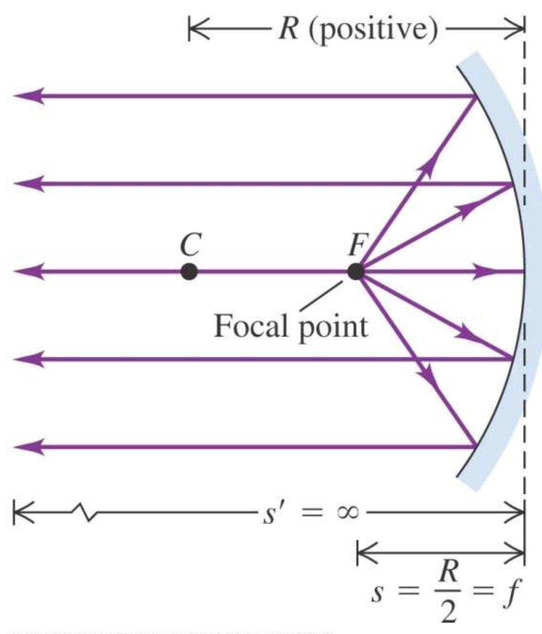
Convex Spherical Mirrors

With *R* and *f negative*, parallel rays falling upon a convex mirror will *diverge* as if emanating from a *virtual focal point F* behind the mirror. Rays aiming toward this virtual focal point will be reflected back toward infinity as parallel rays.

convex



concave



Summary for Spherical Mirrors

The following are valid for both concave and convex spherical mirrors if we follow the proper sign conventions.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ (object-image relation, spherical mirror)

$$f = \frac{R}{2}$$ (focal length, spherical mirror)

$$m = -\frac{s'}{s}$$ (lateral magnification, spherical mirror)

Note: these equations agree with results for a flat mirror if we take $R = \infty$.