Special Processes

6. Adiabatic (no heat exchange, $Q = 0$):

$$\Delta U = Q - W \xrightarrow{\quad Q=0 \quad} \Delta U = -W$$

Initial volume

Maximum compression

$V_2 = \frac{1}{15} V_1$

V_1

Note: The compression stroke in an internal combustion engine is quick and it can be well approximated by an *adiabatic* process.
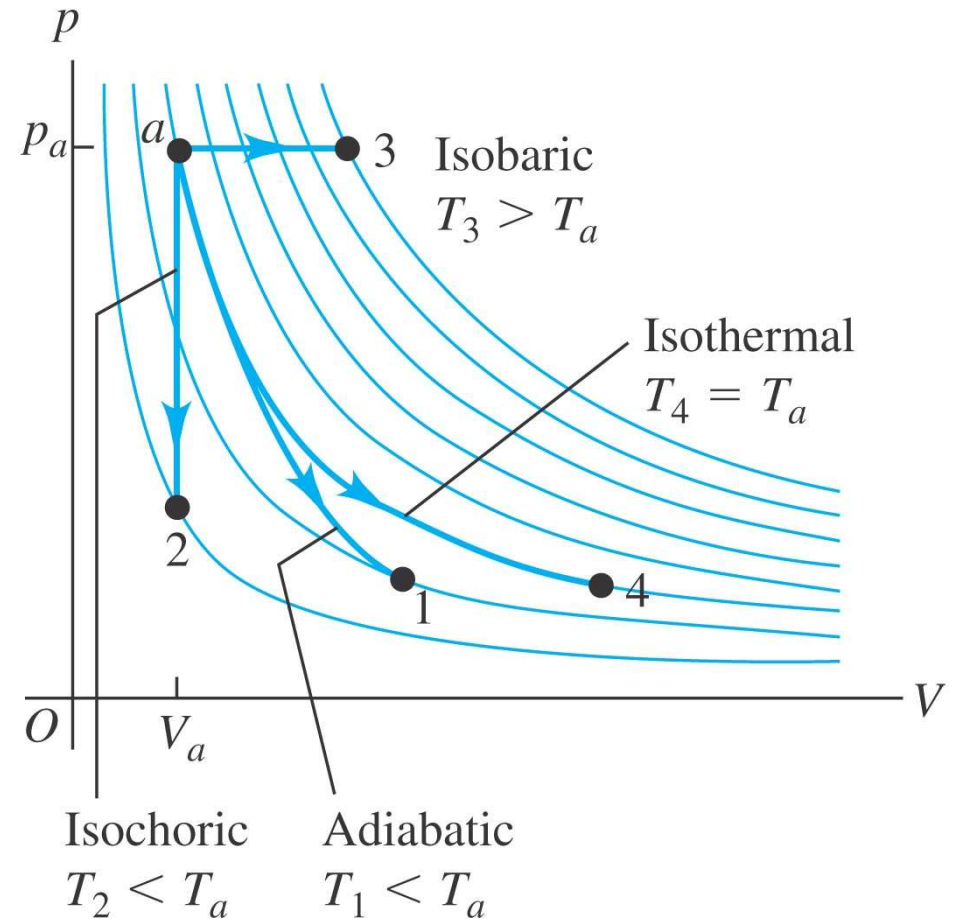
(will come back to this example later)

