Qualifying exam - January 2012

Statistical Mechanics

You can use one textbook. Please write legibly and show all steps of your derivations.

Problem 1 [20 points]
Consider a substance for which

$$E = AVT^n, \tag{1}$$

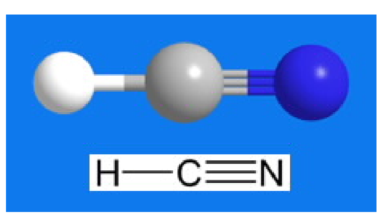
where E is energy, V is volume, T is temperature and $A > 0$ and $n > 1$ are constants.

1. What is the entropy of this substance? [5 points]
2. Calculate the pressure p of this substance as a function of temperature. [5 points]
3. Show that pV/E is a constant and determine this constant. [5 points]
4. Is this substance thermodynamically stable if $n < 1$? [5 points]

Problem 2 [35 points]
Consider a system of N localized non-interacting identical molecules, each having an electric dipole moment $\mathbf{p}$. The system is placed in an electric field $\mathbf{E}$ at a temperature T. Assuming that the system is classical and disregarding the kinetic energy of the molecules, calculate the following properties:

1. Partition function of the system. [7 points]
2. Average potential energy $\bar{\varepsilon}$ per molecule. [7 points]
3. Average dipole moment $\bar{p}$ per molecule. [7 points]
4. The dielectric susceptibility $(\partial\bar{p}/\partial E)_T$. [7 points]
5. The specific heat $(\partial\bar{\varepsilon}/\partial T)_E$. [7 points]

Problem 3 [20 points]
Calculate the internal energy (in J/mole) and specific heat at a constant volume (in J/mole/K) of hydrogen cyanide HCN at the temperature of 800 K. Consider HCN as an ideal gas and treat the molecular rotations and vibrations in the classical limit. The HCN molecule has a linear structure H$-$C$\equiv$N (see figure below). The gas constant is $R = 8.314$ J/mole/K.



Problem 4 [25 points]

Consider a cavity containing black-body radiation at a temperature T_1. Suppose the volume of the cavity increases in an equilibrium adiabatic process from an initial value V_1 to a final value $V_2 = 5V_1$.

1. What is the final temperature T_2 in the cavity? [5 points]
2. If the initial radiation pressure was p_1, what is the final pressure p_2? [5 points]
3. If the cavity initially contained a total of N_1 photons, what is the final number N_2 of photons in the cavity? Explain the physical meaning of this result. [15 points]