Classical Electrodynamics Qualifying Exam: January 19, 2011

1. [30] A line charge with uniform charge density lies along the z-axis between $z = 0$ and $z = b$ and has total charge Q.
a) [10] Find an exact expression for the electrostatic potential $\Phi(r, z)$ in cylindrical coordinates.
b) [5] Show that your result in part (a) has the correct asymptotic form as $\sqrt{r^2 + z^2}/b \to \infty$.
c) [10] Find the potential $\Phi(r, \theta)$ in spherical coordinates (r, θ, ϕ) as a series involving Legendre polynomials and powers of r, for $r > b$.
d) [5] Show that your results in parts (a) and (c) are equivalent for observation points on the z-axis with $z > b$. Recall the Taylor series

$$\ln(1 - x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n + 1} \quad . \tag{1}$$

2. [25] In this problem, you will find the electrostatic potential $\Phi(r, z)$ inside a circular cylinder with radius a and height L, adopting cylindrical coordinates (r, ϕ, z). The bottom of the cylinder is at $z = 0$ and the cylinder's axis is the z-axis. The potential is zero on the surface at $z = 0$ and on the curved surface and is a constant V_0 on the surface at $z = L$.
a) [15] Show that the potential inside the cylinder can be expressed in the form

$$\Phi(r, z) = \sum_{n=1}^{\infty} A_n \sinh\left(\frac{x_{0n} z}{a}\right) J_0\left(\frac{x_{0n} r}{a}\right) \tag{2}$$

where x_{0n} is the n^{th} zero of the Bessel function $J_0(x)$.
b) [10] Find the coefficients A_n. Recall that

$$\frac{d}{du}\left[u^{\nu}\, J_{\nu}(u)\right] = u^{\nu}\, J_{\nu-1}(u) \tag{3}$$

3. [15] A sphere has radius a, is centered on the origin, and is made up of a uniform, linear dielectric material with dielectric constant ϵ/ϵ_0. A point charge q is located at the origin. Find the surface and volume bound charge densities.

4. [30] An infinite, conducting plane at $z = 0$ carries a uniform current per unit transverse length, $K\hat{y}$.
a) [10] Find the magnetic induction $\vec{B}$ everywhere outside of the plane.
b) [10] A second infinite, conducting plane at $z = -d$ carries a uniform current per unit transverse length, $-K\hat{y}$. Use the Lorentz force law to find the pressure that the first plane exerts on the second. Is it attractive or repulsive?
c) [10] Repeat part (b), this time using the Maxwell stress tensor.