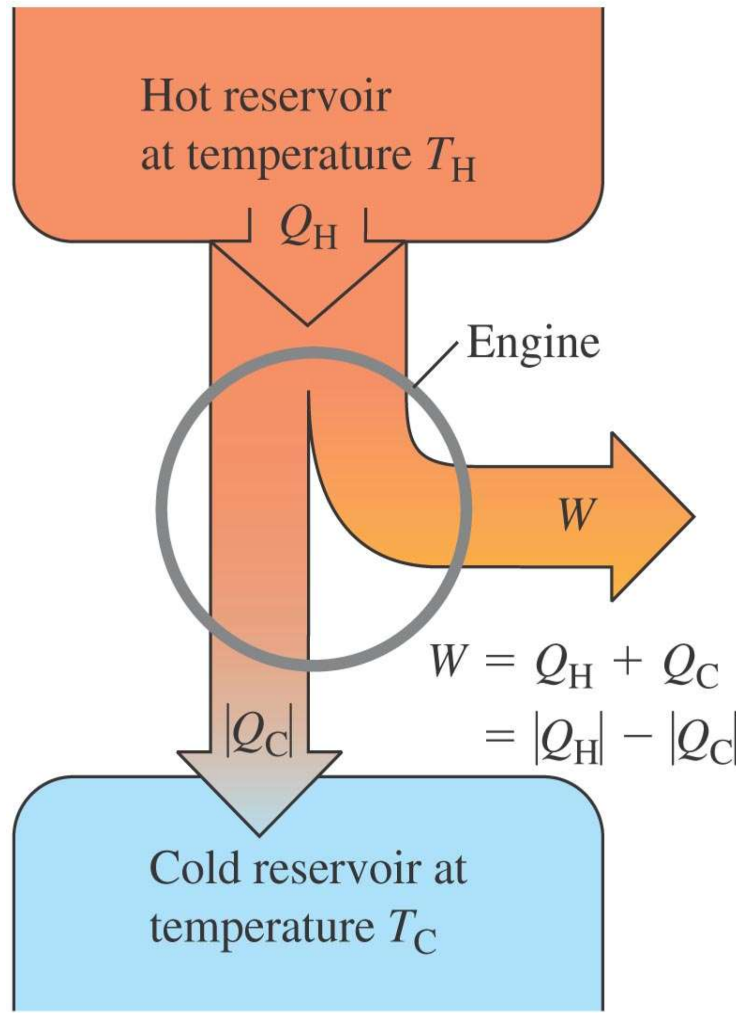
Heat Engines

- Definition: A device that converts a given amount of *heat* into *mechanical energy*.
- All heat engines carry some *working substance* thru a *cyclic process*:

Engine absorbs heat from hot reservoir at T_H

Mechanical work is done by engine

Engine releases residual heat to cold reservoir at T_C



Dstering

Work Done by a Heat Engine

The heat engine works in a *cyclic* process,

$$\Delta U = 0$$

1st Law gives, $$\Delta U = Q_{net} - W = 0$$

$$\boxed{Q_{net} = W}$$

where, $$Q_{net} = Q_H + Q_C = |Q_H| - |Q_C|$$

explicit signs for heats

Efficiency for a Heat Engine

- Thermal Efficiency *e* is defined as the *ratio* of the mechanical energy output to the heat energy input,

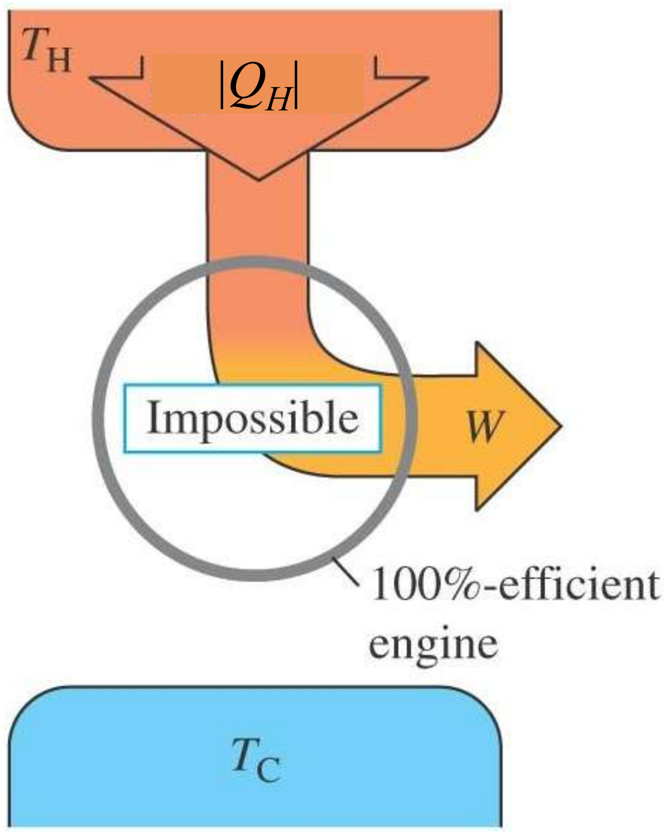
$$e = \frac{W}{Q_H} = \frac{\text{what you get out}}{\text{what you put in}}$$

Substituting $W = Q_H + Q_C$, we have

using explicit signs here

$$e = \frac{W}{Q_H} = \frac{Q_H + Q_C}{Q_H} = 1 + \frac{Q_C}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$$

A “perfect” (100% efficient) heat engine



A “perfect” heat engine means 100% efficiency (e=1). This means that

$$e = 1 - \frac{|Q_C|}{|Q_H|} \quad \rightarrow \quad 1 \quad \text{means} \quad Q_C = 0$$

All heat absorbed from reservoir T_H is converted into mechanical work W. No residual heat is released back.

The Kelvin-Planck’s statement of the 2nd Law does *not* allow this !