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$D_{\text{fire piston}}$
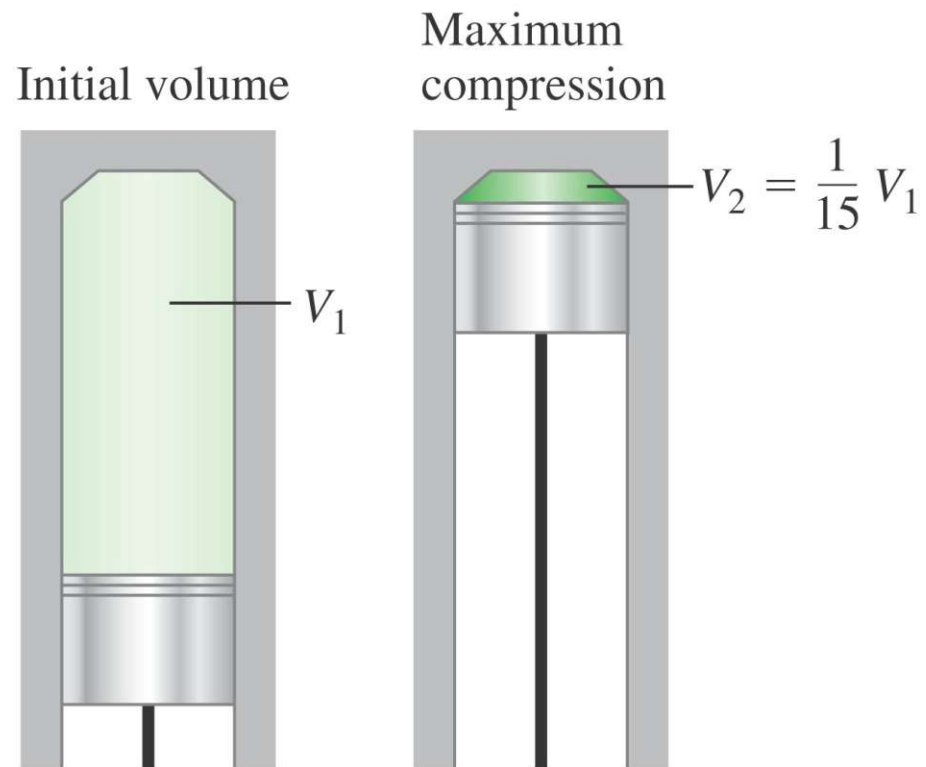
Summary

Four Reversible (Quasi-static) processes:

1. Adiabatic (Q=0)
2. Isochroic (ΔV=0)
3. Isobaric (ΔP=0)
4. Isothermal (ΔT=0)

p

p_a — a 3   Isobaric $T_3 > T_a$

Isothermal $T_4 = T_a$

2

1   4

O V_a V

Isochoric $T_2 < T_a$ Adiabatic $T_1 < T_a$
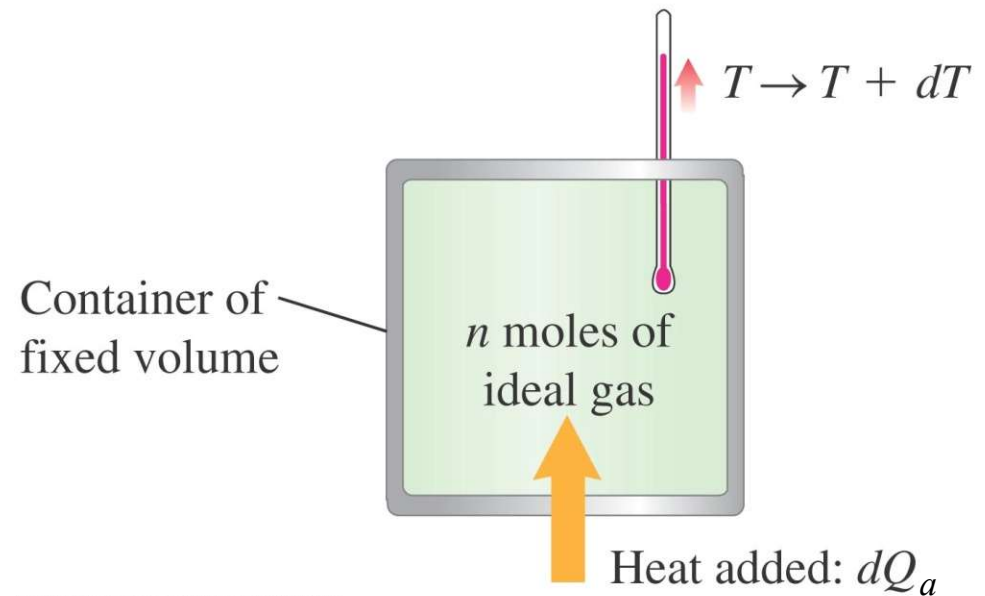
1$^{\text{st}}$ Law for infinitesimal changes: $dU = dQ - dW = dQ - PdV$

