Quantum Mechanics Qualifying Exam

August 2011

1. For a complex number z, define the so-called coherent state

$$|z\rangle = e^{-|z|^2/2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |n\rangle$$

where $|n\rangle$ is the n-th eigenstate of the one dimensional harmonic oscillator. Show that $|z\rangle$ is an eigenstate of the annihilation operator a which satisfies $a|n\rangle = \sqrt{n}|n-1\rangle$. Find the corresponding eigenvalue of a. [10 pts]

2. Let ψ_{nlm} be the energy eigenstate wave functions of the hydrogen atom, with n the principle quantum number and l, m labeling the angular momentum eigenstates. Neglect the spin degree of freedom and assume the atom is isolated. At $t = 0$, the wave function of the atom is

$$\psi(t=0) = \frac{1}{\sqrt{10}}(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1}).$$

a). What is the expectation value of the energy of the atom in electron volts? [10 pts]
b). Find $\psi(t)$, the wave function of the atom for $t > 0$. [10 pts]
c). What is the probability of finding the system with angular momentum $L_z = -\hbar$ at time t? [10 pts]
d). An angular momentum measurement on the atom yields $L_z = -\hbar$, $(l = 1, m = -1)$. Immediately after the measurement, what are the expectation values of angular momentum L_x and L_x^2 in terms of $\hbar$ and $\hbar^2$ respectively? [15 pts]

3. Consider an electron moving on a (smooth) ring of radius R in the xy plane. In cylindrical coordinates, its position is uniquely specified by the azimuthal angle ϕ. There is a homogeneous magnetic field B in the z direction. The Hamiltonian is given by (in the symmetric gauge and CGS units)

$$H = \frac{1}{2m_e}\left(-i\frac{\hbar}{R}\frac{d}{d\phi} - \frac{eBR}{2c}\right)^2$$

where e is the electron charge, m_e is the electron mass, and c is the speed of light. Show that angular momentum L_z is conserved. Find the energy eigenvalues and eigenstates. [15 pts]

4. A particle of mass m is constrained to move along the x axis and is subject to a potential modeled by Dirac's delta function $(V_0 > 0)$

$$V(x) = -V_0\delta(x)$$

a). Show that there exists only one bound state. Find the bound state energy. [15 pts]
b). Find the value x_0 such that the probability of finding the particle in the bound state within $-x_0 < x < x_0$ is exactly 1/2. [15 pts]