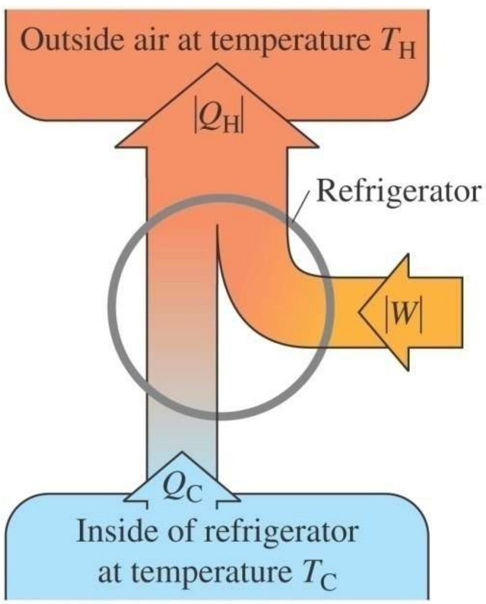
$\cancel{e_{perfect} = 1}$ and $e_{realistic} < 1$

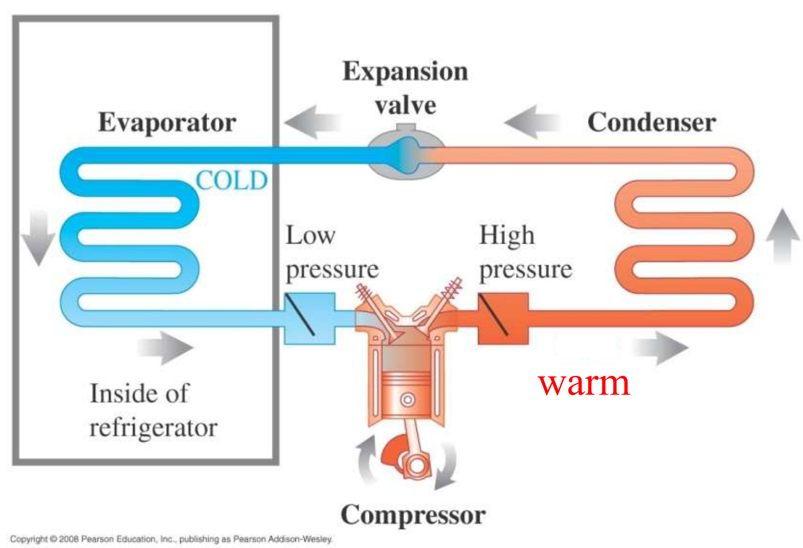
D drinking bird

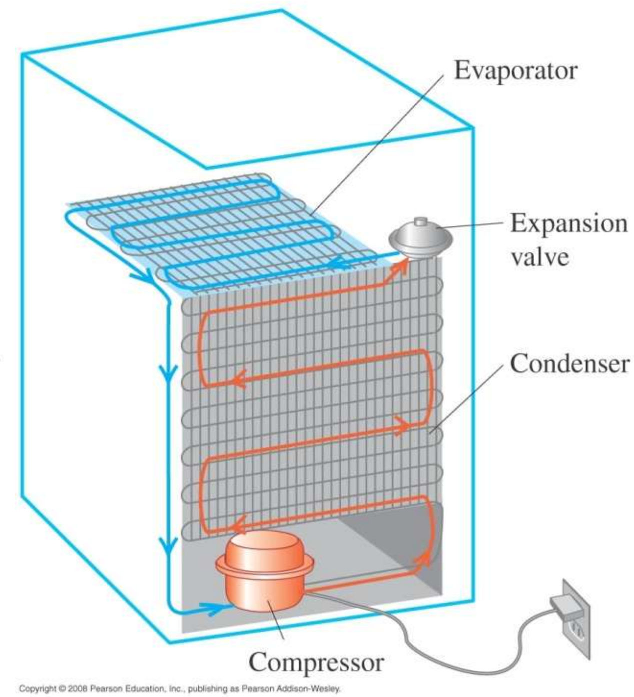
Refrigerators

Refrigerators are basically heat engine running in *reverse*.

- Heat from inside the refrigerator (cold T reservoir) is absorbed and released into the room (high T reservoir) with the *input* of mechanical work.







Refrigerators

From 1st Law, $\Delta U_{cycle} = 0 = \left(|Q_C| - |Q_H|\right) - \left(-|W|\right)$

$$|Q_H| = |Q_C| + |W|$$

explicit signs

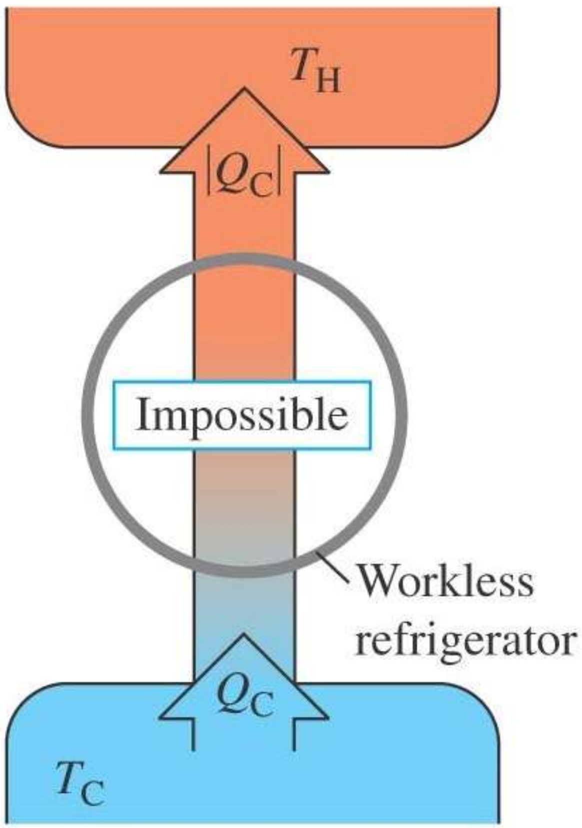
(Note: we have put in the explicit signs according to our sign convention.)

