

Qualifying Exam, Electrodynamics, Jan. 2013

1. [20 pts] In vacuum, an electromagnetic field is described by scalar potential $\Phi(\mathbf{x}, t) = 0$ and vector potential

$$\mathbf{A}(\mathbf{x}, t) = -\frac{qt}{4\pi\epsilon_0} \frac{\mathbf{x}}{|\mathbf{x}|^3}$$

where t is time, \mathbf{x} is the spatial position, and q is a constant. Find the electric field $\mathbf{E}(\mathbf{x}, t)$, the magnetic induction $\mathbf{B}(\mathbf{x}, t)$, the charge density $\rho(\mathbf{x}, t)$, and current density $\mathbf{J}(\mathbf{x}, t)$.

2. [30 pts] An infinitely long hollow pipe has a square cross section of area a^2 . Its four sides are made of ideal conductors with very thin insulating strips at the corners. Three sides of the pipe are grounded, and the remaining side is held at constant potential V . You can leave your answer as sum of a series.

- Find the potential Φ anywhere inside the pipe.
- Find the charge density σ at the side at potential V , and the capacitance (between the side at potential V and the other three sides) per unit length.
- Another side, adjacent to the side at potential V , is now also brought to the same potential. Find the potential at the center of the pipe.

3. [20 pts] Consider a dielectric liquid of permittivity ϵ that extends to infinity. There exists a uniform electric field $E_0\hat{z}$ inside the liquid. Now a spherical bubble of radius a is formed inside the liquid. Assume air inside the bubble has permittivity ϵ_0 . Find the potential and electric field inside the bubble.

4. [30 pts] Total charge Q is uniformly distributed on a very thin, non-conducting disk of radius R . The disk is centered at the origin and lies on the xy plane. It spins around the z axis with angular velocity ω .

- (1) Find the magnetic induction $\mathbf{B}(z)$ along the z -axis. You may need the integral

$$\int \frac{x^3}{(x^2 + a^2)^{3/2}} dx = \frac{x^2 + 2a^2}{(x^2 + a^2)^{1/2}} + C$$

where C is a constant.

- Compute the magnetic dipole moment \mathbf{m} of the system.
- Find the vector potential $\mathbf{A}(r, \theta, \phi)$ for $r \gg R$.