C_p and C_v for an Ideal Gas
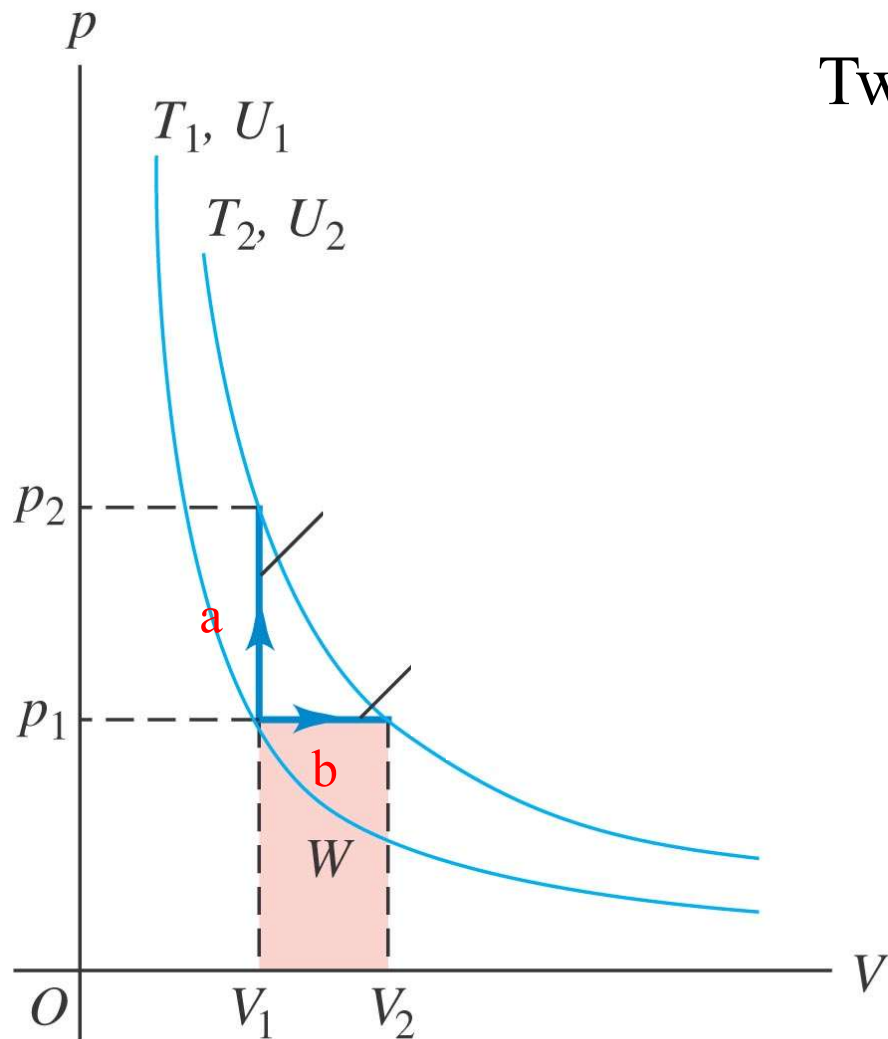