- ✓ $|Q_C|$ (absorbed) → positive
- ✓ $|Q_H|$ (released) → negative
- ✓ $|W|$ (work is done *on* working substance by motor) → negative

Coefficient of Performance for a Refrigerator

$$K = \frac{\text{what you get}}{\text{what you put in}} = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$$

A "perfect" Refrigerator



$$K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$$

A "perfect" refrigerator means ($K = \infty$). This means that

$$|Q_H| = |Q_C| \quad or \quad |W| = 0$$

No mechanical work W is needed to transfer heat from the cold reservoir to the hot reservoir.

The Clausius's statement of the 2nd Law does *not* allow this !

$$\Rightarrow \quad \boxed{K_{realistic} < \infty}$$

2nd Law, Disorder, & Available Energy

Two Forms of Energy in any Thermal Process:

Internal Energy

In the *Kinetic-Molecular Model*, this consists of the KE and PE associated with all the *randomly* moving microscopic molecules.

(One typically cannot control the individual *random* motions of all these molecules.)

Macroscopic Mechanical Energy

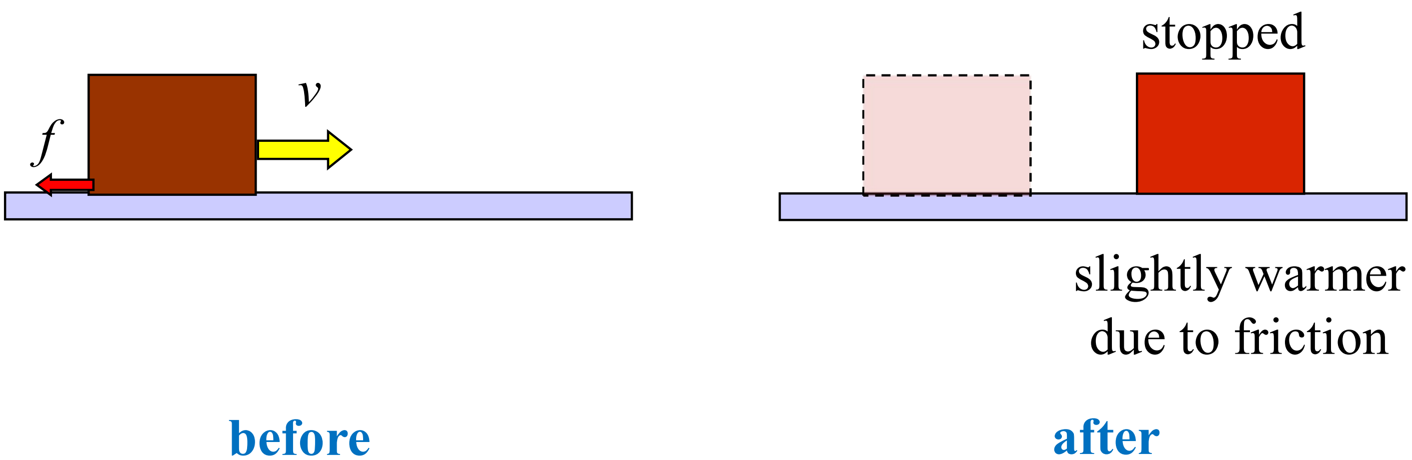
The piston's motion in an automobile engine results from the overall *coordinated* macroscopic motion of the molecules.

(Energy associated with this coordinated [*ordered*] motion can be used for useful work.)

2^{nd} Law, Disorder, & Available Energy

In a natural process (a block sliding to a stop),

e.g.



- The coordinated motion of the block is converted into the slightly *more* agitated random motions of the molecules in the block and table.

- **Macroscopic Mechanical Energy** (KE of the block) is converted into **Internal Energy** through *heat* as a result of friction.

2nd Law, Disorder, & Available Energy

- Now consider the possibility of *reverse* situation… do we typically see a group of randomly moving molecules **all** push a block in a coordinated fashion?
- NO. In other words, one typically cannot *completely* convert the internal energy of a system back into macroscopic mechanical energy.
- However, this does not mean that internal energy is not accessible. An *Heat Engine* is exactly the machine that can perform this conversion but only *partially*.

The **2nd Law of Thermodynamics** is basically a statement limiting the *availability* of internal energy for useful mechanical work.

The Carnot Cycle (Most Efficient Heat Engine)

- A *reversible* cycle described by Sadi Carnot in 1824.
- The Carnot Theorem gives the *theoretical limit* to the thermal efficiency of *any* heat engine.
- The Carnot cycle consists of:
  - A cycle operating between two temperatures: T_H and T_C
  - 2 reversible isothermal processes in which $Q\pm$
  - 2 reversible adiabatic processes in which $T_H \leftrightarrow T_C$
  - An Ideal Gas as its working substance

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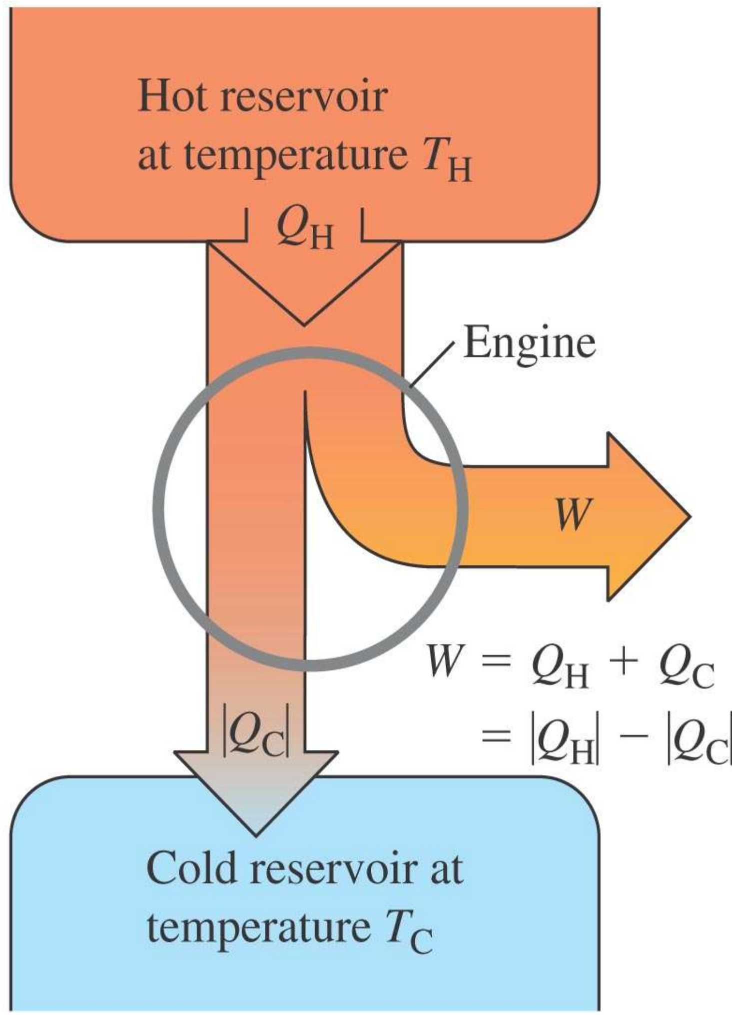

