Classical Electrodynamics Qualifying Exam: January, 2012

1. [10] Show how to obtain the differential form of Gauss's Law from the integral form using the Divergence Theorem.

2. [20] An infinite, hollow, rectangular, conducting pipe runs along the z-axis and extends from $x = 0$ to $x = a$ and $y = 0$ to $y = b$. All the faces are grounded except the face at $x = a$, which is held at constant potential V and insulated from the other faces. Find the potential $\Phi(x, y, z)$ inside the pipe.

3. [20] Two spherical shells with radii a and b ($a < b$) are centered on the origin. The inner shell is held at zero potential, while the outer shell is observed to be at potential $\Phi(r = b, \theta) = V_0\,(3\cos^2\theta - 1)$ where V_0 is a constant. Find the potential between the 2 shells (i.e., for $a \leq r \leq b$).

4. [30] a) [10] A circular loop of radius R lies in the x-y plane, is centered on the origin, and carries a current I. The current flows counterclockwise when viewed from above the x-y plane ($z > 0$). Find the magnetic induction $\vec{B}$ on the axis of the loop.
b) [20] A sphere of radius a is centered on the origin and rotates with angular velocity $\omega\hat{z}$. The sphere carries electric charge with surface-charge density $\sigma(\theta) = \sigma_0 \sin^2\theta$, where θ is the angle with respect to $\hat{z}$. Find the magnetic induction $\vec{B}$ along the z-axis in the limit $z \gg a$. Hint: Use the result from part (a).