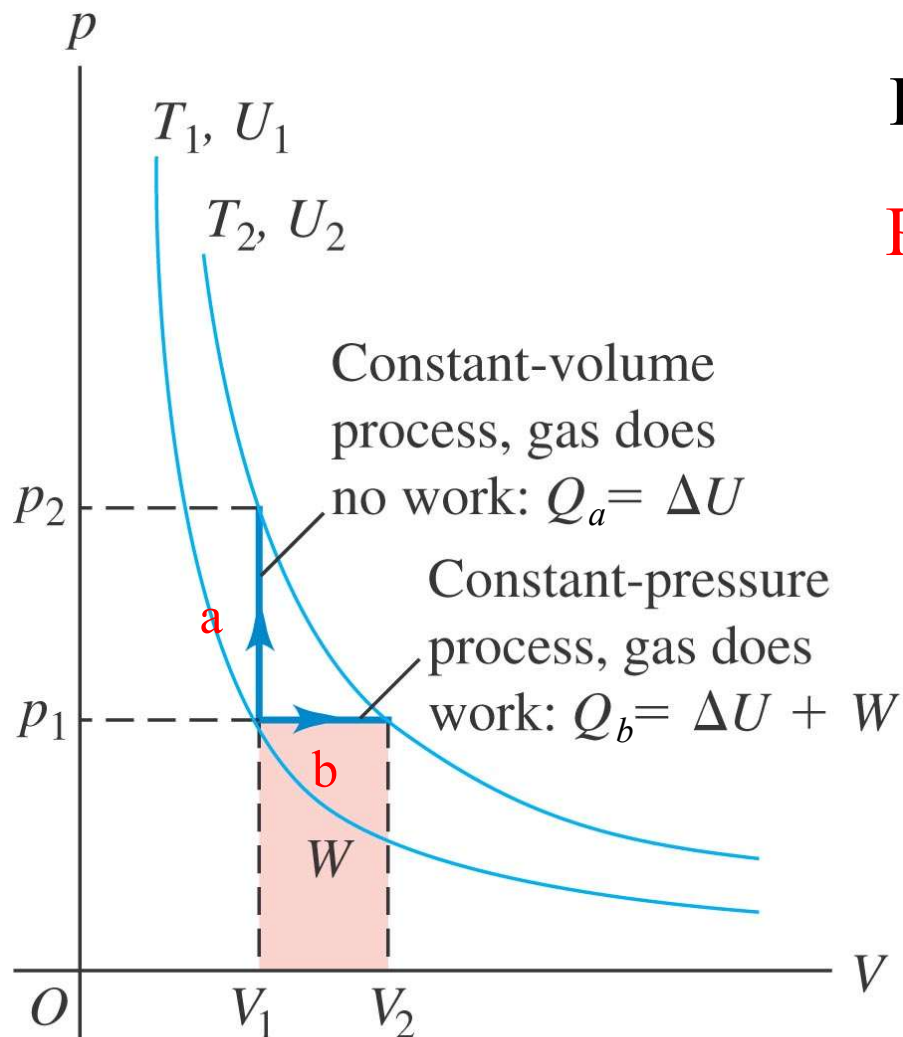