Steps of the Carnot Cycle

[animation](http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/carnot.htm)

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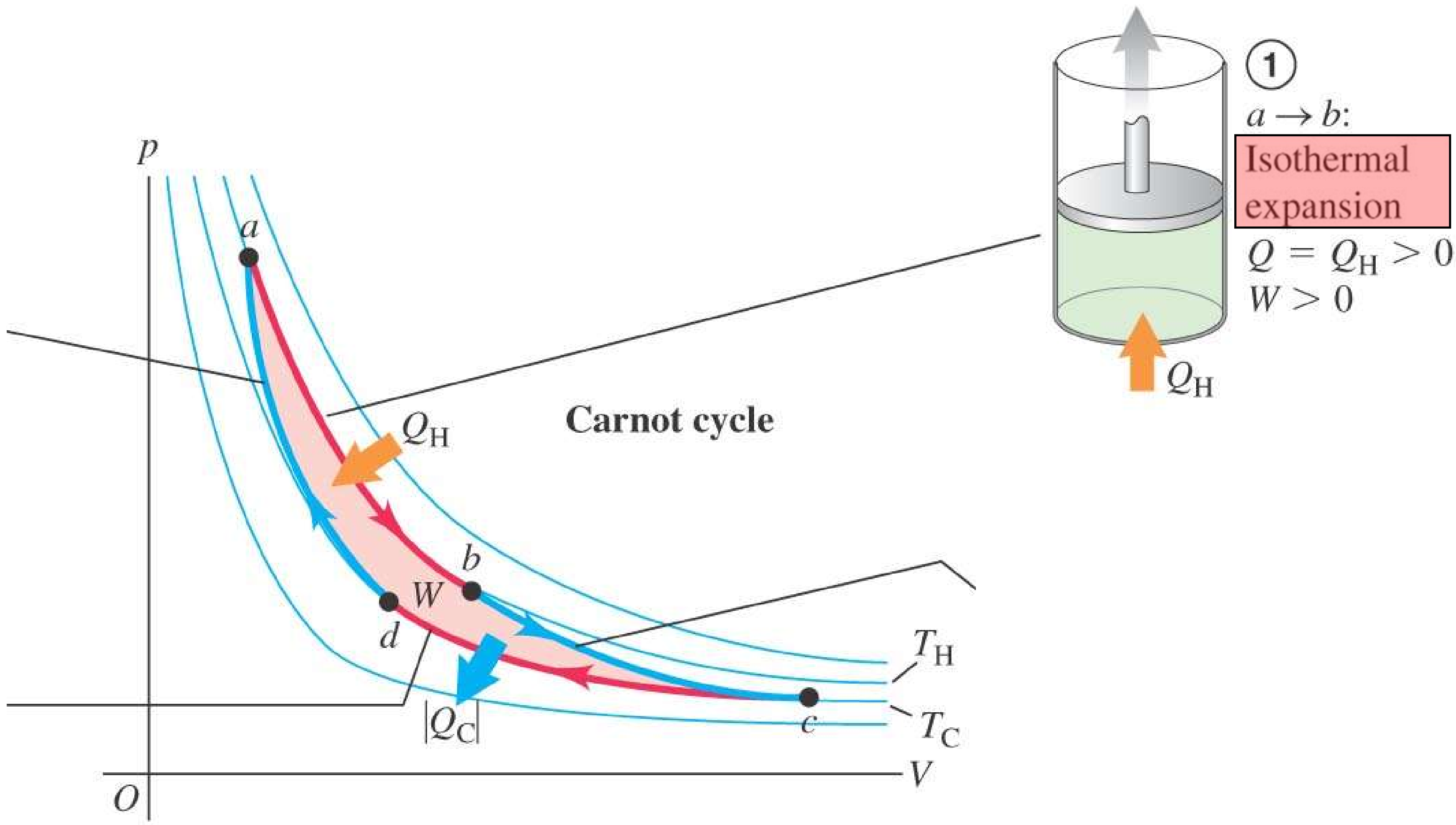