Two different ways to change $dT=T_2 - T_1$:

Process a: Constant V

$T \rightarrow T + dT$

Container of fixed volume

n moles of ideal gas

Heat added: dQ_a

$$dQ_a = nC_v dT$$

C_p and C_v for an Ideal Gas

p

T_1, U_1

T_2, U_2

Constant-volume process, gas does no work: $Q_a = \Delta U$

Constant-pressure process, gas does work: $Q_b = \Delta U + W$

p_2

a

p_1

b

W

O V_1 V_2 V

For the same dT :

Process b: Constant P

$T \to T + dT$

Piston motion

Container with movable piston that applies constant pressure

n moles of ideal gas

Heat added: dQ_b
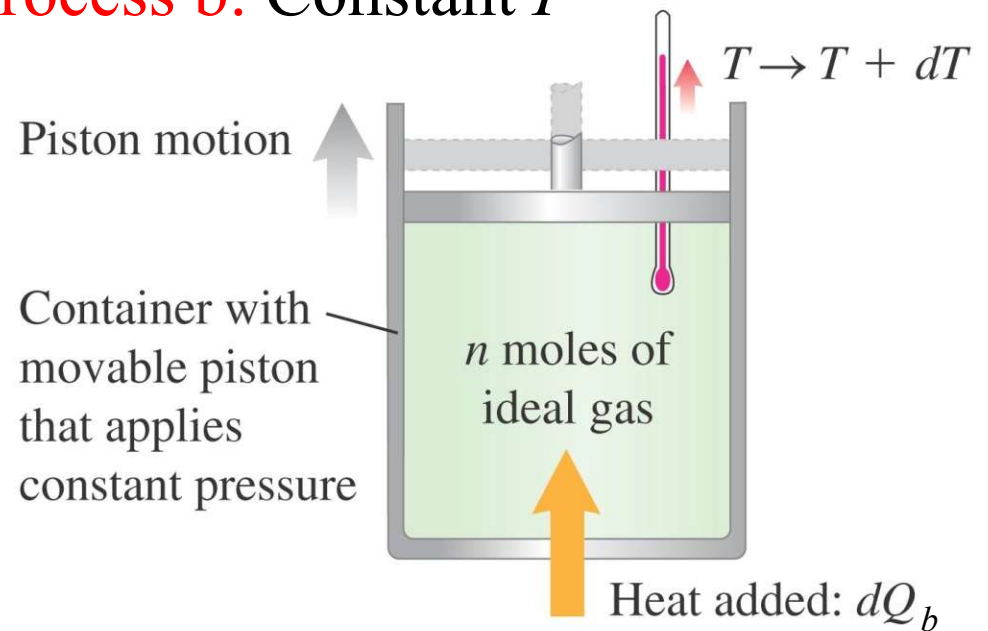
$$dQ_b = nC_P dT$$

Which is bigger: $Q_a < or > Q_b$, $C_v < or > C_P$?

C_p and C_v for an Ideal Gas

First let consider the constant *volume* process (a):

➢ No work done by/on gas → $W = 0$
➢ Then, 1st law gives,

$$dU_a = dQ_a = nC_v dT$$

Now, for the constant *pressure* process (b) with the *same dT*:

➢ We have $dW = PdV$ and $dQ_b = nC_p dT$ (by definition)
➢ Substitute them into the 1st law again gives,

$$dU_b = dQ_b - dW = nC_p dT - PdV$$

C_p and C_v for an Ideal Gas

First let consider the constant *volume* process (a):

➤ No work done by/on gas → $W = 0$
➤ Then, 1st law gives,

$$dU_a = dQ_a = nC_v dT$$

Now, for the constant *pressure* process (b) with the *same dT*:

➤ We have $dW = PdV$ and $dQ_b = nC_p dT$ (by definition)
➤ Substitute them into the 1st law again gives,

$$dU_b = dQ_b - dW = nC_p dT - PdV$$

C_p and C_v for an Ideal Gas

Taking the differential on both sides of the Ideal Gas Law, we have,

$$d(PV) = nRdT \qquad nR \text{ is constant with } n \text{ fixed}$$
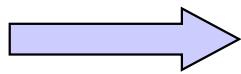
With P constant, this gives,

$$d(PV) = PdV = nRdT$$

Substitute this into the last Eq on the previous page, we have:

$$dU_b = nC_p dT - nRdT$$

$$= n(C_p - R)dT$$

C_p and C_v for an Ideal Gas

Important point: Since U for an ideal gas is a function of T only and process a and process b have the same $dT = T_2 - T_1$,

$$dU_a = dU_b$$

$$\Longrightarrow \quad nC_v dT = n(C_p - R)dT$$

Finally, this gives, $\boxed{C_p = C_v + R}$ (True for all Ideal Gases)

(Note: This fact is very useful. Basically, one can use $dU = nC_v dT$ to calculate the internal energy change in an ideal gas for any given dT whether V is constant or not).

The Ratio of Heat Capacities

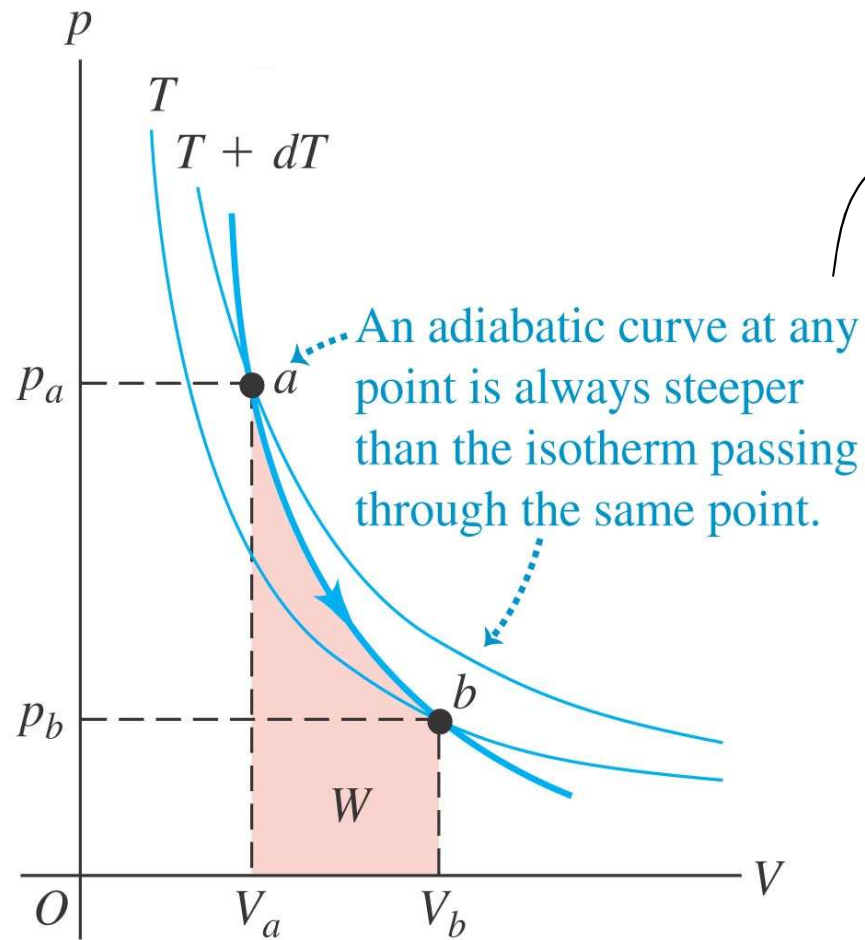☐ A useful ratio γ can be defined:

$$\gamma = \frac{C_p}{C_v}$$

Recall that for a *monoatomic* Ideal Gas, $\quad C_v = \frac{3}{2}R$

so that, $\quad C_p = C_v + R = \frac{5}{2}R \quad$ *and* $\quad \gamma = \frac{5}{3} = 1.67$

For a *diatomic* Ideal Gas, $\quad C_v = \frac{5}{2}R$

and, $\quad C_p = C_v + R = \frac{7}{2}R \quad$ *and* $\quad \gamma = \frac{7}{5} = 1.40$

Adiabatic Processes for an Ideal Gas

p

T

$T + dT$

..... An adiabatic curve at any point is always steeper than the isotherm passing through the same point.

p_a ⟶ a

p_b ⟶ b

W

O | V_a V_b ⟶ V

adiabatic expansion → T drops

Isothermal → $P \sim \dfrac{1}{V}$
$\Delta T = 0$

Adiabatic → $P \sim \dfrac{1}{V^{\gamma}}$
$Q = 0$

$(\gamma > 1)$

Start from definition, $dQ=0$ for any adiabatic process,

Then, from 1st Law, we have,

$$dU = -dW = -PdV$$

Adiabatic Processes for an Ideal Gas

Now, we use the "trick" that for an ideal gas, dU is the *same* for all processes with the *same* $dT = T_f - T_i$. As stated previously, we can calculate dU using,

$$dU = nC_v dT$$

Putting this into the 1st law $dU = -dW$ gives,

$$nC_v dT = -PdV = -\frac{nRT}{V}dV$$

(In the last step, we used Ideal Gas Law: $PV=nRT$)

Adiabatic Processes for an Ideal Gas

Rearrange terms, we have,

$$\frac{dT}{T} + \frac{R}{C_v}\frac{dV}{V} = 0$$

Using the relations for the molar specific heats,

$$\frac{R}{C_v} = \frac{C_p - C_v}{C_v} = \frac{C_p}{C_v} - 1 = \gamma - 1$$

Then, we have,
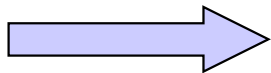
$$\frac{dT}{T} + (\gamma - 1)\frac{dV}{V} = 0$$

Adiabatic Processes for an Ideal Gas

Integrating this equation, we have,

$$\int \frac{dT}{T} + (\gamma - 1) \int \frac{dV}{V} = \text{constant}$$
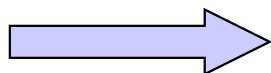
$$\ln T + (\gamma - 1) \ln V = \text{constant}$$

$$\ln \left(TV^{\gamma - 1} \right) = \text{constant}$$

$$TV^{\gamma - 1} = \text{constant}$$

Note: T has to be in K.