Steps of the Carnot Cycle

animation

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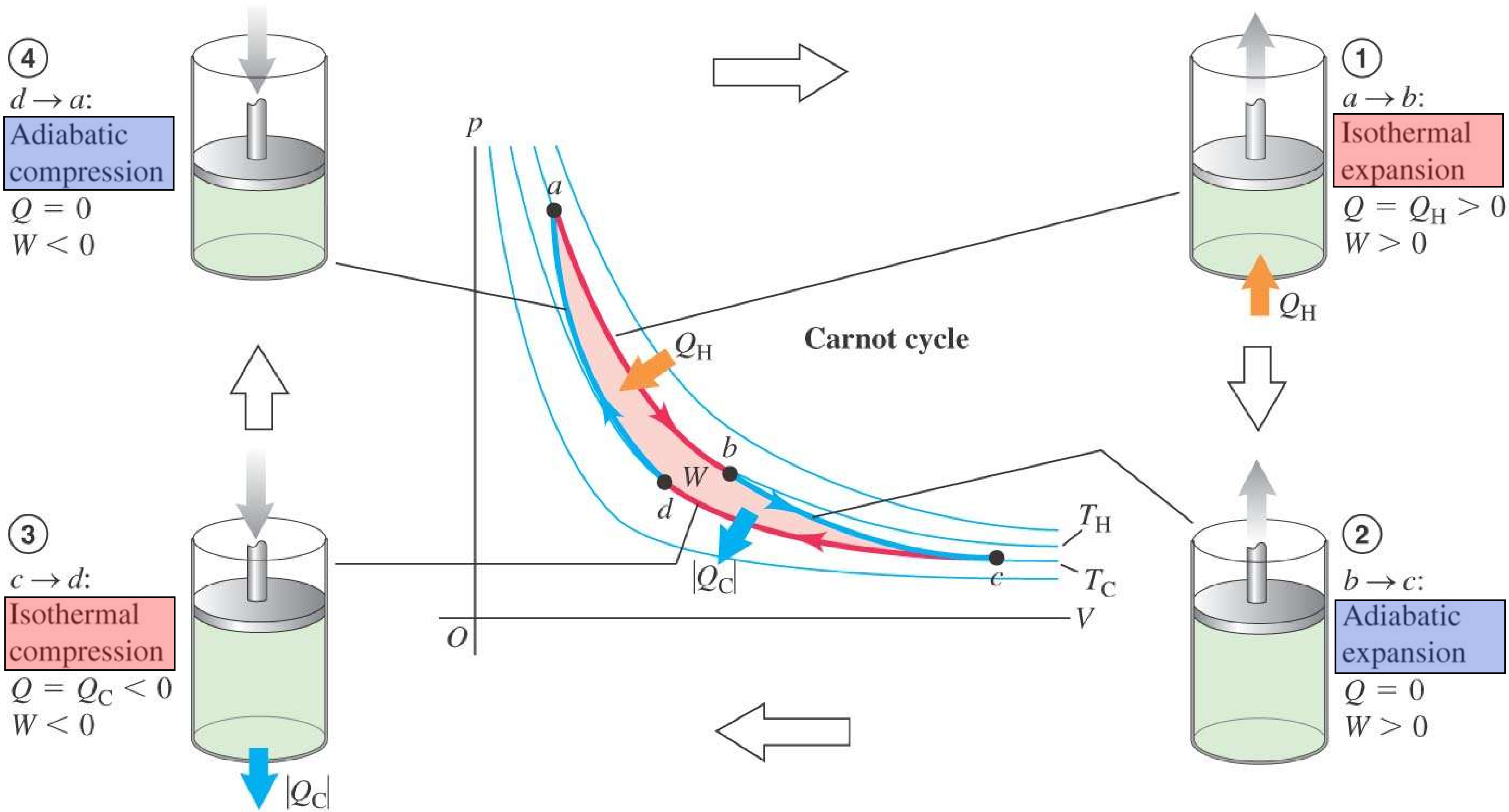


Details on the Carnot Cycle

The isothermal expansion ($a \rightarrow b$) and compression ($c \rightarrow d$):

$$\Delta U_{isothermal} = 0$$

(T is constant and $U(T)$ is a function of T only for an Ideal Gas.)

$$|Q_H| = W_{ab} = nRT_H \ln\left(\frac{V_b}{V_a}\right)$$

($a \rightarrow b$: isothermal expansion)

$$|Q_C| = |W_{cd}| = \left| nRT_C \ln\left(\frac{V_d}{V_c}\right) \right| = nRT_C \ln\left(\frac{V_c}{V_d}\right) \quad (V_c > V_d)$$

($c \rightarrow d$: isothermal compression)

Details on the Carnot Cycle

The adiabatic expansion ($b \rightarrow c$) and compression ($d \rightarrow a$):

$$Q_{adiabatic} = 0 \qquad \text{(by definition)}$$

From Section 19.8, we learned that $TV^{\gamma-1} = const$ for adiabatic processes.

$$T_H V_b^{\gamma-1} = T_C V_c^{\gamma-1} \qquad (b \rightarrow c\text{: adiabatic expansion})$$

$$T_H V_a^{\gamma-1} = T_C V_d^{\gamma-1} \qquad (d \rightarrow a\text{: adiabatic compression})$$

Dividing these two equations gives,

$$\frac{V_b}{V_a} = \frac{V_c}{V_d}$$

Efficiency of the Carnot Cycle

From definition, we have $e = 1 - \dfrac{|Q_C|}{|Q_H|}$

Using our results for Q_C and Q_H from the *isothermal* processes,

$$e = 1 - \frac{nRT_C \ln\left(V_c/V_d\right)}{nRT_H \ln\left(V_b/V_a\right)} = 1 - \frac{T_C \ln\left(V_c/V_d\right)}{T_H \ln\left(V_b/V_a\right)}$$

Then, from the *adiabatic* processes, we have $V_b/V_a = V_c/V_d$

$$\Rightarrow \quad \frac{\ln\left(V_c/V_d\right)}{\ln\left(V_b/V_a\right)} = 1 \quad \Rightarrow \quad \boxed{e \equiv 1 - \frac{|Q_C|}{|Q_H|} = 1 - \frac{T_C}{T_H}}$$

(T must be in K)

(Carnot Cycle only)

Efficiency of the Carnot Cycle

$$e_{carnot} = 1 - \frac{T_C}{T_H} \quad \text{(Carnot Cycle)}$$

General Comments:

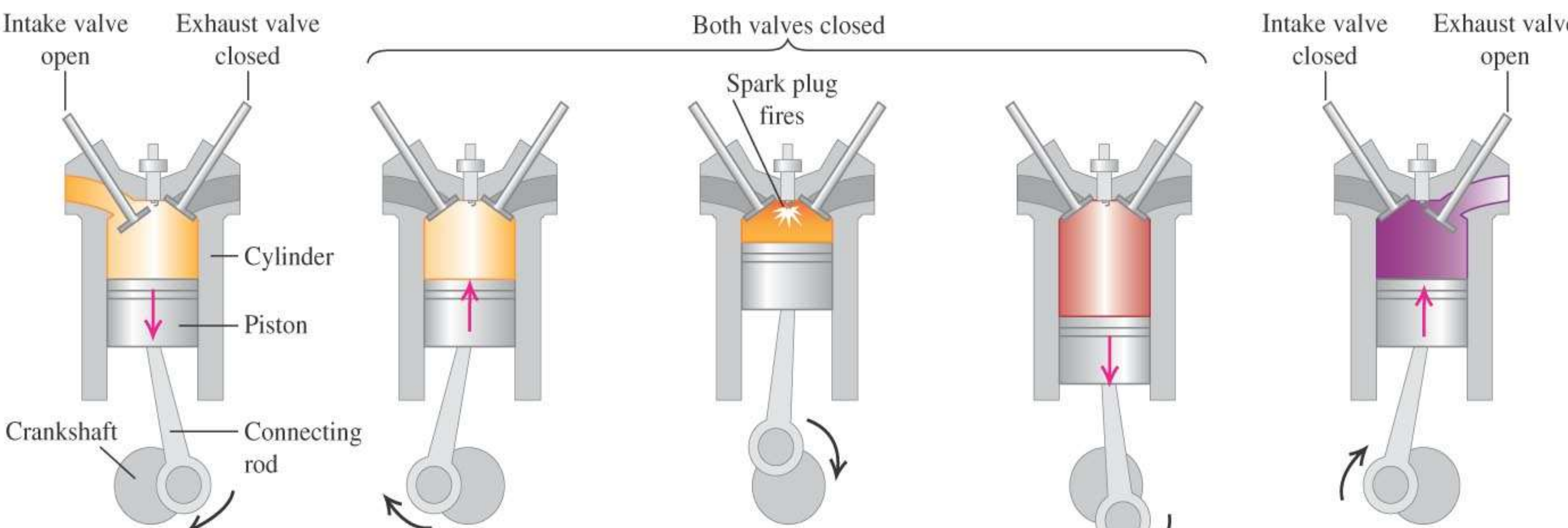
- Higher efficiency if either T_C is lower and/or T_H is higher.
- For any *realistic* thermal process, the cold reservoir is far above absolute zero, i.e., $T_C > 0$.
- Thus, a *realistic e* is *strictly less* than 1! (No 100% efficient heat engine)
- *Realistic* heat engines must take in energy from the high T reservoir for the work that it produces AND some heat energy must be *released* back to the lower T reservoir.

(Kelvin-Planck's Statement)

skip

Internal Combustion Engine
The Otto Cycle

A fuel vapor can be compressed, then detonated to rebound the cylinder, doing useful work.



Intake stroke: Piston moves down, causing a partial vacuum in cylinder; gasoline–air mixture enters through intake valve.

Compression stroke: Intake valve closes; mixture is compressed as piston moves up.

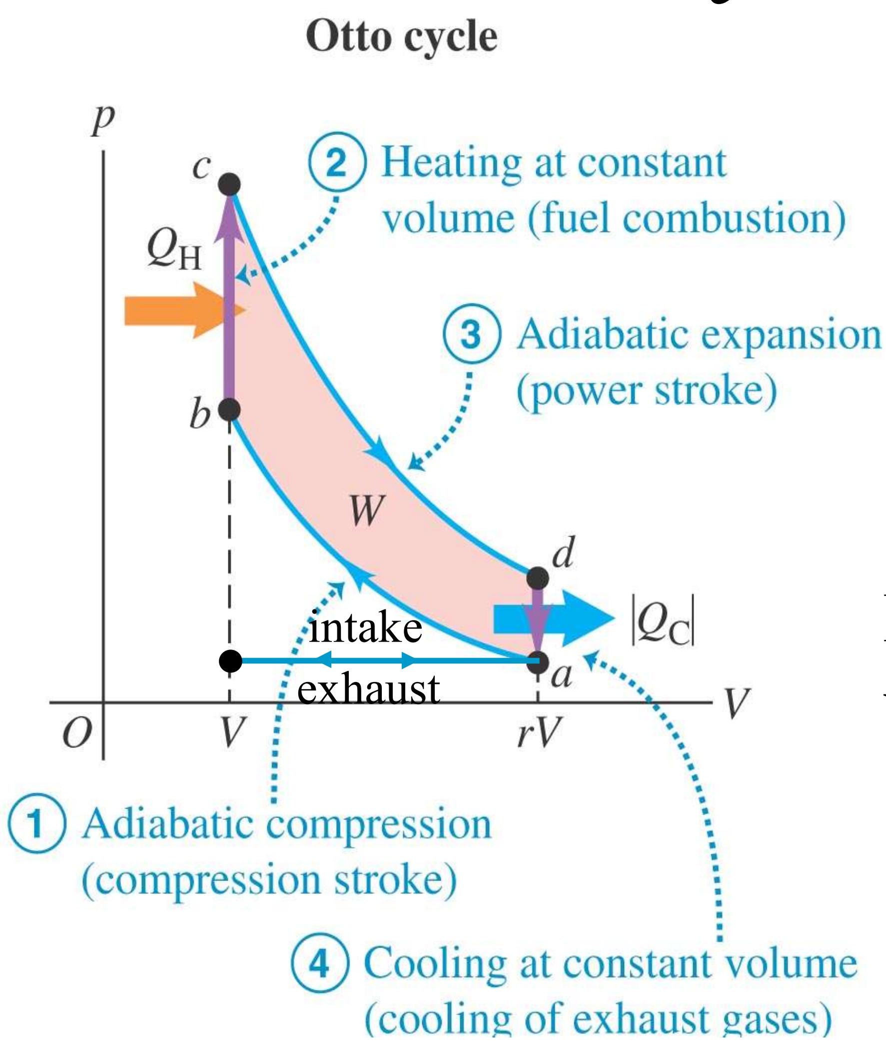
Ignition: Spark plug ignites mixture.