Using the Ideal Gas Law again, we can replace T with $\dfrac{PV}{nR}$,

$$\frac{PV}{nR} V^{\gamma - 1} = \text{constant} \quad \rightarrow \quad PV^{\gamma} = \text{constant}$$

(alternate form)

Work in an Adiabatic Process (Ideal Gas)

We know that, $dW = -dU \; (dQ = 0)$

Using the same "trick" on dU, we can calculate the work done in an adiabatic process if we know the changes in temperature.

$$dW = -dU = -nC_v dT$$

$$W = -nC_v(T_2 - T_1)$$

or $$W = -\frac{C_v}{R}(P_2 V_2 - P_1 V_1)$$

Examples

- Fire Piston (demo)

- Example 19.68 (Comparison of processes)

Fire piston calculations

Example 19.68 calculations

Given initial state $P_1, V_1 \longrightarrow$ final V_2

3 diff ways:
    a) Isothermal
    b) Adiabatic
    c) Isobaric
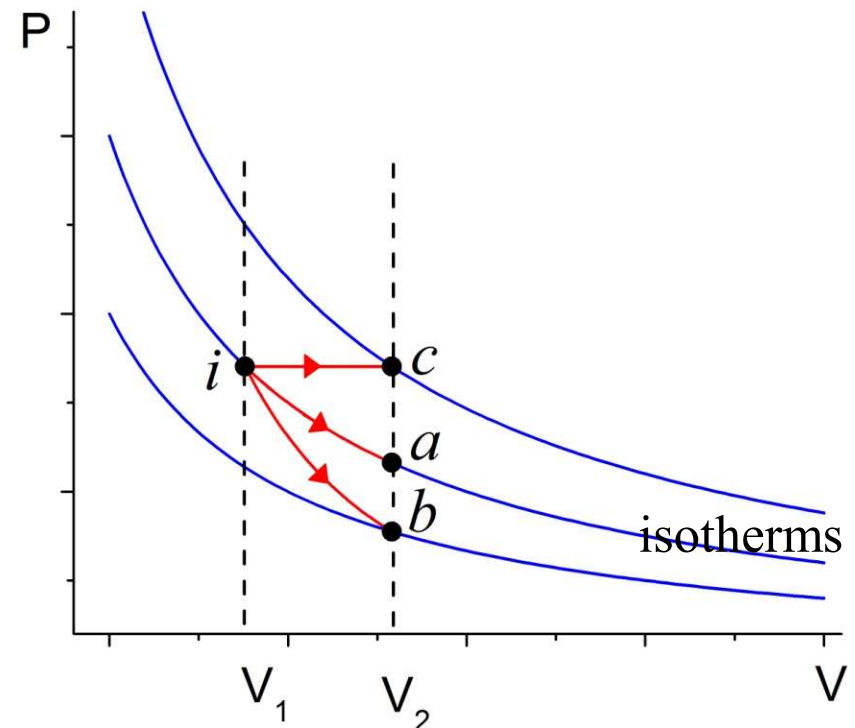
Adiabatic Expansion (reversible & nonreversible)

- **_Reversible_ adiabatic expansion** (quasi-static) :
  - Expanding gas push piston up → work is done *by* gas → $W > 0$ → $\Delta U < 0$ (energy flows out of gas)
  - For an Ideal Gas, U is a function of T only,
  - So, $\Delta U < 0$ also implies $\Delta T < 0$ (temperature drops!)

- **Adiabatic _free_ expansion** (non-quasi-static /nonreversible):
  - Gas expands into vacuum → no work done $W=0$
  - Adiabatic → $Q = 0$
  - 1st law gives $\Delta U = 0$
  - U remains unchanged and T is a constant!

insulation

State 1   State 2

5.0 L

2.0 L

State 1   Insulation   State 2

Vacuum

5.0 L

2.0 L

Breakable partition   Gas at 300 K