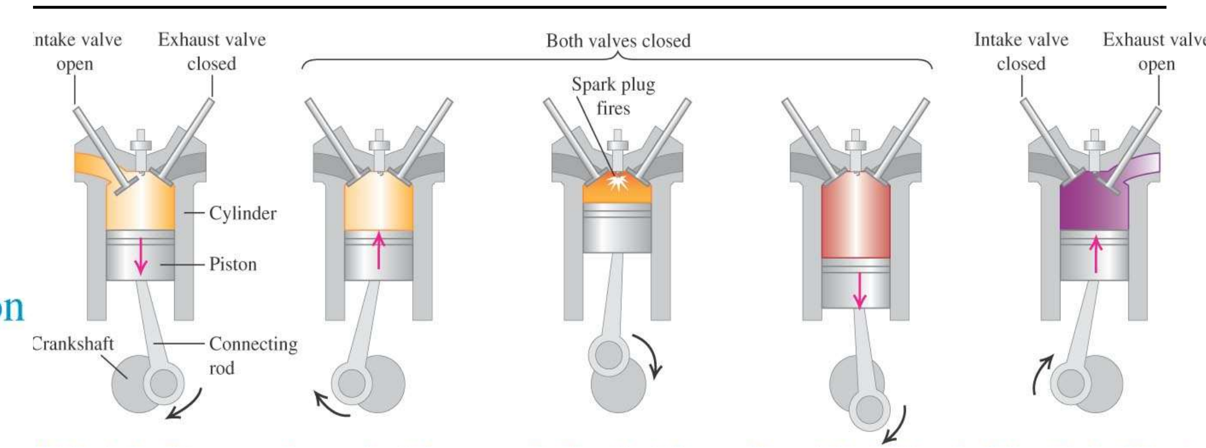
Power stroke: Hot burned mixture expands, pushing piston down.

Exhaust stroke: Exhaust valve opens; piston moves up, expelling exhaust and leaving cylinder ready for next intake stroke.

animation
http://auto.howstuffworks.com/engine1.htm

The Otto Cycle





intake **a → b** **b, c** **c → d** **exhaust**

For the two constant V processes: 2 and 4, we can calculate,

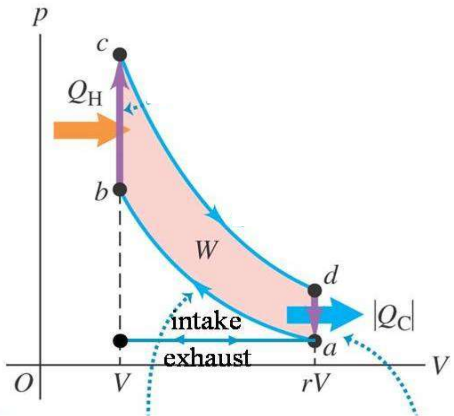
$$Q_H = \Delta U_2 = nC_V(T_c - T_b) > 0$$

$$Q_C = \Delta U_4 = nC_V(T_a - T_d) < 0$$

r is the compression ratio (8 to 13)

The Otto Cycle

Applying the definition of *efficiency*,



$$e = 1 + \frac{Q_C}{Q_H} = 1 + \frac{nC_V\left(T_a - T_d\right)}{nC_V\left(T_c - T_b\right)} = 1 + \frac{T_a - T_d}{T_c - T_b}$$

Now, we can utilize the two adiabatic processes: 1 and 3,

$$T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1} \qquad \text{and} \qquad T_c V_c^{\gamma-1} = T_d V_d^{\gamma-1}$$

$$T_a \left(rV\right)^{\gamma-1} = T_b V^{\gamma-1} \qquad\qquad T_c V^{\gamma-1} = T_d \left(rV\right)^{\gamma-1}$$

$$T_a r^{\gamma-1} = T_b \qquad\qquad T_c = T_d r^{\gamma-1}$$

The Otto Cycle

Substituting T_b and T_c into the efficiency equation, we have,

$$e = 1 + \frac{T_a - T_d}{T_c - T_b} = 1 + \frac{T_a - T_d}{T_d r^{\gamma-1} - T_a r^{\gamma-1}} = 1 - \frac{T_a - T_d}{\left(T_a - T_d\right) r^{\gamma-1}}$$

$$\Rightarrow \quad \boxed{e = 1 - \frac{1}{r^{\gamma-1}}}$$

Using a typical value for the compression ratio $r = 8$ and $\gamma = 1.40$ gives,

$$e = 0.56 \quad \text{(or 56\%)}$$

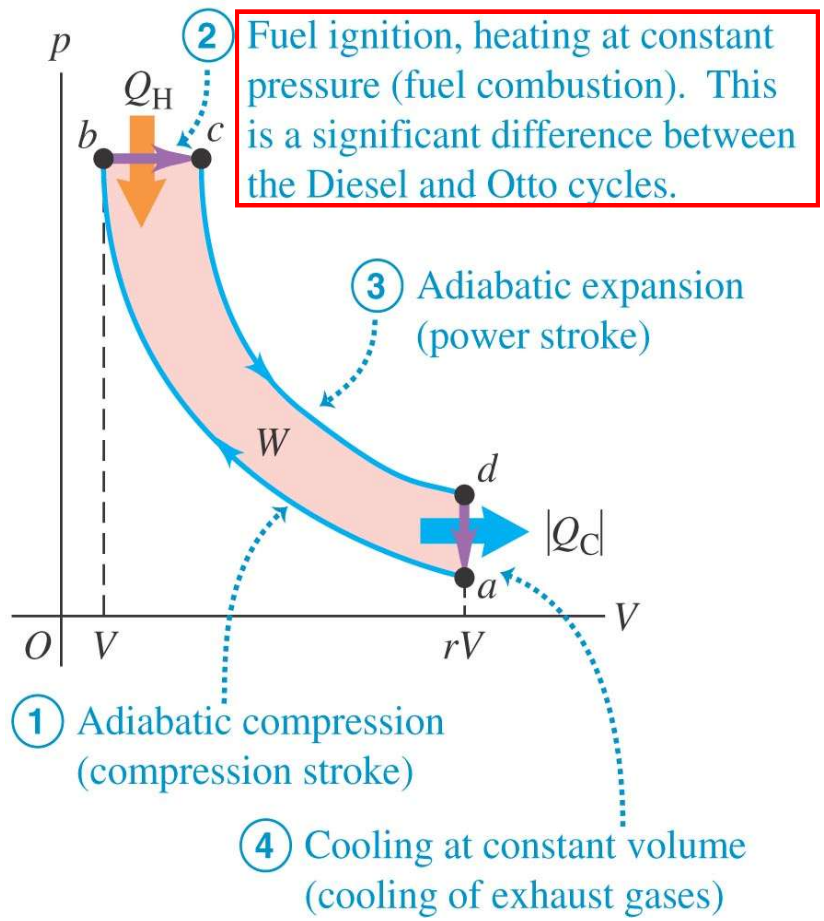
Note: This is a theoretical value. Realistic gasoline engine typically has e ~ 35%.

The Diesel Cycle

Key difference:

- No fuel in cylinder at the beginning of the compression stroke (process 1)
- Fuel is injected only moments before ignition in the power stroke
- No fuel until the end of the adiabatic compression can avoid pre-ignition
- Compression ratio *r* value can be higher (15 to 20)
- Higher temperature can be reached during the adiabatic compression
- Higher *e* and no need for spark plugs

Diesel cycle



$D_{\text{steam engine}}$

Entropy

Recall from a Carnot Cycle, we have derived the following relationship:

$$\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H} \quad \Longrightarrow \quad \frac{|Q_H|}{T_H} + \frac{-|Q_C|}{T_C} = 0$$

Formally, we can rewrite this as,

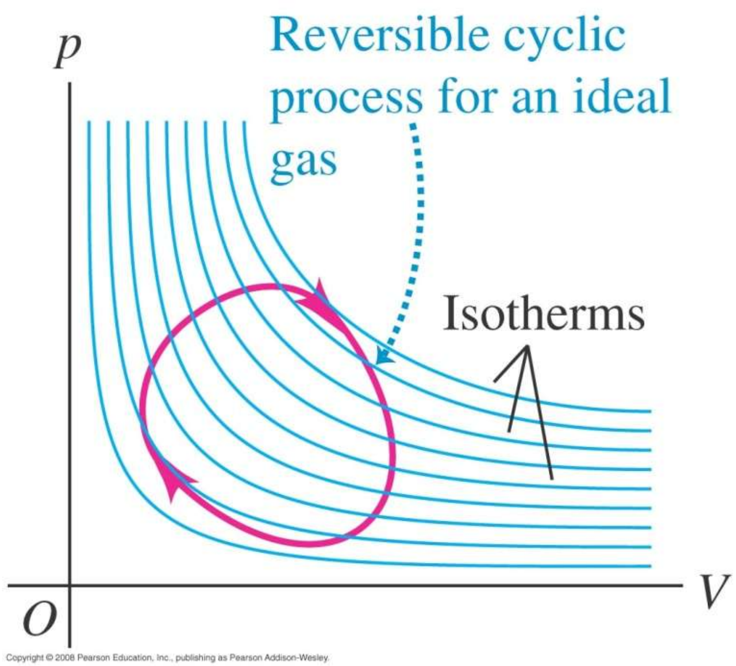
$$\sum_{cycle} \frac{Q}{T} = 0$$

(We have absorbed the explicit sign back into the variable Q.)

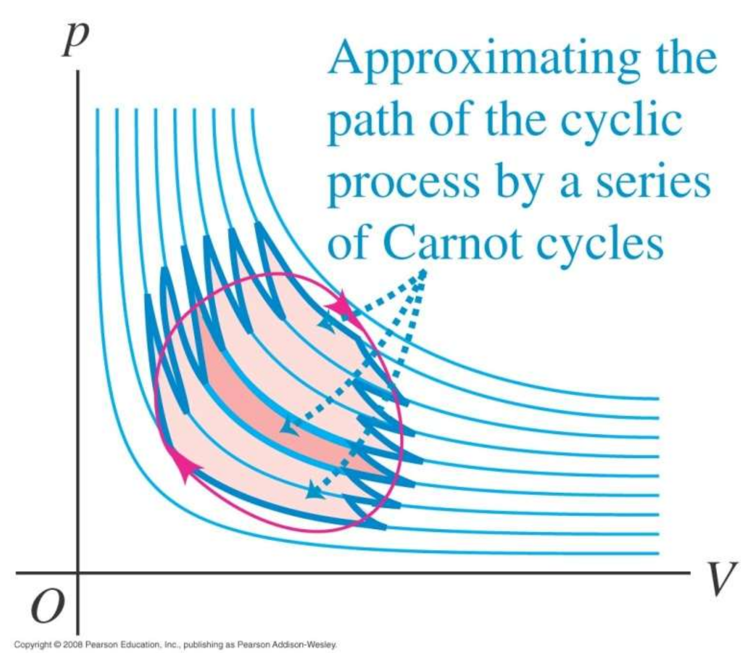
where Q represents the heat absorbed/released along the isotherm at temp T.

Entropy

Now consider any reversible cycles…



Any reversible cycles can be approximated as a series of Carnot cycles !



Entropy

This suggests that the following generalization to be true for any reversible cycles,

$$\oint_{cycle} \frac{dQ_r}{T} = 0$$

where,

dQ_r is the infinitesimal heat absorbed/released by the system at an infinitesimal *reversible* step at temp T.

$\oint_{cycle}$ denotes the integration evaluated over one complete cycle.

Entropy

We have seen this property previously,

$$\oint_{cycle} dU = 0$$

Changes in the internal energy U over a closed cycle is zero!

This is a consequence of the fact that U is a **state variable** and dU for any processes depends on the initial and final states only.

Thus the result $\oint_{cycle} \frac{dQ_r}{T} = 0$ indicates that there is another **state variable** S such that,

$$dS \equiv \frac{dQ}{T} \quad \text{and} \quad \oint_{cycle} dS = 0$$

This new **state variable** S is called the **entropy** of the system.